Advanced Microeconomics 1
Helsinki GSE, Fall 2024
Juuso Välimäki

Problem Set 4

Questions on Choice Under Uncertainty

1. Prove that if a risk averse decision maker rejects a fixed favorable bet at all levels of wealth, then the Bernoulli utility of the decision maker is bounded from above.

2. Consider the savings and consumption model analyzed in lectures. There are two periods, $t = 0, 1$. The decision maker has a strictly concave separable Bernoulli utility function

$$u(c_0, c_1) = u_0(c_0) + \delta u_1(c_1),$$

where c_t denotes consumption in period t. Assume that the consumer receives a certain income w_0 in period 0 and a random income $\tilde{w}_1$ in period 1. The only means for transferring wealth between periods for the consumer is by either borrowing or lending at a risk free rate r.

   (a) Set up the consumer's intertemporal budget constraint and characterize the solution to the savings problem through first order conditions (are these also sufficient conditions?).

   (b) Consider the changes in optimal savings resulting from changes in interest rate r. Can you find an income and a substitution effect in your expression for $\frac{ds}{dr}$?

   (c) Show that when the Arrow-Pratt coefficient of relative risk aversion is less than unity, savings increase in interest rate.

3. Consider the model of the previous exercise. Assume that $u_i(c_i) = \alpha + \beta c_i + (\gamma - c_i)^2$.

   (a) What is the range for possible consumptions where utitlity is increasing in consumption?

   (b) Assume that all the possible realizations from $\tilde{w}_1$ lie in the range found in part a. Does the demand for savings depend on the riskiness of the distribution of $\tilde{w}_1$?

4. Consider the following model of criminal behavior due to Becker. An individual's income is w, and her monetary benefit from crime is b, where b comes from a continuous strictly positive density function on the entire real line $g(b)$. (Benefit b is known to the individual.) If the individual commits a crime, then she will be caught with probability π and in this case she must pay a fine F.

   (a) Show first that there is a unique cutoff level b^* such that the individual commits the crime if and only if $b > b^*$.

(b) Show next that b^* is increasing in π and F.

(c) Suppose next that if caught in a crime, the individual must go to jail for fraction f of her total labor time. Then we have $F = fw$. Show that if the coefficient of relative risk aversion is less than 1, then b^* is increasing in w.

5. Consider an economy where all agents face an independent risk to lose 100 with probability p. N agents decide to create a mutual agreement where the aggregate loss in the pool is equally split among its members.

   (a) Describe the change in the lotteries facing individuals in the pool when N is changed from 2 to 3.

   (b) Show that the risk with $N = 3$ is smaller in the sense of second order stochastic dominance that the risk with $N = 2$.

BONUS: Questions on Entire Course

1. Consider the following short questions.

   (a) In order to aid the poor, the Government introduces a scheme whereby the first 1kg of butter a family buys is subsidized and the remaining amounts are taxed. Consider a family which consumes butter and is made neither better off nor worse off as a result of this scheme. Is it correct to state that the total amount of tax this family pays cannot exceed the subsidy it receives? Explain your answer.

   (b) A consumer buys one unit of a good when its price is €2 and two units when its price is €1. Is it correct to state that he would rather pay €2.80 for two units of the good than go without it altogether? Explain your answer.

   (c) You can only adjust your consumption of x_2 in the long run, but x_1 is flexible in the short run. Is it true that if x_1 is normal, then the demand for x_1 is more elastic in the long run than in the short run? Explain your answer.

2. A consumer in a three-commodity environment (x, y, z) behaves as follows. *i)* When prices are $p_x = 1, p_y = 1$ and $p_z = 1$ the consumer buys $x = 1, y = 2$ and $z = 3$;

   ii) When prices are $p_x = 4, p_y = 6$ and $p_z = 4$ the consumer buys $x = 3, y = 2$ and $z = 1$.

   Does the consumer maximize a strictly quasi-concave utility function?

3. Suppose that the expenditure function of a consumer is of Gorman polar form:

$$e(p, u) = a(p) + ub(p).$$

   Derive the demands for each good and calculate also the income shares that each good receives. Can you find an economic interpretation for your results.

4. Consider the aggregate demand in a model where individual consumers behave in a random manner (and thus do not satisfy any of the rationality criteria that we had for individual choice). To be more specific, assume that a consumer with wealth w facing prices $\boldsymbol{p}$ picks a consumption vector at random from the budget set $B(\boldsymbol{p}, w) = \{x : p \cdot x = w\}$ according to the uniform distribution. Suppose furthermore that there are a continuum of such consumers (and assume that you can apply the law of large numbers for this setting, i.e. the distribution of realized choices in the population coincides with the distribution of a single consumer's choice).

   (a) Denote the individual (random) demand by $x^i(p, w)$. Compute the average demand

   $$\overline{\boldsymbol{x}}(\boldsymbol{p}, w) = \int \boldsymbol{x}^i(\boldsymbol{p}, w)\, di.$$

   (b) Does this average demand satisfy weak axiom of revealed preference?

   (c) Can you find a utility function such that $\overline{\boldsymbol{x}}(\boldsymbol{p}, w)$ is the Walrasian demand function for that utility function?