Advanced Microeconomics 1
Helsinki GSE
Juuso Välimäki

Problem Set 2, 2024

1. Show that the following demand observations satisfy WARP, but not SARP (or GARP)

$$\boldsymbol{p}^1 = (1,1,2),\ \boldsymbol{p}^2 = (2,1,1), \boldsymbol{p}^3 = (1,2,1+\epsilon),$$

$$\boldsymbol{x}(\boldsymbol{p}^1,1) = (1,0,0), \boldsymbol{x}(\boldsymbol{p}^2,1) = (0,1,0), \boldsymbol{x}(\boldsymbol{p}^3,1+\epsilon) = (0,0,1).$$

2. Answer briefly the following questions.

   (a) Are all utility functions representing continuous preferences on $\mathbb{R}^n_+$ continuous?

   (b) A decision maker evaluates the available options in a finite set X according to three separate criteria $i \in \{1,2,3\}$. She assigns to each option $x \in X$ numerical rating $u_i(x)$ for each $i \in \{1,2,3\}$. To aggregate over the criteria, she computes an aggregate score $g(u_1(x), u_2(x), u_3(x))$ using a function $g : \mathbb{R}^3 \to \mathbb{R}$ and chooses the alternative with the highest score in any feasible set $A \subset X$. Under what assumptions on g is the decision maker's choice behavior consistent (in the sense of satisfying Sen's α and β)?

   (c) Sketch the budget budget set of a consumer that divides her income w between food and wine when price of food is normalized to 1 and wine (assumed to be of homogenous quality) can be purchased either domestically at price p_d or at a cheaper foreign rate $p_f < p_d$. The catch is that in order to get the cheaper rate for wine, you have to pay a fixed cost of $F > 0$. What can you say about optimal quantities demanded if the consumer has a strictly monotone, strictly convex and continuously differentiable utility over wine and food?

3. Affordable living for low-income households is a goal for many governments. Three schemes may be tried to help them: i) Lump-sum income subsidy of size $I > 0$. ii) Rent subsidy: a fraction r of a household's rent is paid by the government. iii) Voucher: a fixed amount $s > 0$ is paid from the household's rent if rent exceeds R. The consumer has a Cobb-Douglas utility function for housing services x and other consumption y given by:

$$u(x,y) = \frac{1}{5}\ln x + \frac{4}{5}\ln y,$$

and her monthly income is $w = 500$. Assume that the units of consumption and housing are chosen so that the price of each is 1 per unit. Assume also that the low-income population is small enough so that the schemes ' effect on market prices can be ignored (i.e. they stay fixed at unity).

   (a) Without any subsidies, how much will the household spend on housing?

   (b) Suppose that the government wants the household to live in a unit producing 150 worth services. What is the cost to the government for achieving this target under each of the schemes above?

4. Suppose that a non-satiated consumer has a continuously differentiable demand function $\boldsymbol{x}(\boldsymbol{p}, w)$ that satisfies WARP.

   (a) Show that the Slutsky matrix $S(\boldsymbol{p}, w)$ does not have full rank by showing that
   $$\boldsymbol{p}^\top S(\boldsymbol{p}, w) = 0,$$
   and also
   $$S(\boldsymbol{p}, w)\boldsymbol{p} = 0.$$
   (Hint: use homogeneity and Walras' law.)

   (b) In the case with two goods, use the above result to show that the Slutsky matrix is symmetric. Comment.

5. In many instances, a consumer chooses an indivisible item from a set of differentiated alternatives with different prices. For concreteness, let's think of a growing city located on a lake and the choice of housing. Assume that housing units are identical except in their proximity to the lake denoted by $t \in [0, 1]$. All consumers prefer locations closer to the lake to more distant ones. The consumers vary by their available income $w \in [\underline{w}, \overline{w}]$ and they split their income between choosing a location t at price $p(t)$ and other consumption $y \in \mathbb{R}_{\geq 0}$.

   (a) Formulate the problem for choosing the optimal housing for a consumer with income w assuming that her utility is strictly increasing in y and strictly decreasing in t.

   (b) Assume that utility is additively separable and strictly concave in t and y (it can be written as $u(t, y) = v_t(t) + v_y(y)$ for some functions v_t and v_y). Assume also that the price function $p(t)$ is differentiable, convex and decreasing. Write the first-order conditions for the problem and argue that any point satisfying them is an optimum.

   (c) Use the implicit function theorem to derive comparative statics of t in w.

   (d) What if only locations $\{t_1, t_2, t_3\} \in [0, 1]$ are available. Can you use calculus for solving the problem? Can you determine the comparative statics of t in w for this case by other methods?

   (e) Draw the feasible set for general utility function $u(t, y)$. What assumptions on u do you need to have the same comparative statics results (you can assume the differentiable case)?

6. You need to evaluate the welfare effects of a price change from $\boldsymbol{p}^0$ to $\boldsymbol{p}^1 \neq \boldsymbol{p}^0$ for a given household whose income does not change. Unfortunately you only have data on the realized demand at the current prices, i.e. you know the demand vector $\boldsymbol{x}(\boldsymbol{p}^0, w)$ and you know that the household has has strictly monotonic preferences.

   (a) Show that a sufficient condition for the household to be strictly better off at $\boldsymbol{p}^1$ is that
   $$(\boldsymbol{p}^1 - \boldsymbol{p}^0) \cdot \boldsymbol{x}(\boldsymbol{p}^0, w) < 0.$$

   (b) Can the household be better off at $\boldsymbol{p}^1$ if the reverse strict inequality holds?

(c) Give a sufficient condition for the household preferences so that

$$(\boldsymbol{p}^1 - \boldsymbol{p}^0) \cdot \boldsymbol{x}(\boldsymbol{p}^0, w) \leq 0$$

implies that the household is strictly better off at $\boldsymbol{p}^1$.

7. (Bonus Question) Read Giving According to Garp by Andreoni and Miller in *Econometrica* (2002), pp. 737-753. Answer briefly the following questions.

   (a) How do the authors assess the consistency or rationality of their subjects in the experiment? Is giving to others irrational?

   (b) Could you find a subject whose choice behavior is consistent with WARP but not SARP?

   (c) How do the authors measure the severity of violations to consistent behavior?

8. (Bonus Question) Quite a few recent papers have household-level scanner data on purchases with a revealed preference analysis. An example of such research is The Money Pump as a Measure of Revealed Preference Violations by Federico Echenique, Sangmok Lee, and Matthew Shum,in the *Journal of Political Economy* Vol. 119, No. 6, December 2011, doi:10.1086/665011. Comment on the following points.

   (a) Aggregation of purchases over time. Why not use individual purchases?

   (b) Observation of prices from consumer data. What if prices vary daily and consumers visit the store on different days?

   (c) Separability of food purchases from other consumption: why might this be problematic?

9. (Bonus Question) Le Chatelier's Principle states that if one factor (say capital) is in fixed supply in short run, then the variable factor (say labor) reacts to factor price changes gradually. More concretely, following a change in the price of labor, one sees first a small adjustment in labor, then following a long-run adjustment in capital, a further adjustment in the same direction.

   (a) Consider the following production problem. A firm produces one unit of output at price $p = 10$ by one of two technologies: i) Two units of l or ii) One unit of both l and k. Let the initial labor price be $w = 2$ and capital price $r = 3$. What is the optimal choice of technology? What happens in the short run (i.e. holding k fixed) if labor price changes to $w = 6$. What happens after capital adjusts? What about Le Chatelier's Principle?

   (b) Let's try to find conditions where Le Chatelier's principle holds. Let $f(k, l)$ be a competitive firm's production function and consider parametric changes in the wage rate (i.e. we are fixing p, r throughout. Write the short-run and long-run problems as follows:

   $$l(k; w) = \underset{l \geq 0}{\operatorname{argmax}}\, pf(k, l) - wl - rk,$$

   $$k(w) = \underset{k \geq 0}{\operatorname{argmax}}\, pf(k, l(k; w)) - wl(k; w) - rk.$$

   Assume that the maximizers are unique for both problems and show that if the cross partial derivative satisfies $f_{kl}(k, l) \geq 0$, then for $w' \geq w$, we have:

   $$l(k(w'), w') \leq l(k(w), w') \leq l(k(w), w).$$