Assignment 3

1. A travelling salesman has carpets in his van. He goes from door to door trying to sell them. Each time he sells one he gets excited and is able to be more convincing and each time he is rejected and cannot close a deal he gets less excited and is less convincing when dealing with the next customer. This salesman was blessed with 10 levels of excitement and each interaction either lifts or decreases his excitement by one level. The probabilities for the salesman to sell a carpet depending on how he feels are {0, 0.1, 0.2, 0.3, …0.9}. Write an algorithm to simulate this Markov chain. In b) - f) run simulations for all initial levels of excitement, **except the first level**, because business is downright non-existent when one feels this low for starters. Use one bin per timestep. **Run simulations over the time period (sales events) of $T = 1000$. For statistics, run 10 000 simulations**, which in the present story means that the salesman makes 10 000 trips.
   a) (w=1) **Write down the Markov matrix.**
   b) (w=4) **For every simulation of each initial level of excitement, compute the total number of sold carpets by the end of the simulation (cumulative number of sales). For each initial level of excitement plot as a normalised histogram (density=True) the probability mass density (PMD) of the cumulative number of sales** (9 histograms in total)**.** To make this clear, such a histogram has the cumulative number of sales on the x-axis and the (normalised) frequency with which each number of cumulative sales is realised in **10 000 trials** on the y-axis. Use a **logarithmic scale on the y-axis** to see the PMDs and possible trends as clearly as possible.
   c) (w=2) For each different initial level, compute the average (over the 10 000 trials) number of carpets the salesman can sale within the given time T. **Plot these average numbers of carpets as a function of the salesman's initial level of excitement** (1 plot)**.**
   d) **(i)** (w=3) **Compute and plot the PMFs for time (the number of steps) required for the salesman to sell 5 carpets. Exclude the runs that ended up without selling at least 5 carpets**. At sufficiently high initial levels of excitement the PMFs should start showing a definite functional form. **(ii)** (w=1) **Choose appropriate scale (axes) for plotting so that you can name this functional form – and name it. (Use this scale for all plots in d).)**
   e) (w=2) Compute the **average** times (for the different initial levels) it takes for the salesman to sell 5 carpets. Plot these times as a function of the salesman's initial level of excitement.
   f) (w=1) In point d) and e), excluding the runs that didn't sell enough carpets is underestimating the number of steps needed to sell 5 carpets. **Plot the probability of selling less than 5 carpets in T timesteps as a function of the initial level of excitement.**
   g) (w=1) In d) you obtained some definite functional form(s) for certain level(s) of initial excitement. **Why does the current process produce these functional forms? (**If you did not get these forms, you can still argue for them based on what you know.)

2. Homogeneous (stationary) and inhomogeneous (nonstationary) Poisson processes:
   Let's take the Poisson process to describe radioactive decay. The number of nuclei is initially $N_0 = 10\ 000$. The decay (fission) rate for one nucleus is $\lambda_c = 0.2$ per second.

   a) Consider an ensemble of $N_0 = 10\ 000$ nuclei, where the number of undecayed nuclei is kept constant; in other words, every time a nucleus decays it is replaced by an undecayed nucleus. The differential equation for the process is $\frac{dN}{dt} = -\lambda_c N_0$. Accordingly, the constant decay rate is $\lambda = \lambda_c N_0$. Applying inverse distribution method to the inter-event time distribution for this homogeneous Poisson process, simulate the decay process.
      (i) (w=2) Implement the algorithm. Indicate clearly the different parts and functions.
      (ii) (w=2) Compute mean values by averaging over 1000 simulations of the following: The half time, i.e., the time it takes for $N_0/2$ nuclei to decay, and the variance of this half time.

   b) Next, consider an ensemble of $N_0 = 10\ 000$ nuclei, where decayed nuclei are not replaced. This affects the inter-event time distribution, and the process becomes inhomogeneous as the decay rate will depend on time. The differential equation describing this process reads as $\frac{dN}{dt} = -\lambda_c N(t)$, where the number of undecayed nuclei decreases exponentially, $N(t) = N_0 e^{-\lambda_c t}$. Applying inverse distribution method to the pertinent time-dependent - as we have $\lambda(t)$ - inter-event time distribution for this inhomogeneous Poisson process, simulate the decay process.
      (i) (w=2) Implement the algorithm. Indicate clearly the different parts and functions.
      (ii) (w=2) Compute mean values by averaging over 1000 simulations of the following: The half time, i.e., the time it takes for $N_0/2$ nuclei to decay, and the variance of this half time.

   Note. Above, $N = \langle N \rangle$, i.e. an ensemble average. Likewise, $N(t) = \langle N(t) \rangle$, i.e. an ensemble average at time t.

   Note! Use the definitions of the lecture notes. There exists an alternative definition for the parameters in the exponential distribution; do not use that.