Advanced Microeconomics 1
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Practice Questions for Final Exam

1. Define the terms in italics and answer the following short questions.

   (a) Give an example of a *complete* and *transitive* binary relation $\succeq$ on a choice set X such that no maximal element exist, i.e. such that there is no x such that $x \succeq y$ for all $y \in X$.

   (b) Give an example of a choice function that satisfies *Sen's* α but not *Sen's* β.

   (c) Consider choice from a budget set $B(\boldsymbol{p}, w) \subset \mathbb{R}^L$. Is it the case that if x is *revealed preferred* to y and y is revealed preferred to z, then x is revealed preferred to z?

2. Define the terms in italics and answer the following short questions.

   (a) Suppose that a profit maximizing *competitive firm* chooses $\boldsymbol{y} = (2, -1)$ in its *production set* Y at prices $\boldsymbol{p} = (2, 1)$. What can you say about the optimal production vector at prices $\boldsymbol{p} = (1, 1)$?

   (b) *Rational consumers* divide their monthly budget of 30 between food and other consumption. Normalize the price of other consumption to be 1 and suppose that the consumer has two options for food purchases. The consumers can buy food at the regular supermarket at price $p_s = 2$ (per unit of food) of pay a fixed monthly fee of 10 to buy at a cash and carry shop at unit price $p_c = 1$. What can you say about the monthly food purchases for those who buy at the supermarket and for those that buy at the cash and carry?

   (c) Suppose that a consumer has *convex preferences.* If $\boldsymbol{x} \sim \boldsymbol{y}$, is it true that $\frac{1}{2}\boldsymbol{x} + \frac{1}{2}\boldsymbol{y} \succeq \lambda\boldsymbol{x} + (1 - \lambda)\boldsymbol{y}$ for all $\lambda \in [0, 1]$?

3. Analyze the Walrasian demands for the following cases.

   (a) The indirect utility function is $v(p_1, p_2, w) = \left(wp_1^{-\alpha}p_2^{-1-\alpha}\right)^\gamma$, where $\gamma > 0$ and $0 < \alpha < 1$.

   (b) The expenditure function is given by $e(p_1, p_2, u) = u \min\{p_1, p_2\}$

   (c) Show that the expenditure function in part b. is concave in prices.

4. A consumer has quasilinear preferences in (l, x), where l denotes leisure and x denotes consumption and her utility function can be written as

$$u(l, x) = l + 2x^{\frac{1}{2}}.$$

In each period, the consumer allocates her total time endowment $\bar{l}$ between work and leisure, i.e. if she consumes $l \leq \bar{l}$ units of leisure, she works for $\left(\bar{l} - l\right)$units of time. Let w_l denote the wages per unit.

(a) Formulate the consumer's maximization problem and write the first order necessary conditions for maxima.

(b) Argue that the first order conditions are also sufficient and solve the problem.

(c) Assume that a child is born to the consumer, and now she divides her time between leisure l, child care c and work $\left(\bar{l} - l - c\right)$. Suppose also that her new utility function is

$$u\left(l, c, x\right) = l + 2c^{\frac{1}{2}} + 2x^{\frac{1}{2}}.$$

How does your solution to the problem change?

5. Consider a manager of a firm whose compensation package includes a bonus for good performance. More specifically, assume that the wage w of the manager depends on the firm's profit x as follows.

$$w(x) = \begin{cases} \underline{w} & \text{if } x \leq \underline{x}, \\ \underline{w} + .05(x - \underline{x}) & \text{if } x > \underline{x}. \end{cases}$$

Suppose that the manager's Bernoulli utility function is $u(w) = w$, in other words, the manager is risk neutral in his wage.

(a) Is the manager's payoff function concave or convex in the firm's profit?

(b) Suppose that the profit net of the manager's wage is distributed to the shareholders in dividends. Suppose also that the shareholders' payoff function is linear in the dividends. Is the shareholders' payoff function concave or convex in x?

(c) Suppose that the manager chooses the projects that the firm undertakes. Projects can be viewed as lotteries on possible profit levels. Will the manager choose the correct projects from the shareholder's point of view?

6. An investor can bet on N mutually exclusive events E_i such that $\Pr\left(E_i\right) = p_i$ and $\Sigma_{i=1}^{N} p_i = 1$. Betting α euros on event E_i yields αX_i euros if E_i takes place and 0 otherwise.

(a) Suppose first that the decision maker must invest w in these bets to maximize her expected utility from final wealth. Assume also that $u_i\left(x\right) = \ln\left(x\right)$. Write the expected utility from portfolio $\left(\alpha_1, ..., \alpha_N\right)$ where α_i denotes the amount bet on event E_i and $\Sigma_i \alpha_i = w$.

(b) Find the optimal portfolio for the decision maker.

(c) Allow now for the possibility that the decision maker can save an amount s with $0 \leq s \leq w$. The final wealth is then $s + \alpha_i X_i$ in state i and thus $\Sigma_i \alpha_i = w - s$. When is it optimal for the decision maker to set $s = 0$? (Hint: your condition should not depend on the p_i, but only on the X_i)

7. A consumer divides her total time endowment L between leisure l and working time y. She gets utility from leisure and consumption c. The prices of the consumption good are normalized to 1 and the wage rate per unit of time worked is w. Assume that the utility function of the consumer takes the form $u(c, l) = cl$.

   (a) Assume that labor income is taxed at a constant rate t. Formulate the consumer's utility maximization problem and analyze her labor supply as a function of w and t.

   (b) Fix w throughout this part, and consider the case of progressive taxes. For concreteness, assume that the tax rate for income below $\frac{1}{3}wL$ is 40%, and for the part of income exceeding this level, it is 60%. Draw the budget set for this problem and find the optimal labor supply. (Hint: I accept arguments based on marginal rates of substitutions as well as full Kuhn-Tucker analysis).

   (c) Fix the tax rate t in part a. in such a way that the total tax revenue collected is equal in the two cases. Which of the two schemes allows the consumer to reach a higher level of utility?

8. An investor must decide how to allocate her initial wealth w between two alternative investments, x_1 and x_2. The (random) return per unit of investment is $(1 + \widetilde{r}_1)$ in investment x_1 and $(1 + \widetilde{r}_2)$ in x_2. Her final wealth is thus $\widetilde{w} = x_1 (1 + \widetilde{r}_1) + x_2 (1 + \widetilde{r}_2)$. Assume that the investor has a strictly concave utility function $u(w)$ for final wealth.

   (a) Assume first that $\widetilde{r}_1 = r$ with probability 1. In other words, x_1 is a safe investment with rate of return r. Denote the density of $\widetilde{r}_2$ by $f_2(r_2)$. Write the expected utility from an investment portfolio (x_1, x_2), with $x_2 = w - x_1$. Write also the first order condition for the optimal portfolio choice between the two investments and argue that it is also a sufficient condition.

   (b) Under the assumptions in part a., is the optimal x_1 always increasing in r? Find a necessary and sufficient condition for $x_1 < w$.

   (c) Assume next that x_1 and x_2 are statistically independent and have identical return densities $f(r_i)$ for $i = 1, 2$. Find the optimal investment portfolio.

9. A strictly risk averse agent with a strictly increasing and twice differentiable Bernoulli utility function $u(x)$ owns a house worth w. The initial level of wealth for the decision maker is

$x_0 > w$. The is a probability ρ of the house burning down. In the event of fire, w is lost. The decision maker can, however, purchase fire insurance at price p per unit of money insured. In other words, by buying y units of insurance, the decision maker makes an upfront payment of py that entitles her to receive y in case there is fire.

(a) Write the decision maker's final wealth and expected utility as a function of her choice of insurance y.

(b) Derive the first order condition for the optimal choice of insurance.

(c) Derive the optimal level of insurance for the case where $p = \rho$, i.e. insurance is actuarily fair (e.g. because the insurance industry is perfectly competitive).

(d) Assume that $p > \rho$ and analyse how the optimal insurance purchase depends on the absolute risk aversion of the decision maker.