Advanced Microeconomics 1
Helsinki GSE
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Problem Set 1, 2024

1. A decision maker makes choices in a domain consisting of five alternatives: $X = \{a, b, c, d, e\}$. A choice function assigns to any non-empty subset A of X a choice $c(A) \in A$.
   (a) How many different choice functions could the decision maker have?
   (b) How many different coherent choice functions satisfying the Independence of Irrelevant Alternatives can she have?
2. Determine for the following binary relations if they are *reflexive*, *symmetric* or *transitive*:
   (a) xRy on natural numbers if x divides y.
   (b) xRy on natural numbers if $x + y$ is even.
   (c) xRy on natural numbers if xy is even.
3. Show that if $\succ$ is asymmetric and negatively transitive, then $\succeq$ derived from $\succ$ by setting
$$x \succeq y \iff \neg(y \succ x),$$
is a rational preference relation.
4. Start with a strict preference relation $\succ$ that is asymmetric and transitive, (but not negatively transitive). Define weak preference $\succeq$ by
$$x \succeq y \iff \neg(y \succ x),$$
and indifference $\sim$ by
$$x \sim y \iff \neg(x \succ y) \text{ and } \neg(y \succ x).$$
Give an example of such a strict preference where the derived $\succeq$ is not a rational preference relation.
5. Two decision makers $i \in \{1, 2\}$ have complete and transitive preferences $\succeq_i$ on X. They agree to for a common preference $\succeq$ satisfying for all $x, y \in X$,
$$x \succeq y \iff x \succeq_i y \text{ for some } i \in \{1, 2\}.$$
   (a) Is $\succeq$ complete?
   (b) Is it transitive?
6. Explain if the following procedures lead to a coherent choice rule (i.e. a choice rule satisfying Sen's α and Sen's β:
   (a) The decision maker chooses the second cheapest wine from any menu with at least two wines on it and the only choice if faced with a single alternative.

(b) Two committee members $i \in \{1, 2\}$ with a strict ordering on a finite set rank alternatives from best (smallest score) to worst (highest score) and choose the alternative with the lowest sum of scores.

(c) Continue with the assumptions of the previous part except that now committee member $i = 2$ gets to make the choice. She maximizes the suffering of member $i = 1$, i.e. $c(A)$ is the worst alternative in $A \subset X$ for $i = 1$.

7. You are an HR manager at a firm in charge of hiring new workers. You conduct job interviews in the following manner. For any set of job applicants A, you invite then for the interview in alphabetical order. You score their interview on a scale from 0 to 100 and the first applicant that gets a score at or above 90 is hired. If none of them gets such a grade, you hire the last applicant in alphabetical order. Is this procedure rational in your opinion? Does it satisfy Independence of Irrelevant Alternatives?

8. (Bonus Question) Read 'A Condition for the Completeness of Partial Preference Relations' by David Schmeidler in *Econometrica* (1971) pages 403-404, and explain the logic of the proof in your own words. Can you find an example of a non-trivial continuous but incomplete preference relation on $\mathbb{R}^2_+$ (i.e. a relation that satisfies ii), iii) and iv) in Schmeidler (1971)) ?

9. (Bonus Question) Suppose that X is a finite set and the decision maker has in mind two one-to-one numerical functions u and v on X. Interpret the following choice procedures in words, and determine whether they lead to choice rules satisfying the Independence of Irrelevant Alternatives.

   (a) Fix a number v^*. For any choice problem $A \subset X$, let a^* be the maximizer of u over A and b^* be the maximizer of v over A. The decision maker chooses a^* if $v(a^*) \geq v^*$ and she chooses b^* otherwise.

   (b) Fix u^*. For any choice problem A, the DM chooses a^* maximizing u if $u(a^*) \geq u^*$, otherwise she chooses b^* maximizing v.

10. (Bonus Question) We say that a collection $\mathcal{P}$ of subsets of X is a nested family of subsets if for any $P, Q \in \mathcal{P}$, either $P \subset Q$ or $Q \subset P$. We say that $x \succ y$ if there is an set $P \in \mathcal{P}$ such that $x \in P$ such that $y \notin P$. Write $x \succeq y$ if $\neg(y \succ x)$.

    (a) Show that $\succeq$ is complete and transitive.

    (b) For any complete and transitive relation on X, show that there exists a family of nested subsets corresponding to the order as above.

    (c) Give an economic interpretation to the nested family of sets in the previous part.