Advanced Microeconomics 1
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Problem Set 3

1. Find the expenditure function in the following cases.

   (a) A consumer with preferences represented by
   $$u(x_1, x_2) = 2x_1 + 3x_2.$$

   (b) A consumer with preferences
   $$u(x_1, x_2) = \min\{2x_1, 3x_2\}.$$

   (c) A consumer with preferences
   $$u(x_1, x_2) = \min\{2x_1 + 3x_2, 3x_1 + 2x_2\}.$$

2. The government finances public expenditure of magnitude g by collecting taxes. In this question, you are invited to think about the optimal ways of collecting taxes.

   (a) Suppose that a consumer splits her income w between two goods x and y. Assume that she has twice differentiable strictly concave utility function $u(x, y)$. The government can finance government expenditures $g > 0$ by choosing either a proportional tax t_w on income or by taxing consumption of good x by rate t_x. The government budget constraint for the two cases reads: $t_w w = g$ and $t_x x(p_x, p_y, t_x) = g$. Show that the consumer prefers an income tax in this case.

   (b) Suppose now that there is no exogenous income in the model and good y is now interpreted as leisure. Assume that he consumer has an initial endowment y^e of leisure that she may sell to buy the other good. Hence the consumer's budget constraint is now:
   $$p_x x = p_y (y^e - y), \text{ or}$$
   $$p_x x + p_y y = p_y y^e.$$
   This last equation gives a way in which all problems with income resulting from sales of any endowments (not just labor endowments) should be thought of. First sell the endowment at market prices and then purchase the desired amounts of the goods with the proceeds. Compare now the effect of taxes on x and y as in the previous part. Can you relate the comparison to the price elasticities of demand?

3. Show that for normal goods, the Hicksian demand for a good as a function of its own price (i.e. with all other prices and target utility fixed) is steeper than the Walrasian demand.

4. Preferences are said to be *additively separable* if they can be represented by a utility function of the form:
   $$u(\boldsymbol{x}) = \sum_{i=1}^{L} u_i(x_i).$$
   Suppose that $u_i(x_i)$ is strictly concave and twice differentiable and that the optimal consumption is interior (so that the demands are differentiable in prices).

(a) Show that all goods are normal (i.e. $\frac{\partial x_i(\boldsymbol{p},w)}{\partial w} \geq 0$ for all i).

(b) Show also that for all i, j, k:

$$\frac{\partial x_i(\boldsymbol{p}, w)/\partial p_k}{\partial x_j(\boldsymbol{p}, w)/\partial p_k} = \frac{\partial x_i(\boldsymbol{p}, w)/\partial w}{\partial x_j(\boldsymbol{p}, w)/\partial w}.$$

5. Formulate the following optimization problems and consider the value functions of the problems.

   (a) A monopolist choosing the profit maximizing prize is facing a linear demand function $q = d(p) = a - p$, where $q = d(p)$ is the maximal quantity that can be sold at output price p. Her fixed cost is given by f and the constant marginal cost is $c > 0$. Solve the problem and find the value function.

   (b) A profit maximizing monopolist facing a downward sloping demand $q = a - p$, and marginal cost $c(\beta)$, where β is the level of investment in cost reduction, and the cost of investment is $\gamma\beta^2$. What would you assume on the shape of $c(\beta)$? Write the first-order condition for the problem and compute the derivative of the value function to the problem (with respect to the parameter).

6. A real valued function $f : \mathbb{R}^L_+ \to \mathbb{R}$ is called superadditive if for all z^1, z^2,

$$f\left(z^1 + z^2\right) \geq f\left(z^1\right) + f\left(z^2\right).$$

   (a) Show that every cost function is superadditive in input prices.

   (b) Using this fact, show that the cost function is nondecreasing in input prices.

7. An expected utility maximizing decision maker has a Bernoulli utility function for final wealth x given by $u(x) = -\frac{1}{x}$. Suppose her initial wealth is w and she is offered a gamble winning g with probability p and losing l with probability $(1 - p)$.

   (a) What is her final wealth and expected utility if she accepts the gamble?

   (b) What is her certainty equivalent to accepting the gamble?

   (c) Compute the certainty equivalent to another gamble that wins $g + \frac{\Delta}{p}$ with probability p and loses $l + \frac{\Delta}{1-p}$ with probability $1 - p$ with $\Delta > 0$. Compare to the previous part.

8. (Bonus Question) A rational preference relation $\succeq$ satisfies betweenness if for all $p, q \in \mathcal{L}$ and all $\alpha \in (0, 1)$, we have

$$p \succ q \;\Rightarrow\; p \succ \alpha p + (1 - \alpha) q \succ q.$$

   Show that for continuous rational preference relations betweenness implies the following condition: For all $p, q \in \mathcal{L}$ and all $\alpha \in (0, 1)$, we have

$$p \sim q \;\Rightarrow\; p \sim \alpha p + (1 - \alpha) q \sim q.$$

   In other words, betweenness implies linearity of indifference curves in the Machina triangle.