Assignment 6

1. Implement the **Metropolis-Hastings** (M-H) algorithm to simulate gamma distribution $Ga(\alpha, \beta)$, using exponential distribution $Exp(\lambda)$ as the proposal distribution. Simulate $Ga(3,1)$. **Use the independence chains version of M-H**. (Note: α in gamma distribution and α denoting the acceptance probability are two different things; standard notation: no reason for my head to roll for this .)

   a) (w=3) **Find the optimal proposal distribution**, in this case, find the value for λ that maximises the acceptance probability. Determine this value with precision $\pm\, 0.05$. To do this, compute acceptance probabilities α for various λ values within an appropriate interval. For instance, choose initially $\lambda_i \in [0.1, 1]$ with a suitable step of 0.1, perform 10^5 samples/iterations ($i = 1, 2, \ldots, 10^5$) of Metropolis-Hastings acceptance evaluations to determine $\lambda = \lambda_{\text{opt}}$. Then narrow down the interval and step for λ_i until you can determine the value $\lambda = \lambda_{\text{opt}}$ with precision 0.05, that is, $\lambda = \lambda_{\text{opt}}^{\text{correct}} \pm 0.05$. Do this even if you should be able to figure out the exact value for the optimum $\lambda = \lambda_{\text{opt}}$. In your solution, **show a plot for an appropriate interval of $\boldsymbol{\lambda}$ proving that you are picking the correct value** $\lambda = \lambda_{\text{opt}}$ (λ on the horizontal axis, α on the vertical axis) and that you can pick it with the given precision ± 0.05. **Of course, print out the value of $\boldsymbol{\lambda_{\text{opt}}}$.**

   b) (w=2) **Use the optimal proposal distribution to simulate $\boldsymbol{Ga(3, 1)}$.** Simulate for 10^6 samples. **Plot the resulting distribution** (10^6 points in a histogram, use a sufficient number of bins) **with the corresponding functional form of the gamma distribution** (line plot for this- not a histogram; for example plt.plot(np.arange..., Gamma(np.arange...)).

   c) (w=1) **Compute and print the numerical values for mean and variance** for the resulting distribution.

   d) (w=1) Which of the two methods, the Gibbs sampler or Metropolis-Hastings, requires less information and so is more generally applicable? State briefly the reason for this (one sentence will suffice).

   Note: The definitions used here for $Ga(\alpha, \beta)$ and $Exp(\lambda)$ are the ones given in the lecture notes. If you use some other definition, make sure you calculate the corresponding values in that parametrisation. (In order to have the same numbers to check for in peergrade.)
   Note also: Distibutions here are densities – not CDFs. This is how the word is typically used, although the “official” definition means cumulative functions, see Lecture 1.