CS-E4800 Artificial Intelligence

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Second Assignment (out on February 3)

- Model your problem in the propositional logic (expressed in a high-level typed syntax with first-order features (Forall, Forsome, Exactlyone))
- Translate into the SMT syntax (translator available from course web page):
  `./grounder solution.fma`
- **There is no easy way to run the translator in Windows. Please get access to (university) Linux computers!!!**
- Test satisfiability with MathSAT http://mathsat.fbk.eu/download.html:
  `./mathsat -model < solution.smt`

Second Assignment (out on February 3)

- Assignments emailed as PDF files on February 3
- Solution document as a PDF or text file, containing
  - your name and student number
  - your solution specification, including comments/documentation
  - MathSAT output with -models option with *false* lines removed
  - All filenames start with your student number!
- Return to the course assignments web page by deadline Thursday February 16 at 23:59
- Feel free to ask questions and help, and try to do it well before the deadline.

Example of the modeling language for logic used in the 2nd assignment

```
type M = { Eng,Spa,Ukr,Nor,Jap };
type C = { OldGold, Kools, Chesterfields, LuckyStrike, Par
type P = { Red, Green, Ivory, Yellow, Blue };
type P = { Dog, Snails, Fox, Horse, Zebra };
type D = { Coffee, Tea, Milk, OrangeJuice, Water };
type H = { 1,2,3,4,5 }; // House numbers
type H4 = { 1,2,3,4 };
```

Zebra Puzzle

Background assumptions about uniqueness

```
forall m : M exactlyone c : C Smokes(m,c) end end;
forall c : C exactlyone m : M Smokes(m,c) end end;

forall m : M exactlyone a : A Pet(m,a) end end;
forall a : A exactlyone m : M Pet(m,a) end end;

forall h : H exactlyone p : P Color(h,p) end end;
forall p : P exactlyone h : H Color(h,p) end end;

forall m : M exactlyone d : D Drinks(m,d) end end;
forall d : D exactlyone m : M Drinks(m,d) end end;

forall m : M exactlyone h : H LivesIn(m,h) end end;
forall h : H exactlyone m : M LivesIn(m,h) end end;
```

Zebra Puzzle

```
// The Englishman lives in the red house.

forsome i : H LivesIn(Eng,i) & Color(i,Red) end;

// The Spaniard owns the dog.

Pet(Spa,Dog);

// Coffee is drunk in the green house.

forsome i : M j : H
        (LivesIn(i,j) & Drinks(i,Coffee) & Color(j,Green))
end;
```

Zebra Puzzle

```
// The Ukrainian drinks tea.

Drinks(Ukr,Tea);

// The green house is immediately to the right of the ivo

forsome i : H4 (Color(i,Ivory) & Color(i+1,Green)) end;

// The Old Gold smoker owns snails.

forsome i : M (Smokes(i,OldGold) & Pet(i,Snails)) end;
```

Zebra Puzzle

```
// Kools are smoked in the yellow house.

forsome i : M j : H ( LivesIn(i,j) & Color(j,Yellow) & Smo

// Milk is drunk in the middle house.

forsome i : M ( LivesIn(i,3) & Drinks(i,Milk)) end;

// The Norwegian lives in the first house.

LivesIn(Nor,1);
```

Zebra Puzzle

```
// The man who smokes Chesterfields lives in the house nex

forsome i : M j : M k : H l : H
                          (Smokes(i,Chesterfields) &
                           Pet(j,Fox) &
                           LivesIn(i,k) &
                           LivesIn(j,l) &
                           (k = l+1 | l = k+1))
end;
```

Zebra Puzzle

```
// Kools are smoked in the house next to the house where t

forsome i : M j : M k : H l : H
                          (Smokes(i,Kools) &
                           Pet(j,Horse) &
                           LivesIn(i,k) &
                           LivesIn(j,l) &
                           (k = l+1 | l = k+1))
end;
```

Zebra Puzzle

```
// The Lucky Strike smoker drinks orange juice.

forsome i : M (Smokes(i,LuckyStrike)
                & Drinks(i,OrangeJuice));

// The Japanese smokes Parliaments.

Smokes(Jap,Parliaments);

// The Norwegian lives next to the blue house.

forsome i : H j : H
         (LivesIn(Nor,i) & Color(j,Blue)
          & (i = j+1 | j = i+1))
end;
```

Nesting of ExactlyOne

```
type A = {a1,a2}; type B = {b1,b2};
exactlyone a:A b:B R(a,b) end;
exactlyone a:A exactlyone b:B R(a,b) end end;
```

The two exactly-one constraints above not equivalent.
The first means that exactly one of the following holds.

```
R(a1,b1)  R(a1,b2)  R(a2,b1)  R(a2,b2)
```

The second that exactly one of following holds:

```
exactlyone b:B R(a1,b) end;
exactlyone b:B R(a2,b) end;
```

This allows R(a1,b1), R(a2,b1), R(a2,b2) to be true.

There is exactly one married couple vs. *There is exactly one woman, who is married to exactly one man (and others are either single or polygamists).*

Grounding of the Specification

- Standard grounding process (similar to PDDL)

$$\begin{aligned}
\text{forall x}: \{a,b,c\}\phi(x) &= \phi(a) \wedge \phi(b) \wedge \phi(c) \\
\text{forsome x}: \{a,b,c\}\phi(x) &= \phi(a) \vee \phi(b) \vee \phi(c) \\
\text{atmostone x}: \{a,b,c\}\phi(x) &= \neg(\phi(a) \wedge \phi(b)) \wedge \neg(\phi(a) \wedge \phi(c)) \\
&\quad \wedge \neg(\phi(b) \wedge \phi(c)) \\
\text{exactlyone x}: \text{X } \phi(x) &= \text{forsome x}: \text{X } \phi(x) \\
&\quad \wedge \text{ atmostone x}: \text{X } \phi(x)
\end{aligned}$$

- Integer equalities simplified to TRUE and FALSE, eliminated.

$$\begin{aligned}
\mathit{TRUE} \wedge \phi &\equiv \phi && (1) \\
\mathit{FALSE} \wedge \phi &\equiv \mathit{FALSE} && (2) \\
\mathit{TRUE} \vee \phi &\equiv \mathit{TRUE} && (3) \\
\mathit{FALSE} \vee \phi &\equiv \phi && (4)
\end{aligned}$$

Solution with the MathSAT solver

1. `./grounder zebra.fma > zebra.smt` produces a 691 line, 68 kB file
2. `./mathsat -stats < zebra.smt` finds a solution in 21 milliseconds, with a couple of search steps

Solution with the MathSAT solver

```
mathsat-5.3.6-linux-x86_64/bin/mathsat -stats < zebra.smt
sat
;; statistics
(
 :sat-checks 1
 :sat-restarts 1
 :sat-decisions 7
 :sat-random-decisions 0
 :sat-propagations 286
 :sat-theory-propagations 0
 :sat-watched-clauses-visited 356
 :sat-binary-watched-clauses-visited 253
 :sat-conflicts 0
 :sat-theory-conflicts 0
...
 :time-seconds 0.021
 :memory-mb 9.777
)
```

Solution with the MathSAT solver

```
(Color_1_Yellow true)
(Color_2_Blue true)
(Color_3_Red true)
(Color_4_Ivory true)
(Color_5_Green true)
(Drinks_Eng_Milk true)
(Drinks_Jap_Coffee true)
(Drinks_Nor_Water true)
(Drinks_Spa_OrangeJuice true)
(Drinks_Ukr_Tea true)
(LivesIn_Eng_3 true)
(LivesIn_Jap_5 true)
(LivesIn_Nor_1 true)
(LivesIn_Spa_4 true)
(LivesIn_Ukr_2 true)
```

```
(Pet_Eng_Snails true)
(Pet_Jap_Zebra true)
(Pet_Nor_Fox true)
(Pet_Spa_Dog true)
(Pet_Ukr_Horse true)
(Smokes_Eng_OldGold true)
(Smokes_Jap_Parliaments true)
(Smokes_Nor_Kools true)
(Smokes_Spa_LuckyStrike true)
(Smokes_Ukr_Chesterfields true)
```

Zebra Puzzle in the Predicate Logic

- The puzzle can be formalized in Predicate Logic
- Requires Domain Closure Assumption and Unique Names Assumption $\Longrightarrow$ finite domain
- Typed quantification missing: clumsier formalization
- Theorem provers for the Predicate Logic usually have no advantage over SAT/SMT solvers with finite domain problems

Zebra Puzzle in the Predicate Logic

DCA, UNA

Domain Closure Assumption

$$\forall x(x = A \vee x = B \vee x = C \vee x = D)$$

Unique Names Assumption

$$\neg(A = B)\ \neg(A = C)\ \neg(A = D)\ \neg(B = C)\ \neg(B = D)\ \neg(C = D)$$

Zebra Puzzle in the Predicate Logic

Typed quantification

```
forsome i : M j :  H
        (LivesIn(i,j) & Drinks(i,Coffee) & Color(j,Green))
end end;
```

in the Predicate Logic would be

$$\exists i \exists j (Man(i) \wedge House(j) \wedge LivesIn(i, j) \wedge Drinks(i, Coffee) \wedge Color(j, Green))$$

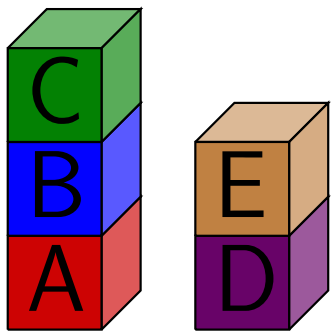
State-Space Search Methods: Summary

1. explicit state-space search
   - breadth-first, depth-first, A*, IDA$*$, greedy best-first
   - forwards in the state space
2. backwards in the space of sets of states represented as formulas, with actions "reversed" e.g. by weakest preconditions
   - same algorithms and heuristics applicable
3. logic-based symbolic methods (today's lecture)
   - satisfiability: iterative deepening search with SAT tests
   - Binary Decision Diagrams: breadth-first with state sets (forwards or backwards)

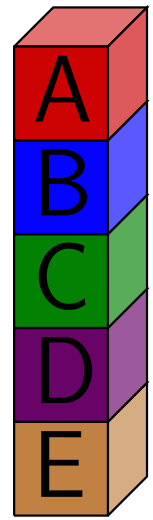
State-space search by logic

Example

initial state



goal state



Problem solved almost without search:

- Formulas for lengths 1 to 4 shown unsatisfiable without any search.
- Search tree for plan length 5 has 3 nodes
- Plans have 5 to 7 operators, optimal plan has 5.

State-space search by logic

Example

	0	1	2	3	4	5
clear(a)	F	F				
clear(b)	F					F
clear(c)	T	T			F	F
clear(d)	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F
on(a,b)	F	F	F			T
on(a,c)	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F
on(b,a)	T	T			F	F
on(b,c)	F	F			T	T
on(b,d)	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F
on(c,b)	T			F	F	F
on(c,d)	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F
ontable(a)	T	T	T			F
ontable(b)	F	F			F	F
ontable(c)	F			F	F	F
ontable(d)	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: ¬clear(b)@1.
3. Branch: clear(a)@3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

State-space search by logic

Example

	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
clear(a)	F	F					F	F	F		T	T	F	F	F	T	T	T
clear(b)	F					F	F	F		T	T	F	F	F	T	T	T	F
clear(c)	T	T			F	F	T	T	T	T	F	F	T	T	T	T	F	F
clear(d)	F	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F
clear(e)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
on(a,b)	F	F	F			T	F	F	F	F	F	T	F	F	F	F	F	T
on(a,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(a,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,a)	T	T			F	F	T	T	T		F	F	T	T	T	F	F	F
on(b,c)	F	F			T	T	F	F	F	F	T	T	F	F	F	F	T	T
on(b,d)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(b,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(c,b)	T			F	F	F	T	T		F	F	F	T	T	F	F	F	F
on(c,d)	F	F	F	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T
on(c,e)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(d,e)	F	F	T	T	T	T	F	F	T	T	T	T	F	F	T	T	T	T
on(e,a)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,b)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,c)	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
on(e,d)	T	F	F	F	F	F	T	F	F	F	F	F	T	F	F	F	F	F
ontable(a)	T	T	T			F	T	T	T	T	T	F	T	T	T	T	T	F
ontable(b)	F	F			F	F	F	F	F		F	F	F	F	F	T	F	F
ontable(c)	F			F	F	F	F	F		F	F	F	F	F	T	F	F	F
ontable(d)	T	T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	F
ontable(e)	F	T	T	T	T	T	F	T	T	T	T	T	F	T	T	T	T	T

1. State variable values inferred from initial values and goals.
2. Branch: ¬clear(b)@1.
3. Branch: clear(a)@3.
4. Plan found:

	0	1	2	3	4
fromtable(a,b)	F	F	F	F	T
fromtable(b,c)	F	F	F	T	F
fromtable(c,d)	F	F	T	F	F
fromtable(d,e)	F	T	F	F	F
totable(b,a)	F	F	T	F	F
totable(c,b)	F	T	F	F	F
totable(e,d)	T	F	F	F	F

State-space search by logic

Example

	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5
clear(a)	F F	F F F T T	F F F T T T
clear(b)	F F	F F T T F	F F T T T F
clear(c)	T T F F	T T T T F F	T T T T F F
clear(d)	F T T F F F	F T T F F F	F T T F F F
clear(e)	T T F F F F	T T F F F F	T T F F F F
on(a,b)	F F F T	F F F F F T	F F F F F T
on(a,c)	F F F F F F	F F F F F F	F F F F F F
on(a,d)	F F F F F F	F F F F F F	F F F F F F
on(a,e)	F F F F F F	F F F F F F	F F F F F F
on(b,a)	T T F F	T T T F F	T T T F F F
on(b,c)	F F T T	F F F F T T	F F F F T T
on(b,d)	F F F F F F	F F F F F F	F F F F F F
on(b,e)	F F F F F F	F F F F F F	F F F F F F
on(c,a)	F F F F F F	F F F F F F	F F F F F F
on(c,b)	T F F F	T T F F F	T T F F F F
on(c,d)	F F F T T T	F F F T T T	F F F T T T
on(c,e)	F F F F F F	F F F F F F	F F F F F F
on(d,a)	F F F F F F	F F F F F F	F F F F F F
on(d,b)	F F F F F F	F F F F F F	F F F F F F
on(d,c)	F F F F F F	F F F F F F	F F F F F F
on(d,e)	F F T T T T	F F T T T T	F F T T T T
on(e,a)	F F F F F F	F F F F F F	F F F F F F
on(e,b)	F F F F F F	F F F F F F	F F F F F F
on(e,c)	F F F F F F	F F F F F F	F F F F F F
on(e,d)	T F F F F F	T F F F F F	T F F F F F
ontable(a)	T T T F	T T T T T F	T T T T T F
ontable(b)	F F F F	F F F F F F	F F F T F F
ontable(c)	F F F F	F F F F F F	F F T F F F
ontable(d)	T T F F F F	T T F F F F	T T F F F F
ontable(e)	F T T T T T	F T T T T T	F T T T T T

1. State variable values inferred from initial values and goals.
2. Branch: ¬clear(b)@1.
3. Branch: clear(a)@3.
4. Plan found:

	0 1 2 3 4
fromtable(a,b)	F F F F T
fromtable(b,c)	F F F T F
fromtable(c,d)	F F T F F
fromtable(d,e)	F T F F F
totable(b,a)	F F T F F
totable(c,b)	F T F F F
totable(e,d)	T F F F F

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State-space search by logic

Example

	0 1 2 3 4 5	0 1 2 3 4 5	0 1 2 3 4 5
clear(a)	F F	F F F T T	F F F T T T
clear(b)	F F	F F T T F	F F T T T F
clear(c)	T T F F	T T T T F F	T T T T F F
clear(d)	F T T F F F	F T T F F F	F T T F F F
clear(e)	T T F F F F	T T F F F F	T T F F F F
on(a,b)	F F F T	F F F F F T	F F F F F T
on(a,c)	F F F F F F	F F F F F F	F F F F F F
on(a,d)	F F F F F F	F F F F F F	F F F F F F
on(a,e)	F F F F F F	F F F F F F	F F F F F F
on(b,a)	T T F F	T T T F F	T T T F F F
on(b,c)	F F T T	F F F F T T	F F F F T T
on(b,d)	F F F F F F	F F F F F F	F F F F F F
on(b,e)	F F F F F F	F F F F F F	F F F F F F
on(c,a)	F F F F F F	F F F F F F	F F F F F F
on(c,b)	T F F F	T T F F F	T T F F F F
on(c,d)	F F F T T T	F F F T T T	F F F T T T
on(c,e)	F F F F F F	F F F F F F	F F F F F F
on(d,a)	F F F F F F	F F F F F F	F F F F F F
on(d,b)	F F F F F F	F F F F F F	F F F F F F
on(d,c)	F F F F F F	F F F F F F	F F F F F F
on(d,e)	F F T T T T	F F T T T T	F F T T T T
on(e,a)	F F F F F F	F F F F F F	F F F F F F
on(e,b)	F F F F F F	F F F F F F	F F F F F F
on(e,c)	F F F F F F	F F F F F F	F F F F F F
on(e,d)	T F F F F F	T F F F F F	T F F F F F
ontable(a)	T T T F	T T T T T F	T T T T T F
ontable(b)	F F F F	F F F F F F	F F F T F F
ontable(c)	F F F F	F F F F F F	F F T F F F
ontable(d)	T T F F F F	T T F F F F	T T F F F F
ontable(e)	F T T T T T	F T T T T T	F T T T T T

1. State variable values inferred from initial values and goals.
2. Branch: ¬clear(b)@1.
3. Branch: clear(a)@3.
4. Plan found:

	0 1 2 3 4
fromtable(a,b)	F F F F T
fromtable(b,c)	F F F T F
fromtable(c,d)	F F T F F
fromtable(d,e)	F T F F F
totable(b,a)	F F T F F
totable(c,b)	F T F F F
totable(e,d)	T F F F F

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Encoding of state-space search as formulas

Atomic propositions for state variables

clear(x)@i	$x \in \{a, b, c, d, e\}, i \in \{0, \ldots, 5\}$
on(x,y)@i	$x \in \{a, b, c, d, e\}, y \in \{a, b, c, d, e\},$ $x \neq y, i \in \{0, \ldots, 5\}$
ontable(x)@i	$x \in \{a, b, c, d, e\}, i \in \{0, \ldots, 5\}$

Atomic propositions for actions

fromtable(x,y)@i	$x \in \{a, b, c, d, e\}, y \in \{a, b, c, d, e\},$ $x \neq y, i \in \{0, \ldots, 4\}$
totable(x,y)@i	$x \in \{a, b, c, d, e\}, y \in \{a, b, c, d, e\},$ $x \neq y, i \in \{0, \ldots, 4\}$
move(x,y,z)@i	$x \in \{a, b, c, d, e\}, y \in \{a, b, c, d, e\},$ $z \in \{a, b, c, d, e\}, x \neq y, x \neq z, y \neq z,$ $i \in \{0, \ldots, 4\}$

Encoding of state-space search as formulas

Preconditions of actions

move(A,B,C)

- move block a from block b to block c.
- precondition $\text{on}(a, b) \wedge \text{clear}(c)$
- effects $\{\neg\text{on}(a, b), \text{on}(a, c), \text{clear}(b), \neg\text{clear}(c)\}$

Formulas for precondition

for all $i \in \{0, \ldots, 4\}$

$$\text{move}(a, b, c)@i \rightarrow \text{on}(a, b)@i \wedge \text{clear}(c)@i$$

Encoding of state-space search as formulas

Effects of actions

move(A,B,C)

- move block a from block b to block c.
- precondition $\text{on}(a, b) \wedge \text{clear}(c)$
- effects $\{\neg\text{on}(a, b), \text{on}(a, c), \text{clear}(b), \neg\text{clear}(c)\}$

Formulas for effects

for all $i \in \{0, \ldots, 4\}$

$$\text{move}(a, b, c)@i \rightarrow \neg\text{on}(a, b)@(i + 1)$$
$$\text{move}(a, b, c)@i \rightarrow \text{on}(a, c)@(i + 1)$$
$$\text{move}(a, b, c)@i \rightarrow \text{clear}(b)@(i + 1)$$
$$\text{move}(a, b, c)@i \rightarrow \neg\text{clear}(c)@(i + 1)$$

Encoding of state-space search as formulas

Frame axioms

- When does a state variable not change?
  Often written in different ways:
  - $x@i \wedge \neg x@(i+1) \rightarrow \text{causesForChangeToFalse}@i$
  - $\neg\text{causesForChangeToFalse}@i \wedge x@i \rightarrow x@(i+1)$
  - $\neg x@i \vee x@(i+1) \vee \text{causesForChangeToFalse}@i$
- Called frame axiom for historical reasons

Frame axioms

for all $i \in \{0, \ldots, 4\}$

$$\begin{aligned} \text{on}(a,b)@i \wedge \neg\text{on}(a,b)@(i+1) \quad \rightarrow \quad & \text{totable}(a,b)@i \\ & \vee \text{move}(a,b,c)@i \\ & \vee \text{move}(a,b,d)@i \\ & \vee \text{move}(a,b,e)@i \end{aligned}$$

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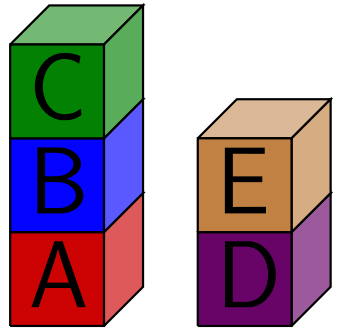
Encoding of state-space search as formulas

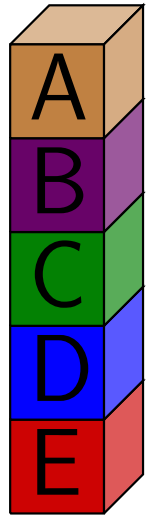
Initial state

$\text{on}(c, b)@0$	$\text{on}(b, a)@0$	$\text{on}(e, d)@0$
$\text{clear}(c)@0$	$\text{clear}(e)@0$	$\text{ontable}(a)@0$
$\text{ontable}(d)@0$	$\neg\text{on}(a, b)@0$	$\neg\text{on}(a, c)@0$
$\neg\text{on}(a, d)@0$	$\neg\text{on}(a, e)@0$	$\neg\text{on}(b, c)@0$
$\neg\text{on}(b, d)@0$	$\neg\text{on}(b, e)@0$	$\neg\text{on}(c, a)@0$
$\neg\text{on}(c, d)@0$	$\neg\text{on}(c, e)@0$	$\neg\text{on}(d, a)@0$
$\neg\text{on}(d, b)@0$	$\neg\text{on}(d, c)@0$	$\neg\text{on}(d, d)@0$
$\neg\text{on}(e, a)@0$	$\neg\text{on}(e, b)@0$	$\neg\text{on}(e, c)@0$
$\neg\text{ontable}(c)@0$	$\neg\text{ontable}(b)@0$	$\neg\text{ontable}(e)@0$
$\neg\text{clear}(a)@0$	$\neg\text{clear}(b)@0$	$\neg\text{clear}(d)@0$



Goal states

$$\text{on}(a, b)@5 \wedge \text{on}(b, c)@5 \wedge \text{on}(c, d)@5 \wedge \text{on}(d, e)@5$$



Relations in the Propositional Logic

A binary relation $R \subseteq X \times X$ is a set of pairs $(a, b) \in X \times X$.

Problem: How to represent the pair $(0001, 1100)$?
Solution: To index the bits use variables
$X_{01} = \{A@0, B@0, C@0, D@0, A@1, B@1, C@1, D@1\}$
instead of $X = \{A, B, C, D\}$.
Pair $(0001, 1100)$ is hence represented as 00011100.

Pair $\begin{pmatrix} A@0 & B@0 & C@0 & D@0 & A@1 & B@1 & C@1 & D@1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$ therefore corresponds to the valuation that assigns 1 to $D@0$, $A@1$ and $B@1$ and 0 to all other variables.

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Relations as Formulas

Example

$(A@0 \leftrightarrow A@1) \wedge (B@0 \leftrightarrow B@1) \wedge (C@0 \leftrightarrow C@1) \wedge (D@0 \leftrightarrow D@1)$
represents the identity relation of 4-bit bit-vectors.

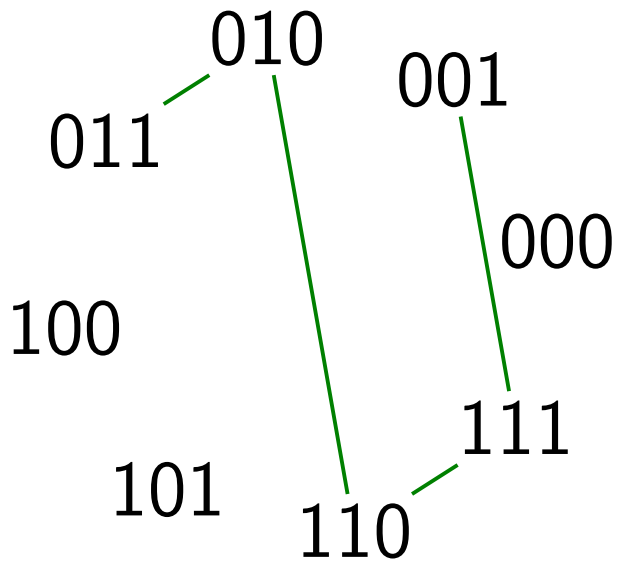
Example

$$\begin{aligned} inc_{01} \quad = \quad & (\neg C@0 \wedge C@1 \wedge (B@0 \leftrightarrow B@1) \wedge (A@0 \leftrightarrow A@1)) \\ & \vee (\neg B@0 \wedge C@0 \wedge B@1 \wedge \neg C@1 \wedge (A@0 \leftrightarrow A@1)) \\ & \vee (\neg A@0 \wedge B@0 \wedge C@0 \wedge A@1 \wedge \neg B@1 \wedge \neg C@1) \\ & \vee (A@0 \wedge B@0 \wedge C@0 \wedge \neg A@1 \wedge \neg B@1 \wedge \neg C@1) \end{aligned}$$

represents the successor relation of 3-bit integers
$\{(000, 001), (001, 010), (010, 011), (011, 100),$
$(100, 101), (101, 110), (110, 111), (111, 000)\}$.

Relations as Formulas

$$\{(001, 111), (010, 110), (011, 010), (111, 110)\}$$



$a@0$	$b@0$	$c@0$	$a@1$	$b@1$	$c@1$	ϕ
⋮						
0	0	1	1	1	1	1
⋮						
0	1	0	1	1	0	1
⋮						
0	1	1	0	1	0	1
⋮						
1	1	1	1	1	0	1
⋮						

$$(\neg a@0 \wedge \neg b@0 \wedge c@0 \wedge a@1 \wedge b@1 \wedge c@1) \vee$$
$$(\neg a@0 \wedge b@0 \wedge \neg c@0 \wedge a@1 \wedge b@1 \wedge \neg c@1) \vee$$
$$(\neg a@0 \wedge b@0 \wedge c@0 \wedge \neg a@1 \wedge b@1 \wedge \neg c@1) \vee$$
$$(a@0 \wedge b@0 \wedge c@0 \wedge a@1 \wedge b@1 \wedge \neg c@1)$$

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Applications

Propositional formulas can represent the transition relation (graph) of any finite discrete system.
Multi-step reachability by chaining the transition relation,.

Example

Make several copies of inc_{01}, and change the subscripts of the variables correspondingly:

$$inc_{01} \wedge inc_{12} \wedge inc_{23} \wedge inc_{34}.$$

Applications

Example

Can bit-vector 100 be reached from bit-vector 000 by four steps?
This is equivalent to the satisfiability of the following formula.

$$\neg A@0 \wedge \neg B@0 \wedge \neg C@0 \wedge$$
$$inc_{01} \wedge inc_{12} \wedge inc_{23} \wedge inc_{34} \wedge$$
$$A@4 \wedge \neg B@4 \wedge \neg C@4$$

Applications

Interesting with choice of actions at each step!

Multiply by 2 (left shift, losing the most significant bit):

$$ml2_{01} = (A@1 \leftrightarrow B@0) \wedge (B@1 \leftrightarrow C@0) \wedge \neg C@1$$

Example

Can bit-vector 111 be reached from bit-vector 000 by four steps, by using operations *inc* and *ml2*?

$$\neg A@0 \wedge \neg B@0 \wedge \neg C@0 \wedge$$
$$(inc_{01} \vee ml2_{01}) \wedge (inc_{12} \vee ml2_{12}) \wedge (inc_{23} \vee ml2_{23}) \wedge (inc_{34} \vee ml2_{34}) \wedge$$
$$A@4 \wedge B@4 \wedge C@4$$

Translation of Actions to Formulas

We consider two systematic translations

1. Transition relations with one action at each step
2. Transition relations with multiple actions at each step

Remark: Translations use equivalences $x@i \leftrightarrow F(\ldots)$ which express new value of state variable in terms of previous state. (Rather than separating this to effect formulas and frame axioms.)

Translation of Actions to Formulas

Some notation:

- set X of Boolean state variables, subscripted as X_i with time @i
- Θ_{ij} with $j = i + 1$ is transition relation formula for change between times i and j, with atomic propositions in $X_i \cup X_{i+1}$

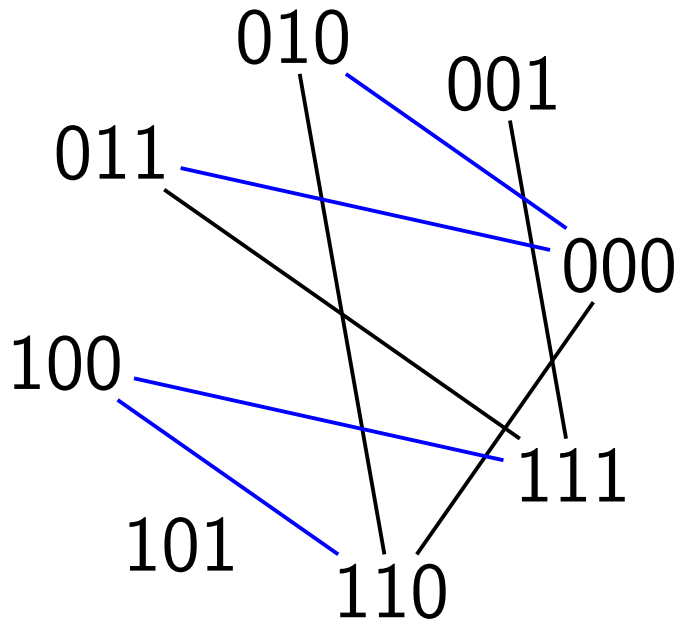
Translation of Actions to Formulas

PRECONDITION: $a = 0$ EFFECTS: $a := 1, b := 1$

PRECONDITION: $b = 1$ EFFECTS: $b := 0, c := 0$

$$(\neg a@0 \wedge a@1 \wedge b@1 \wedge (c@0 \leftrightarrow c@1))$$
$$\vee(b@0 \wedge (a@0 \leftrightarrow a@1) \wedge \neg b@1 \wedge \neg c@1)$$

$\{(000, 110),$
$(001, 111),$
$(010, 000),$
$(010, 110),$
$(011, 000),$
$(011, 111),$
$(110, 100),$
$(111, 100)\}$



Translation of Actions to Formulas

effect	τ_x
$x := 1;$	$x@1$
$x := 0;$	$\neg x@1$
IF ϕ THEN $x := 0;$	$(x@0 \wedge \neg\phi@0) \leftrightarrow x@1$
IF ϕ' THEN $x := 1;$	$(x@0 \vee \phi'@0) \leftrightarrow x@1$
IF ϕ THEN $x := 0;$ IF ϕ' THEN $x := 1;$	$((x@0 \wedge \neg\phi@0) \vee \phi'@0) \leftrightarrow x@1$ (See note below!)
-	$x_o \leftrightarrow x_1$

$\phi@t = \phi$ with atomic propositions subscripted with t

Note: x will be *true* iff it was *true* and doesn't become *false*, or, it becomes *true*.

Translation of Actions to Formulas

Action as a Formula

An action is represented as a formula

$$\varphi@0 \wedge \tau_{x^1} \wedge \tau_{x^2} \wedge \cdots \wedge \tau_{x^m}$$

where φ is the precondition of the action and τ_x are as on the previous slide for all state variables $x^1, \ldots, x^m$.

Set of Actions as a Formula

Let $\phi^1, \phi^2, \ldots, \phi^n$ be formulas for actions $a^1, a^2, \ldots, a^n$. Choice between actions is represented by

$$\Theta_{01} = \phi^1 \vee \phi^2 \vee \cdots \vee \phi^n.$$

Reachability as Satisfiability

Theorem

A state s' such that $s'(G) = 1$ is reachable from a state s such that $s(I) = 0$ by a sequence of T actions if and only if $I@0 \wedge \Theta_{01} \wedge \cdots \wedge \Theta_{(T-1)T} \wedge G@T$ is satisfiable.

Plans with Multiple Simultaneous Actions

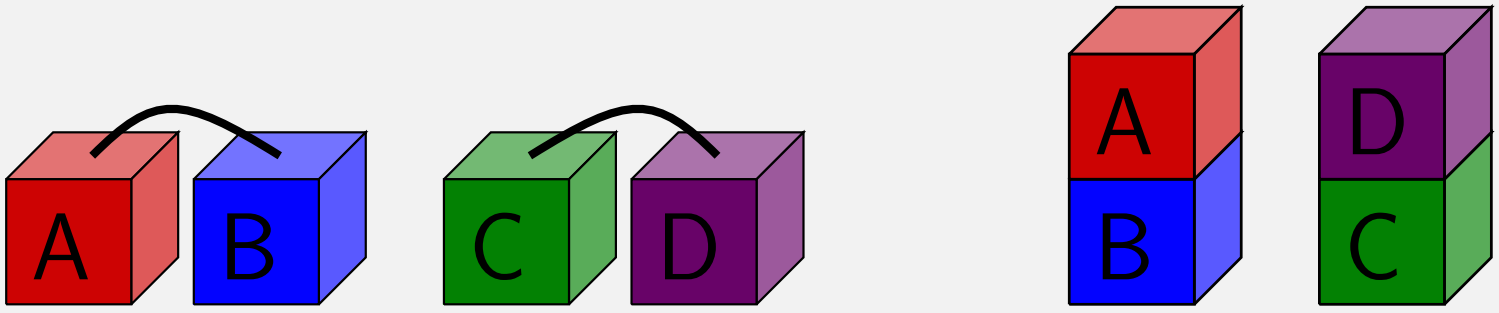
More actions per step $\Rightarrow$ Fewer steps $\Rightarrow$ Smaller formulas $\Rightarrow$ Far lower runtimes

- Dynamics of every state variable separately: $x@(i+1) \leftrightarrow ((x@i \wedge$ no action produces $\neg x) \vee$ some action produces $x)$
- $a@i \rightarrow$ the precondition of a
- $\neg(a_1@i \wedge a_2@i)$ whenever a_1 and a_2 interfere, i.e. either
  - a_1 falsifies the precondition of a_2 (or vice versa), or
  - a_1 and a_2 can have conflicting effects.

  This guarantees that a set of simultaneous actions can always be executed in any order.

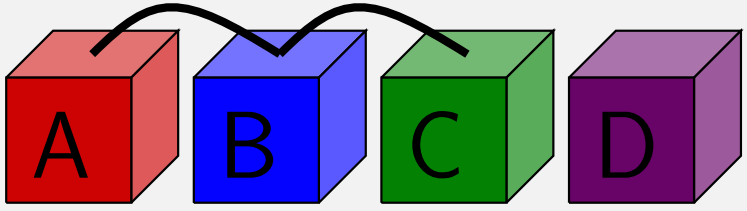
Plans with Multiple Simultaneous Actions

Actions do not interfere



Actions can be taken simultaneously.

Actions interfere



If A moved on B first, B won't be clear and can't be moved.

Example: Robots moving objects

```
type package = { p1,p2,p3,p4,p5 };
type location = { room1, room2};
type robot = { robot1, robot2 };

forall r : robot l : location i : [1..11]
   (at(r,l,i) <-> ((at(r,l,i-1)
                     & forall l2 : location not move(r,l,l2,i-1) end)
                   | forsome l2 : location move(r,l2,l,i-1) end))
end;
```

Example: Robots moving objects

```
forall p : package l : location i : [1..11]
   (at(p,l,i) <-> ((at(p,l,i-1)
                    & forall r : robot not load(r,p,l,i-1) end)
                   | forsome r : robot unload(r,p,l,i-1) end))
end;

forall p : package r : robot i : [1..11]
   (holds(r,p,i) <-> ((holds(r,p,i-1)
                       & forall l : location not unload(r,p,l,i-1) e
                      | forsome l : location load(r,p,l,i-1) end))
end;
```

Example: Robots moving objects

```
 forall r : robot i : [1..11]
   (free(r,i) <->
      ((free(r,i-1)
        & forall p : package l : location not load(r,p,l,i-1) end)
       | forsome p : package l : location unload(r,p,l,i-1) end))
end;
```

Example: Robots moving objects

```
// Preconditions of actions

forall r : robot i : [0..10] p : package l : location
         (load(r,p,l,i) -> (at(r,l,i) & at(p,l,i) & free(r,i))) &
       (unload(r,p,l,i) -> (at(r,l,i) & holds(r,p,i)))
end;

forall r : robot i : [0..10] l1 : location l2 : location
     (move(r,l1,l2,i) -> ((l1 != l2) & at(r,l1,i)))
end;
```

Example: Robots moving objects

```
// Exclusive actions:
// At most one move from a location at a time
forall r : robot i : [0..10] l1 : location
    atmostone l2 : location move(r,l1,l2,i)
end end;

// At most one robot can load for each object at a time
forall p : package i : [0..10] l : location
    atmostone r : robot load(r,p,l,i)
end end;
```

Example: Robots moving objects

```
// Robot cannot move and load or unload at the same time
forall r : robot i : [0..10] l1 : location l2 : location
    move(r,l1,l2,i) -> forall p : package (not load(r,p,l1,i)
                                          & not unload(r,p,l1,i))
                       end
end;
```

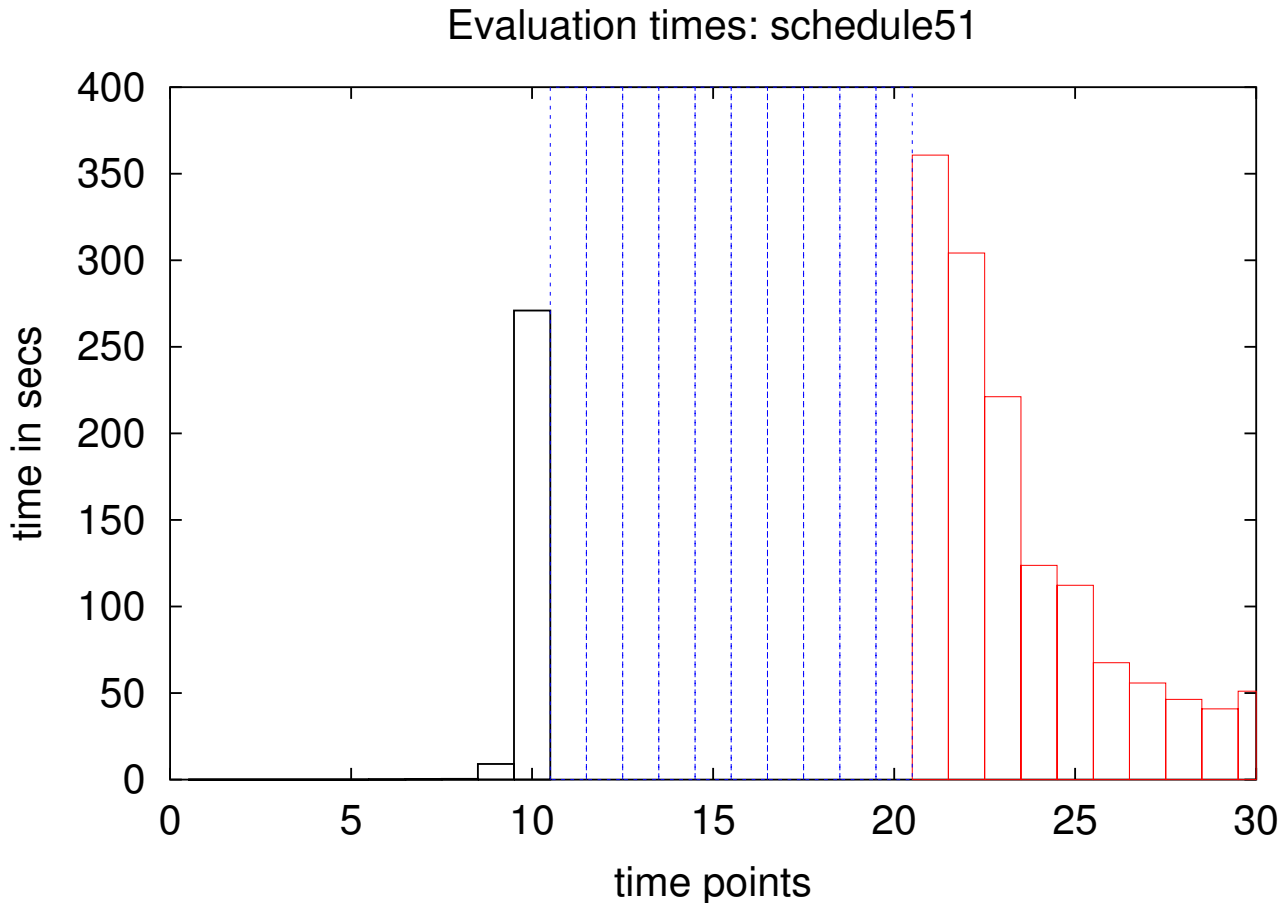
Example: Robots moving objects

```
// Initial state

forall r : robot free(r,0) end;
forall r : robot at(r,room1,0) end;
forall r : robot not at(r,room2,0) end;
forall p : package at(p,room1,0) end;
forall p : package not at(p,room2,0) end;
forall p : package not holds(robot1,p,0) end;

// Goal state

forall p : package at(p,room2,11) end;
```

Why Finding Shortest Solutions is Hard?

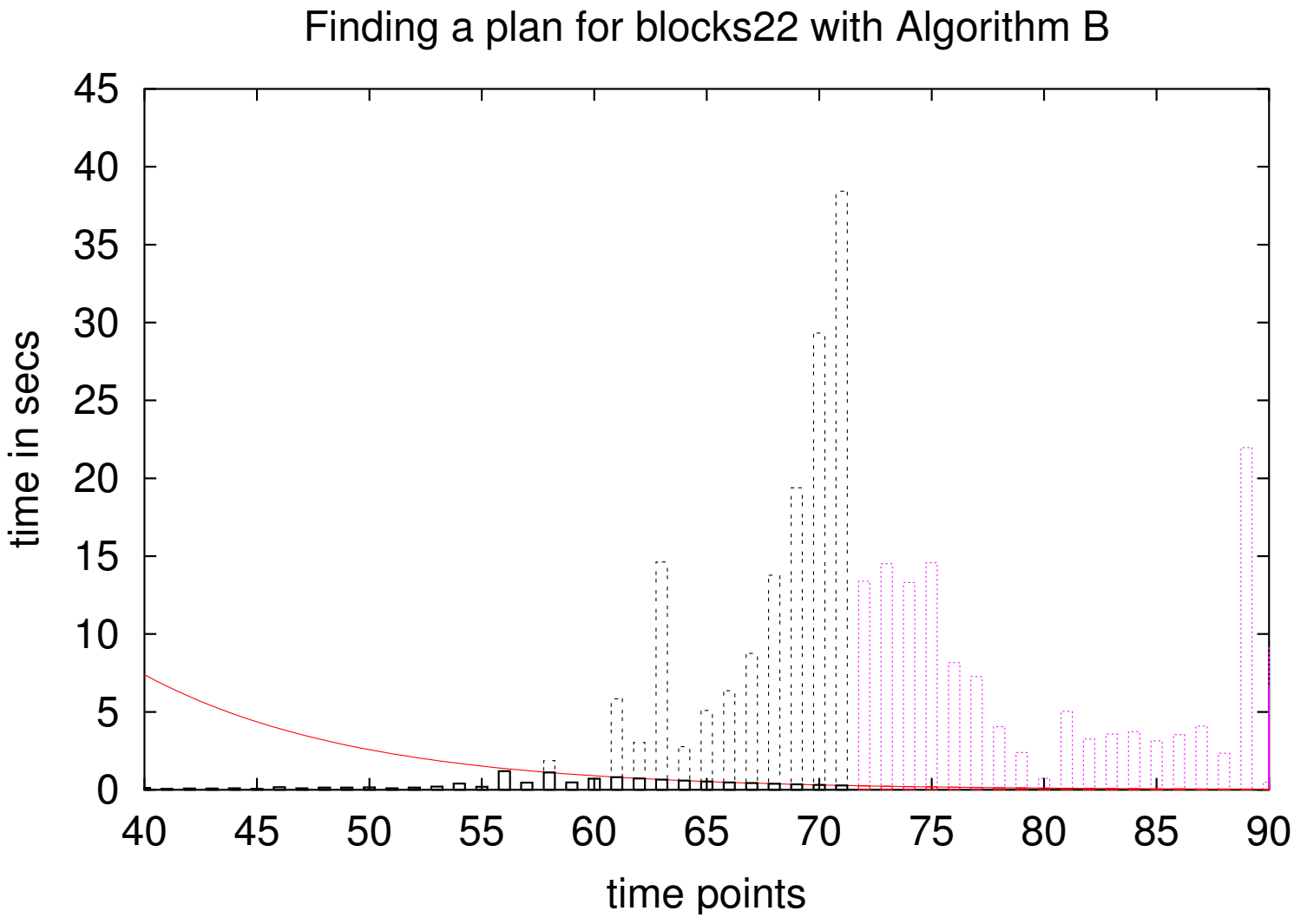
- Finding one (arbitrary) solution can be easy
- Proving that solution is best involves testing that no better solutions exist
- Need to exhaustively cover a potentially very large search space
- Hence, finding optimal solutions is generally much more expensive than finding any solution
- This is analogous to A* vs. suboptimal algorithms (Best First, WA*)

SAT Solver Runtime Profiles

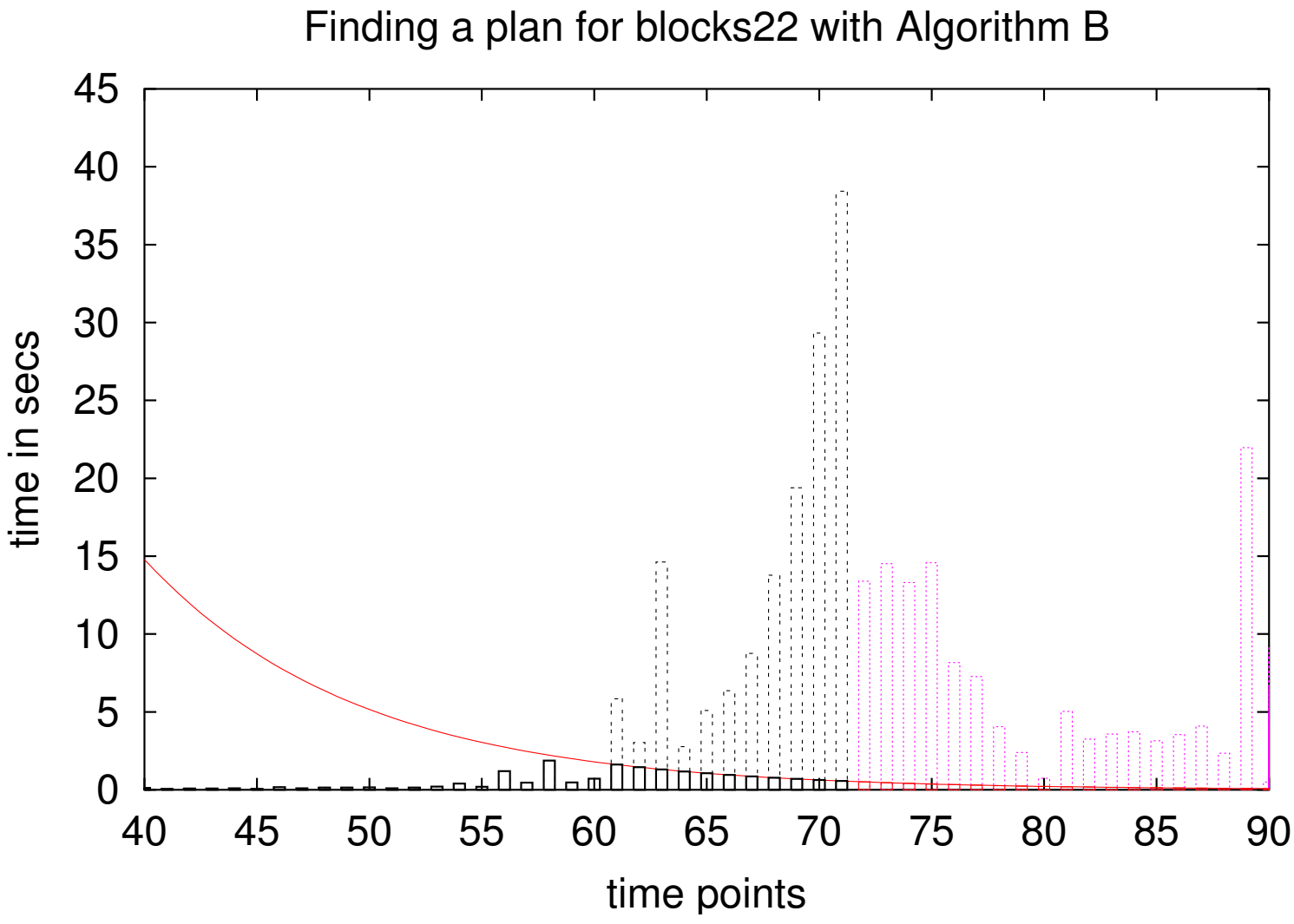


(black = unsatisfiable, blue = unknown, red = satisfiable)

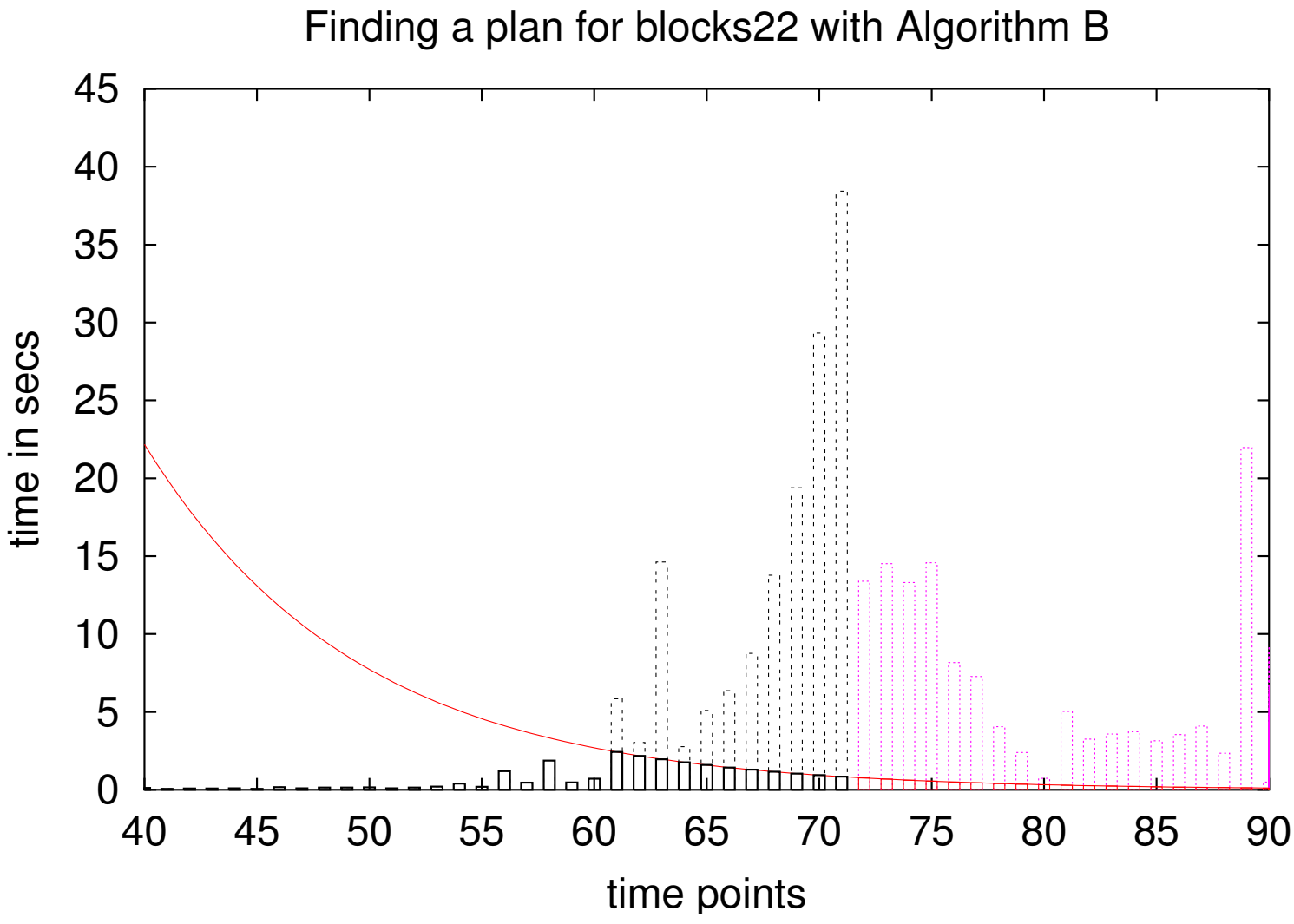
Running SAT Solvers in Parallel



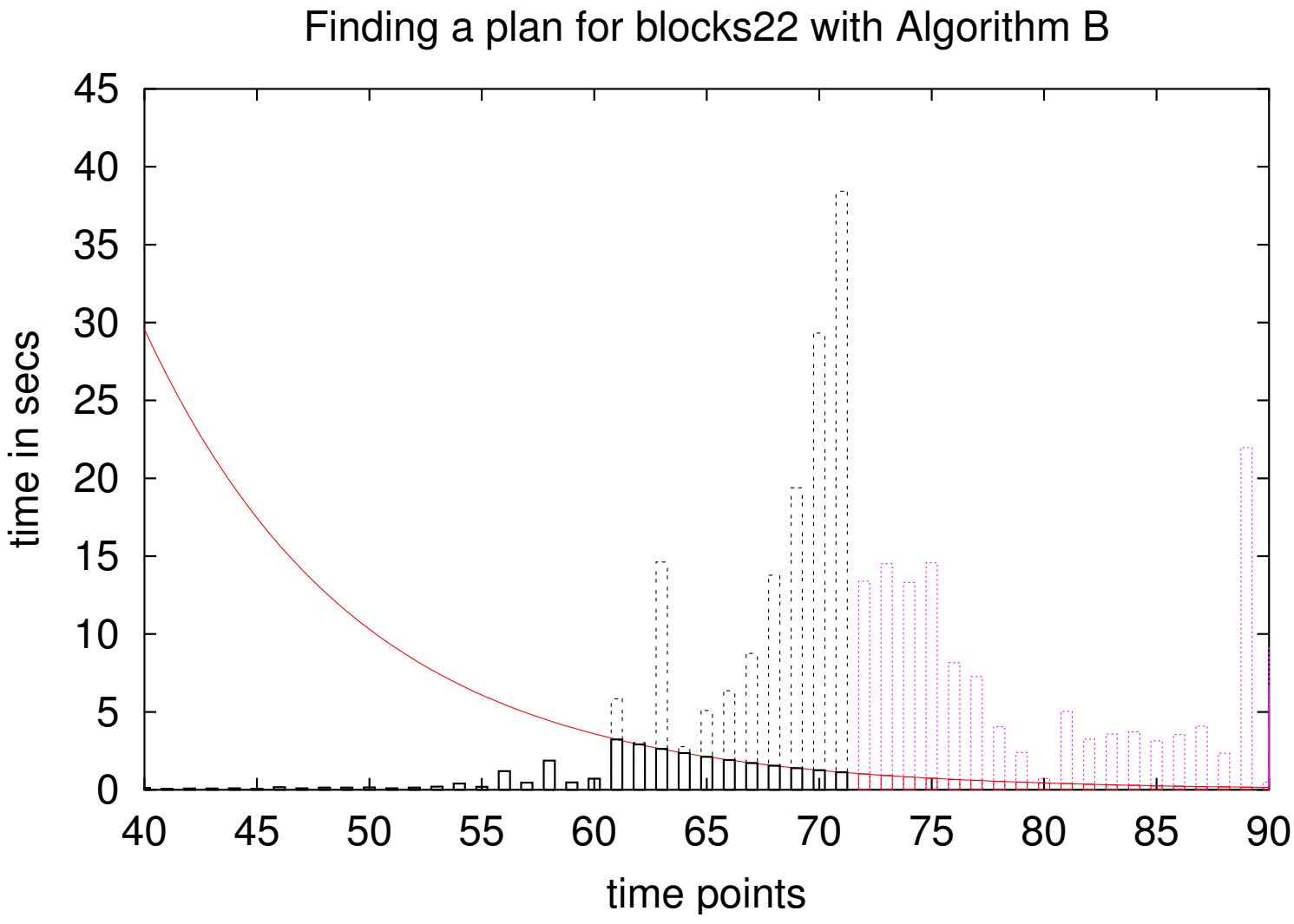
Running SAT Solvers in Parallel



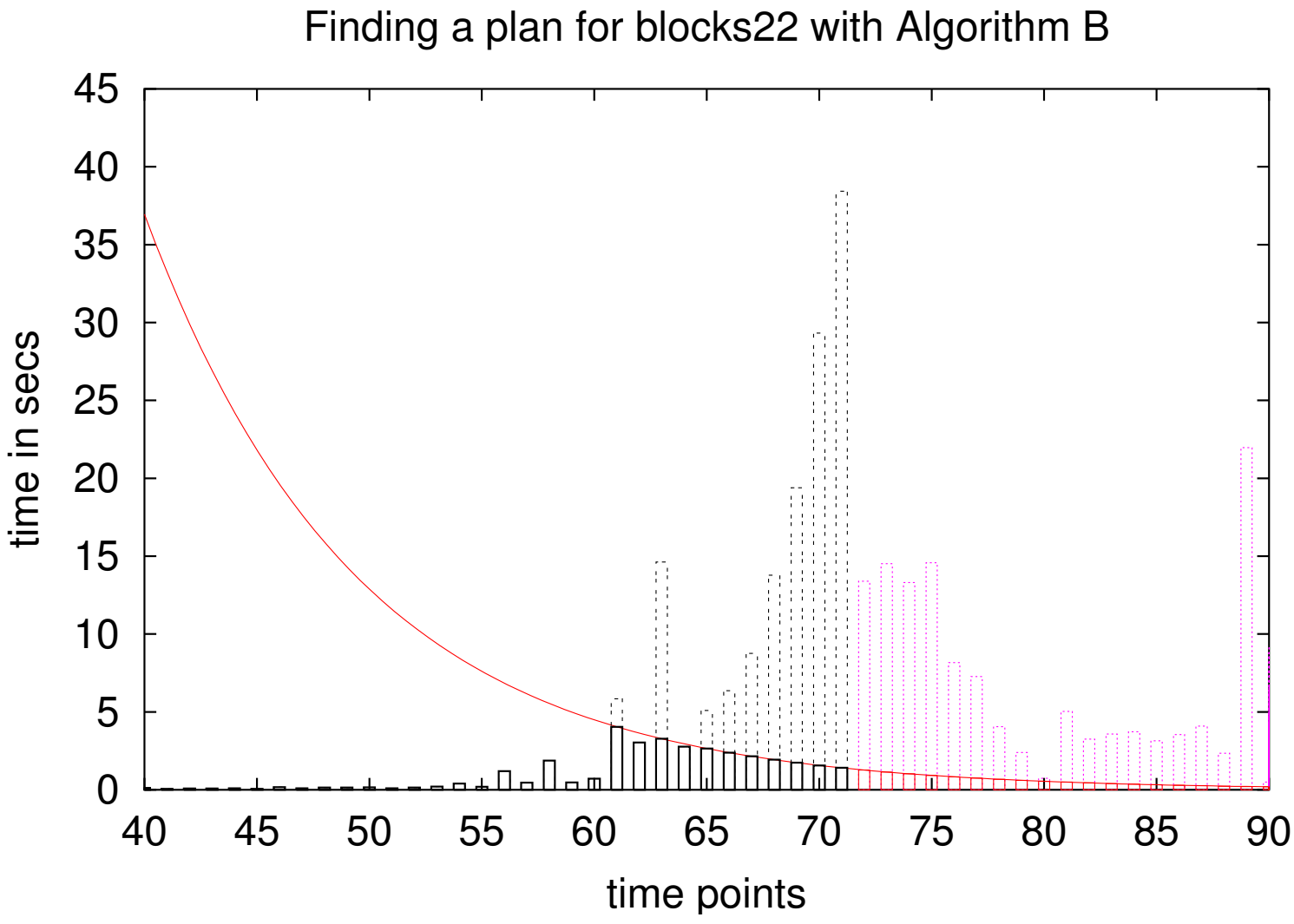
Running SAT Solvers in Parallel



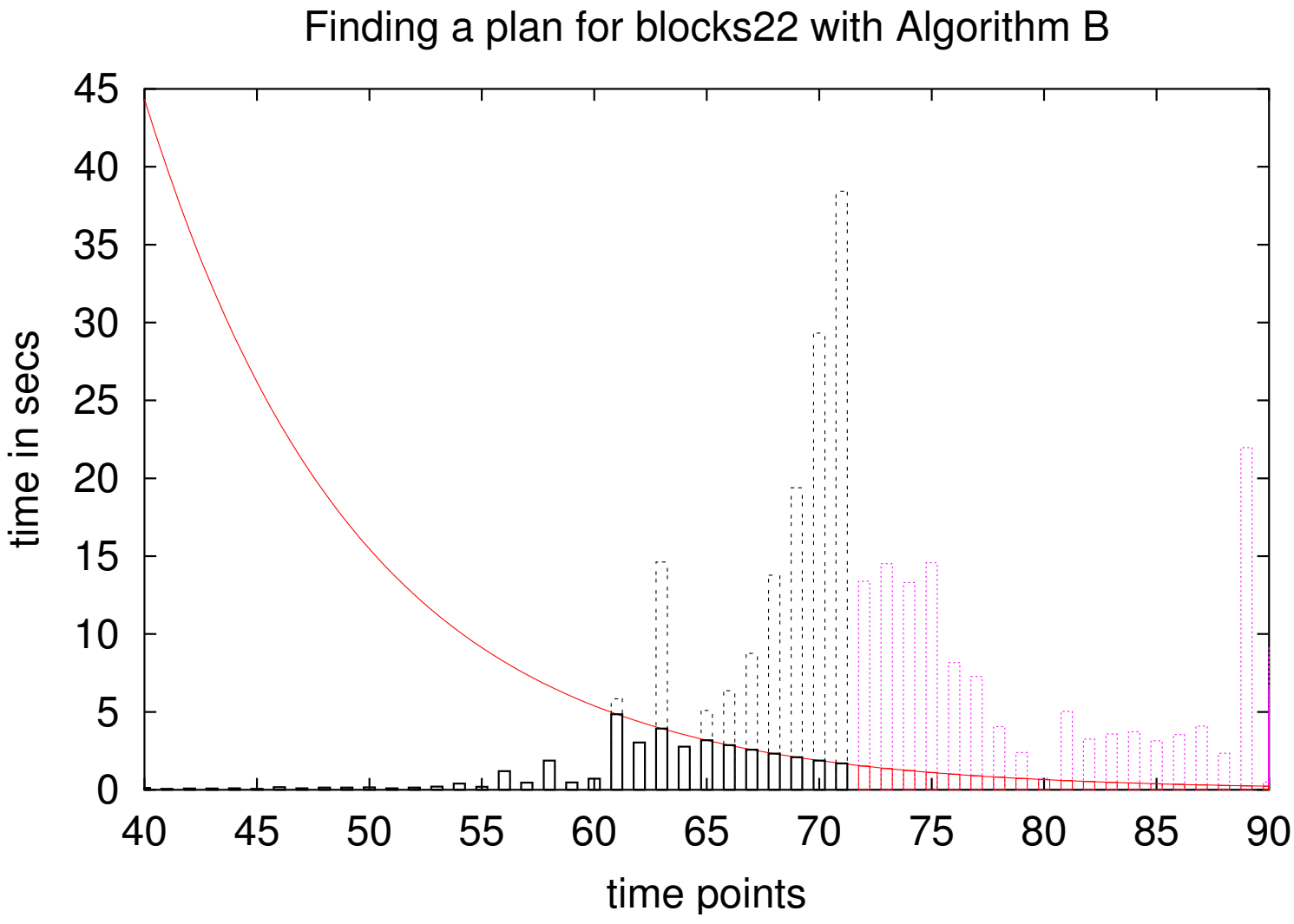
Running SAT Solvers in Parallel



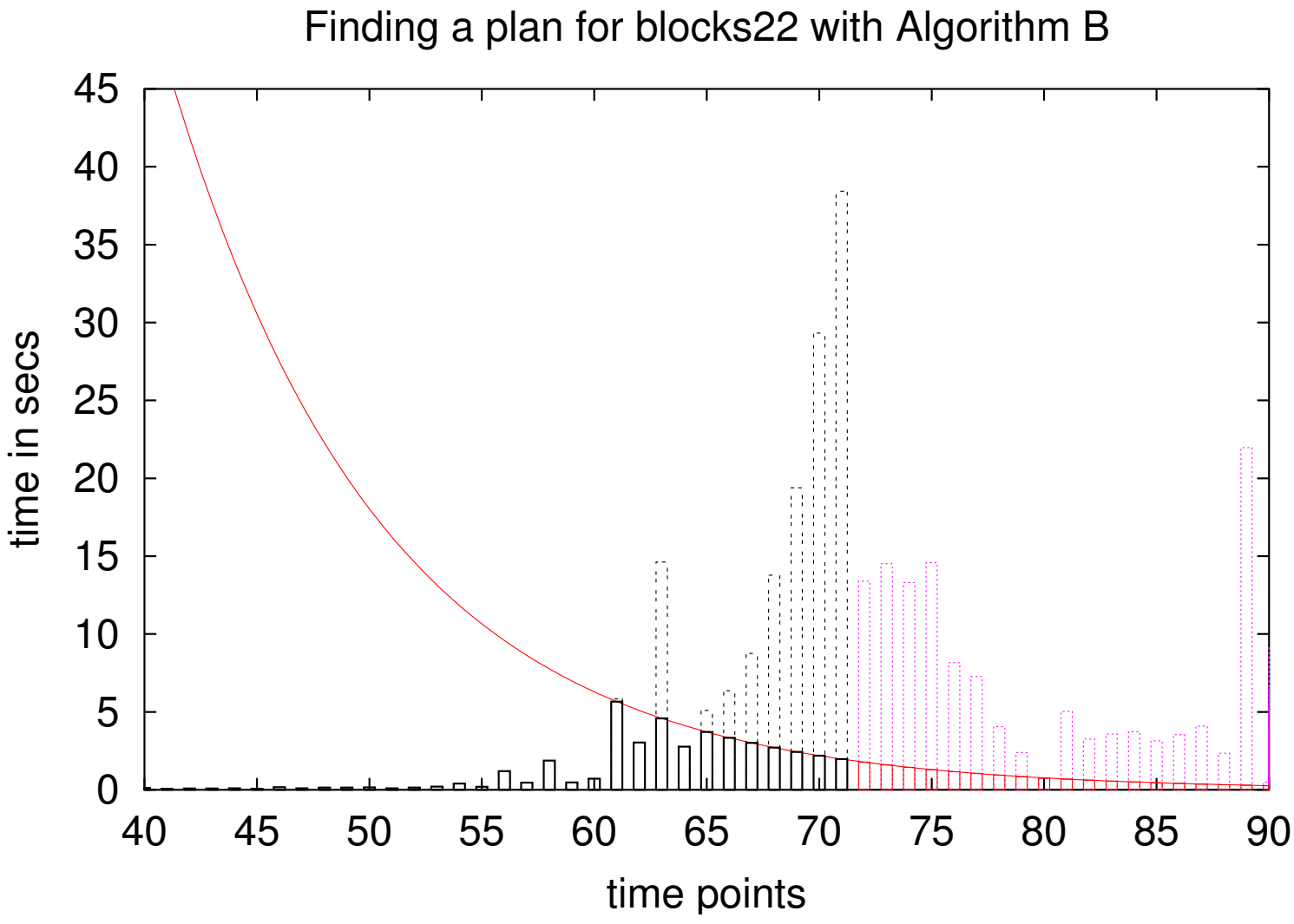
Running SAT Solvers in Parallel



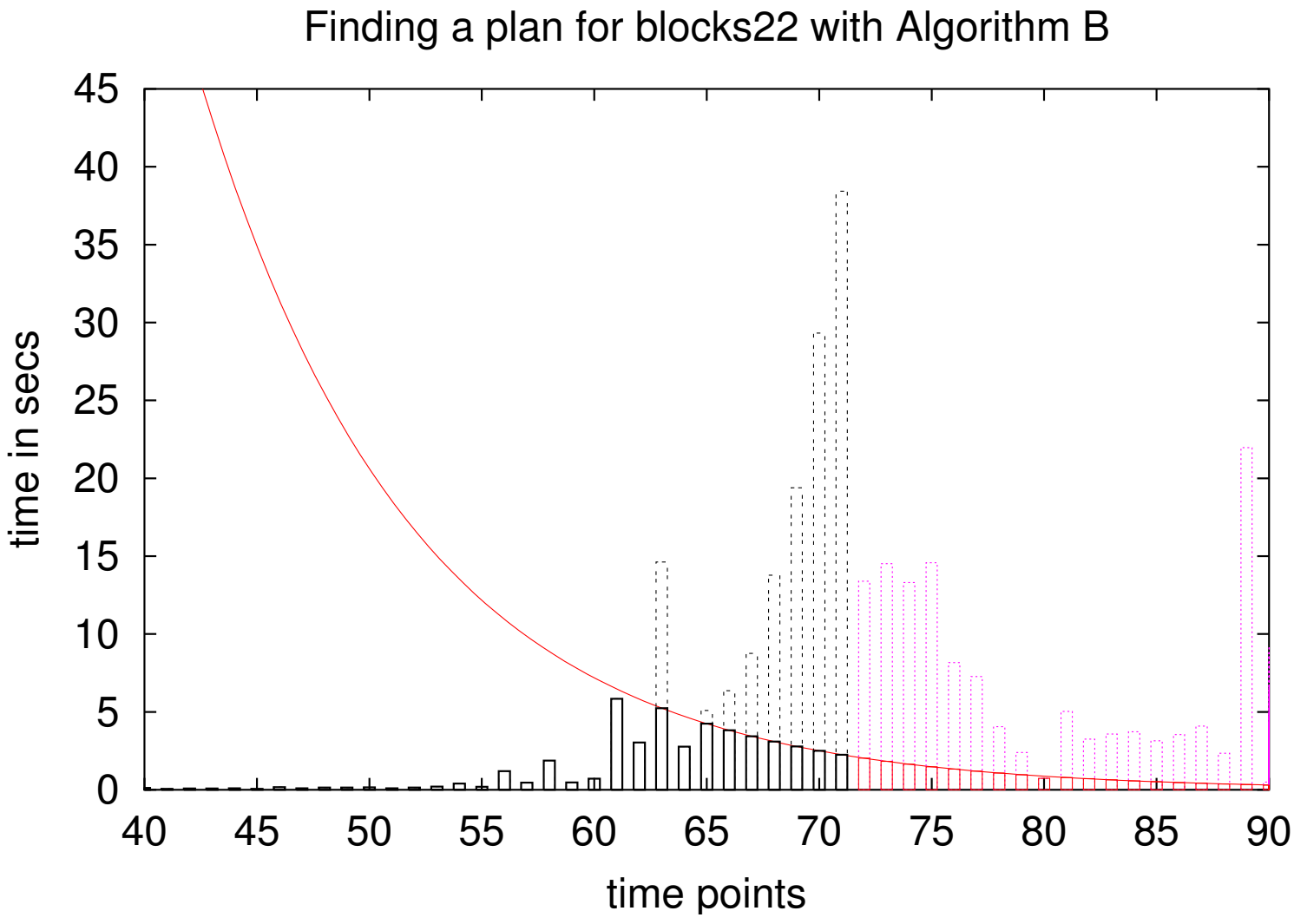
Running SAT Solvers in Parallel



Running SAT Solvers in Parallel



Running SAT Solvers in Parallel



Reachability by Satisfiability

This was a break-through application for satisfiability testing in AI and other areas of computer science:

- planning, control in AI (Kautz & Selman 1996)
- sequential circuit verification (Bounded Model Checking) (Biere et al. 1999)
- software verification (Clarke et al. 2005)
- many other applications

An important part of this development has been the fast progress in algorithms for satisfiability testing. (*ICS-E5050 Advanced Course in Boolean Satisfiability*)