CS-E4800 Artificial Intelligence

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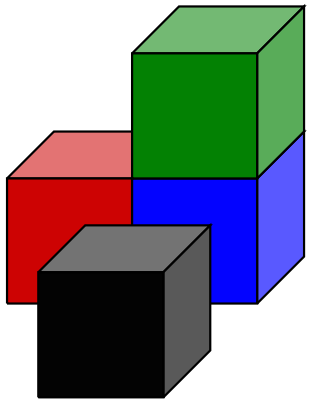
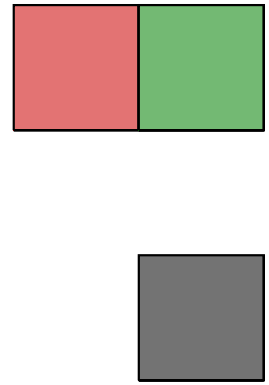
What difference does observability make?

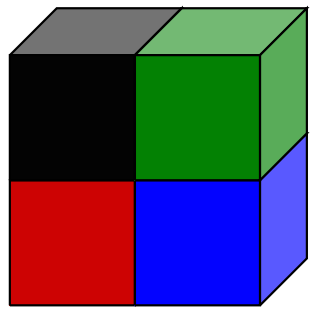
Camera A



Camera B



Goal

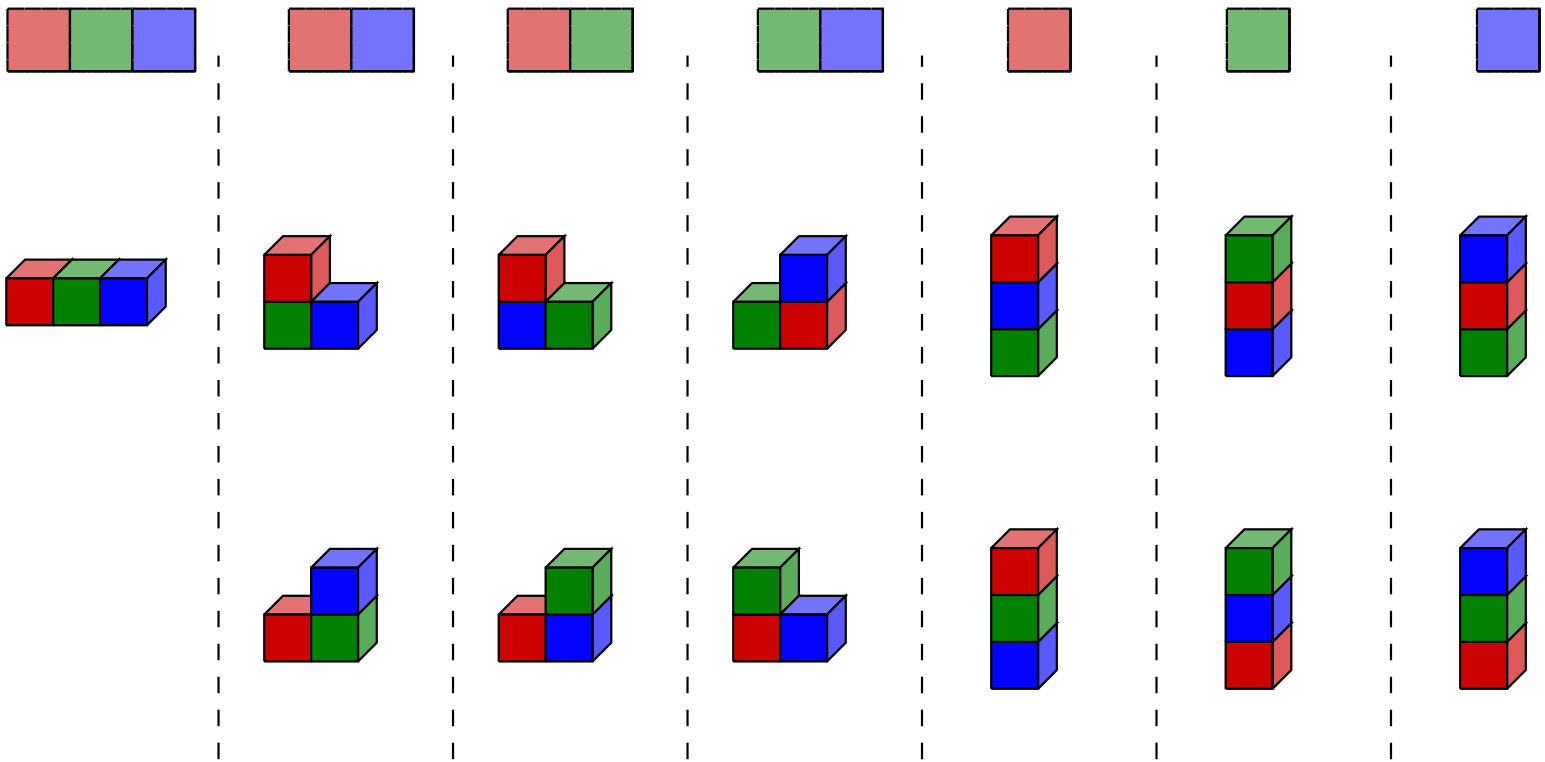


Partial Observability

- Not all features of the state can be observed
  $\Rightarrow$ Cannot determine current state unambiguously
  $\Rightarrow$ Many possible current states
- Sensors incomplete, imprecise, not enough sensors
- Games:
  - Mastermind (1970ies code-breaking game)
  - Card games: Poker, Bridge
- Memory (= remembering observation history) becomes necessary (think about Chess vs. Poker)

Observability

Deterministic observations partition the state space



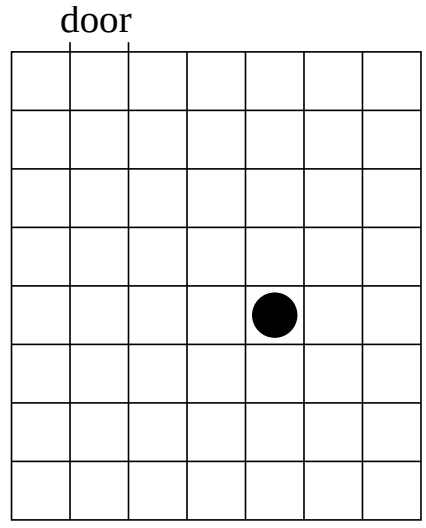
Belief State

- Belief State
  - with probabilities: probability distribution over states (discussed later)
  - without probabilities: set of possible current states
- belief state of size 1: state known unambiguously
- large belief state $\rightarrow$ state very incompletely known
- small belief state $\rightarrow$ state known more accurately
- observations can reduce the size
- nondeterministic actions can increase or reduce
- deterministic actions can reduce

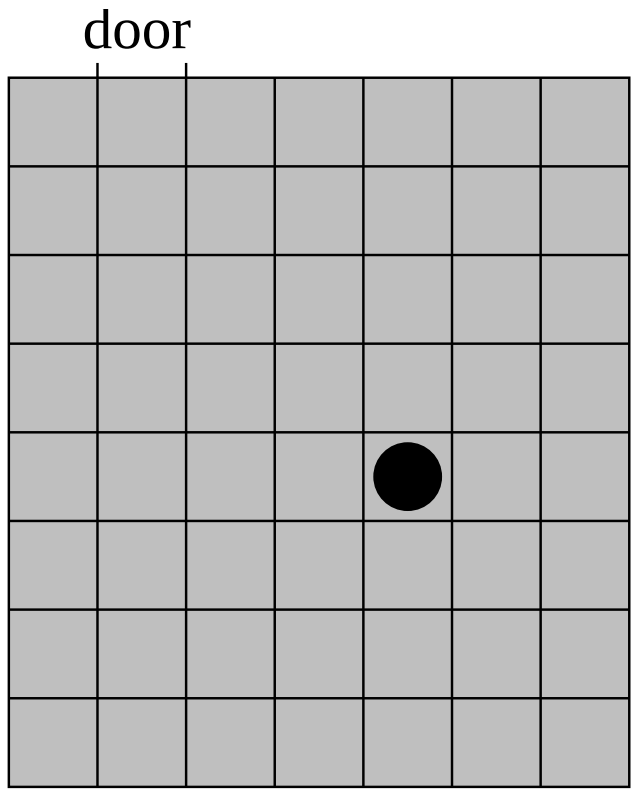
The belief space

Example

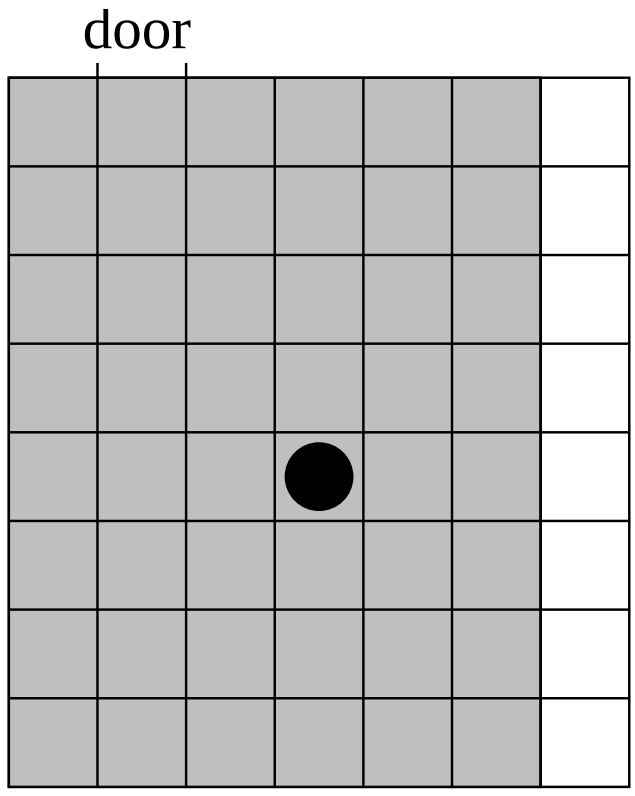
- Robot without sensors, in a 7×8 room, trying to exit
- Position unknown, orientation known
- Actions: North, South, East, West
- Next slides: *One possible* location of the robot ($\bullet$) and the change in the belief state at every execution step.



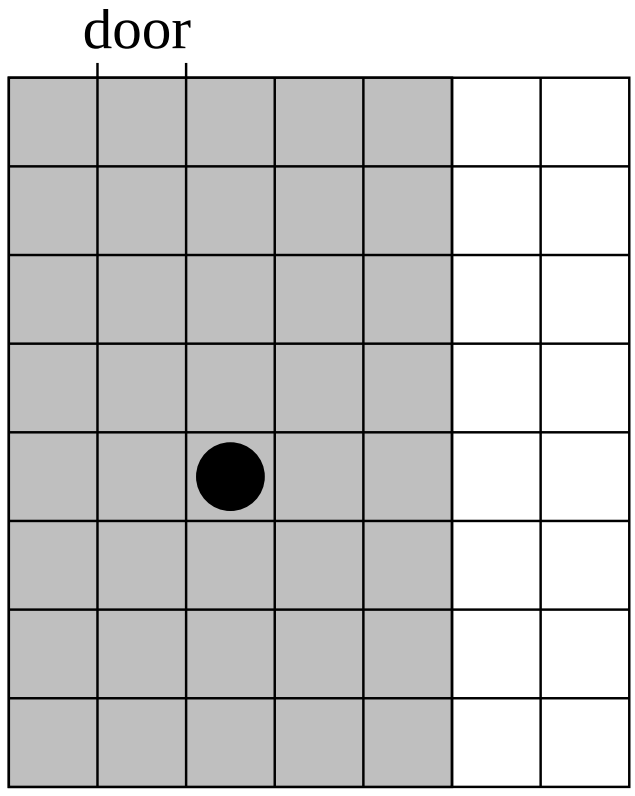
Example: belief state initially



Example: belief state after W



Example: belief state after WW



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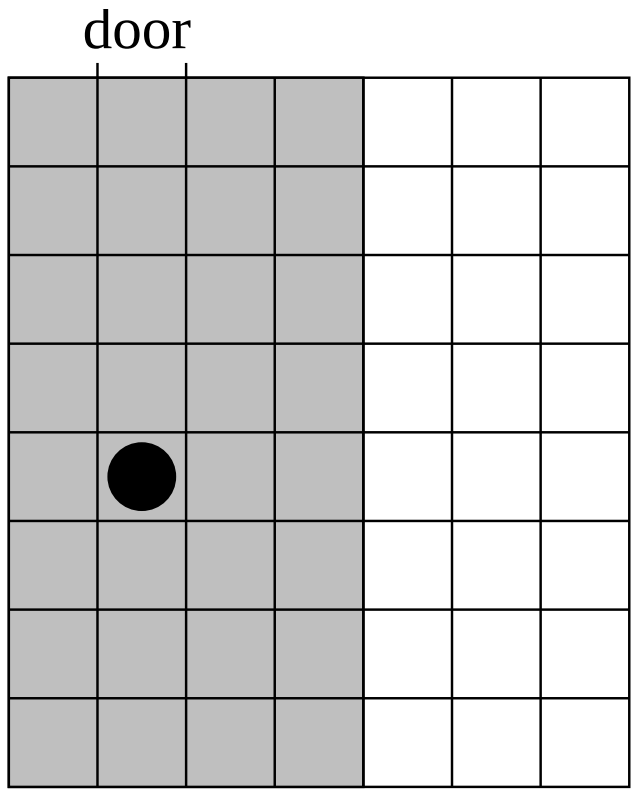
Example: belief state after WWW



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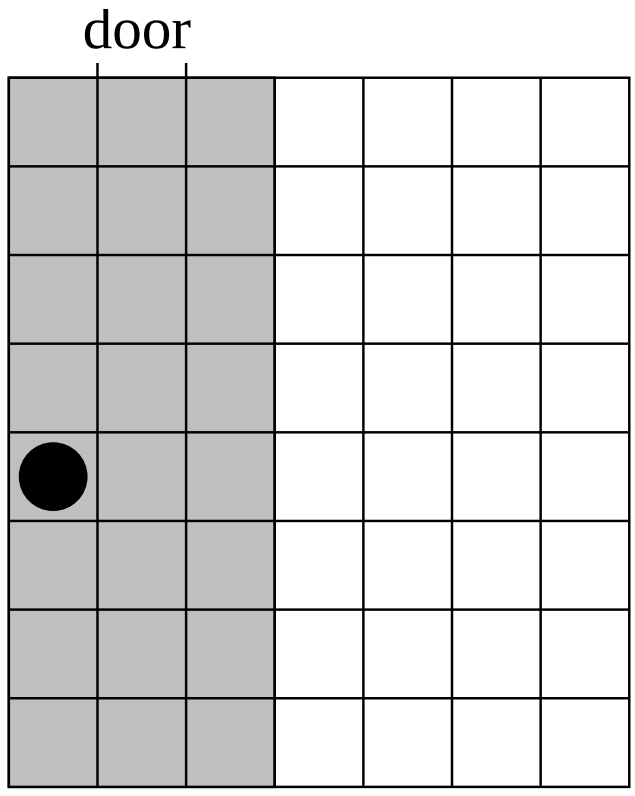
Example: WWWW



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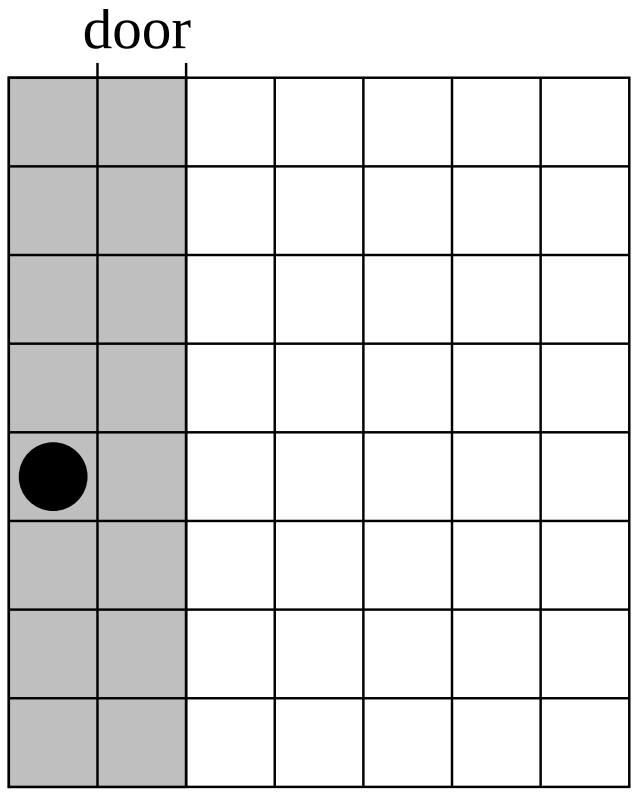
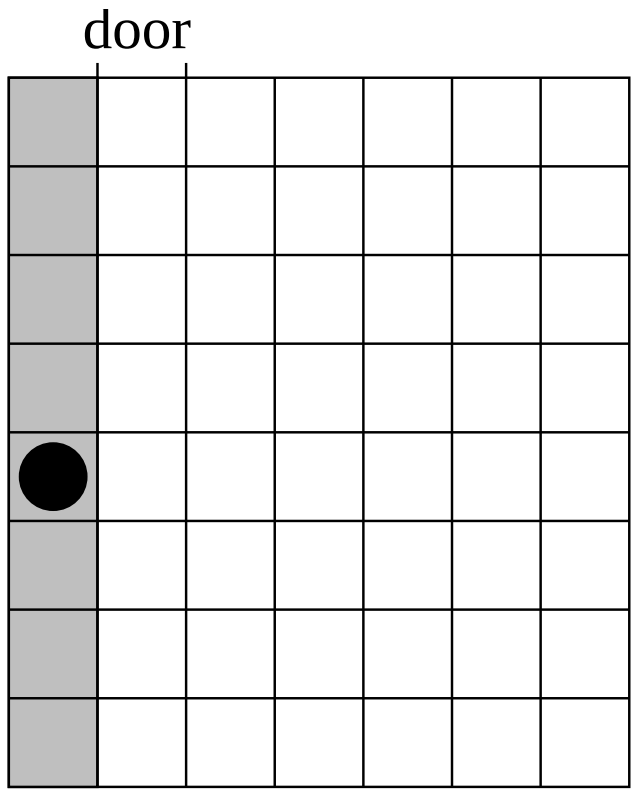
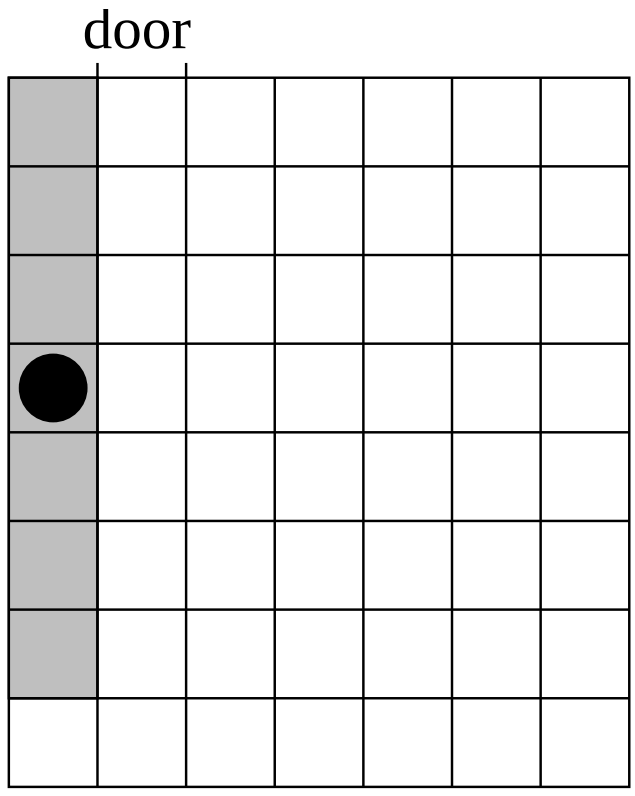
Example: WWWWW



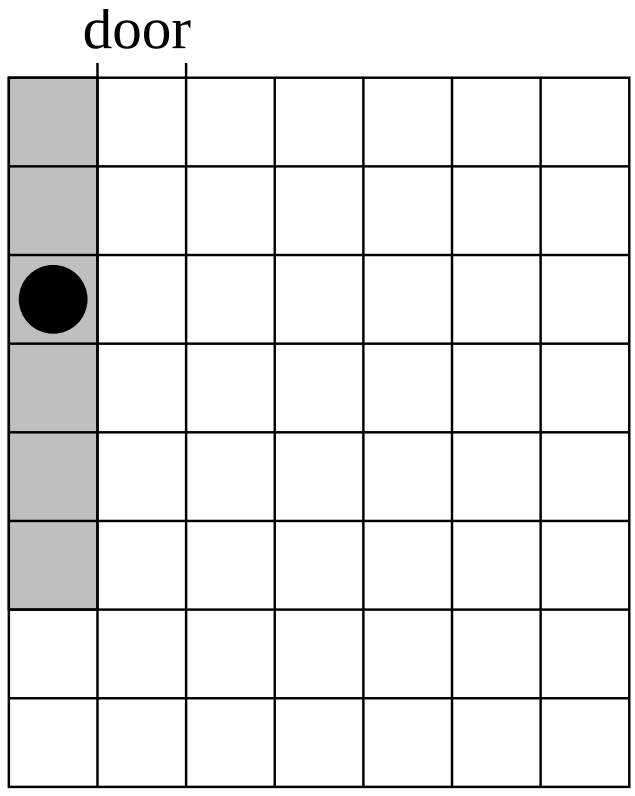
Example: WWWWWW



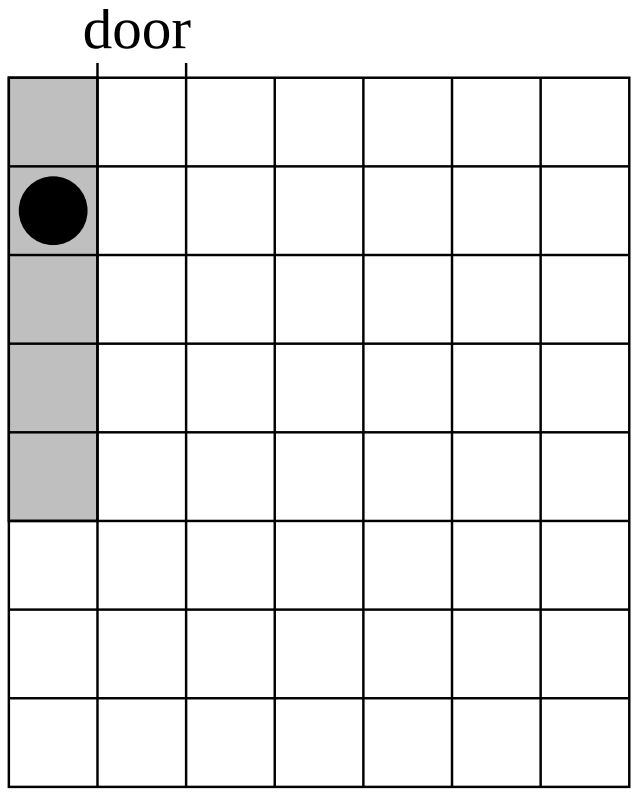
Example: WWWWWWN



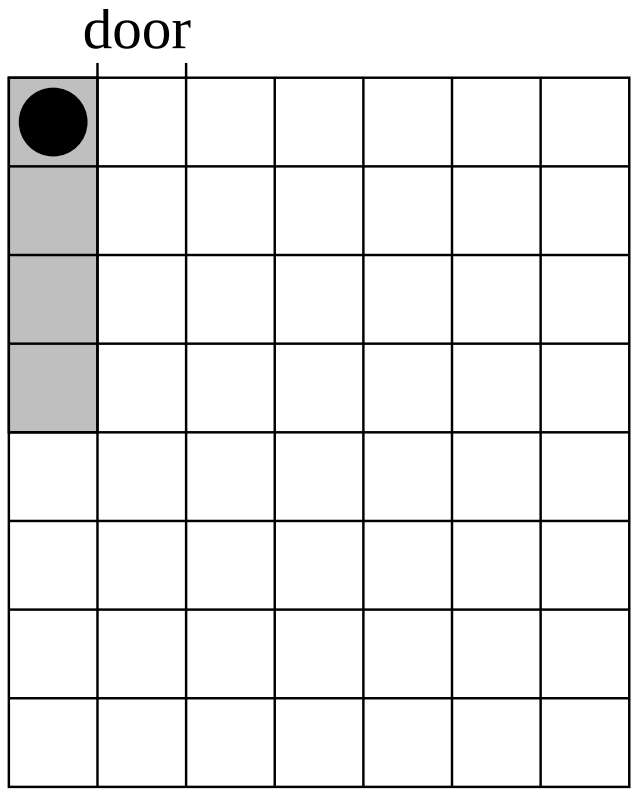
Example: WWWWWWNN



Example: WWWWWWNNN



Example: WWWWWWNNNN



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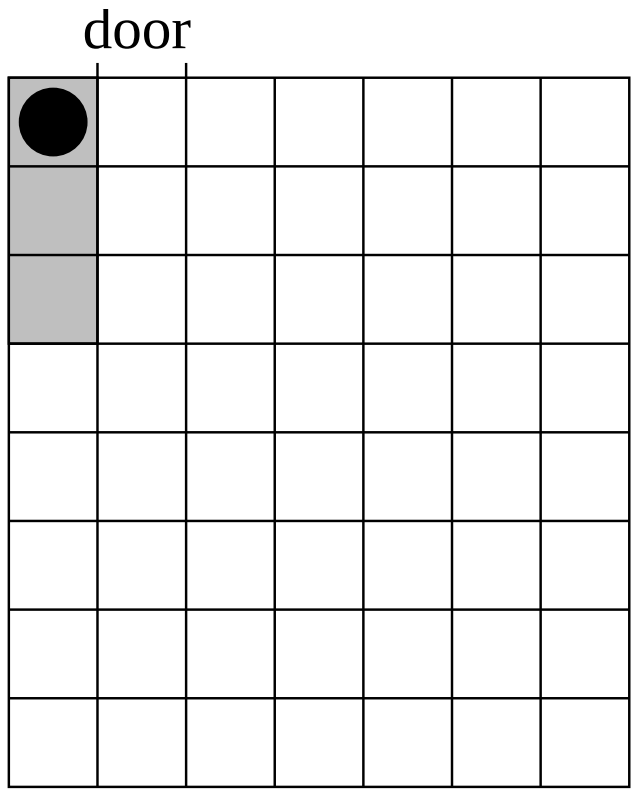
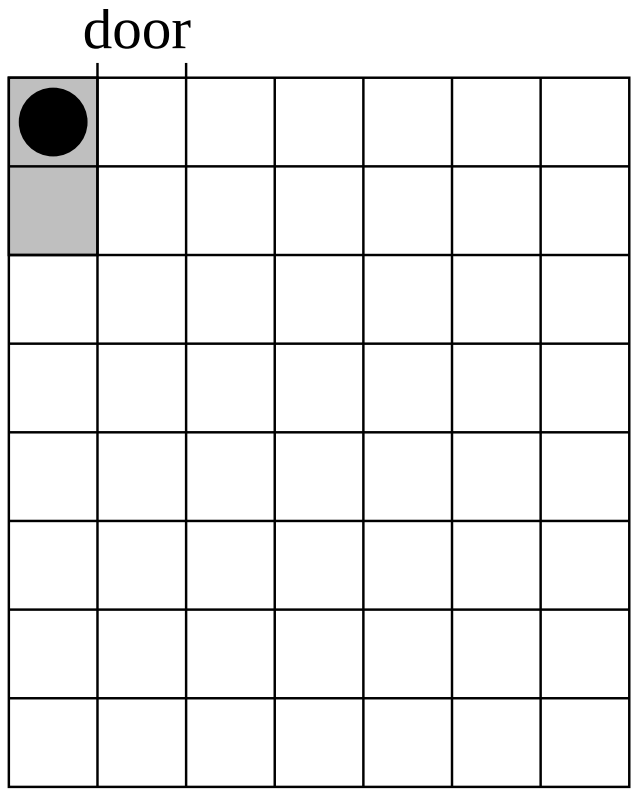
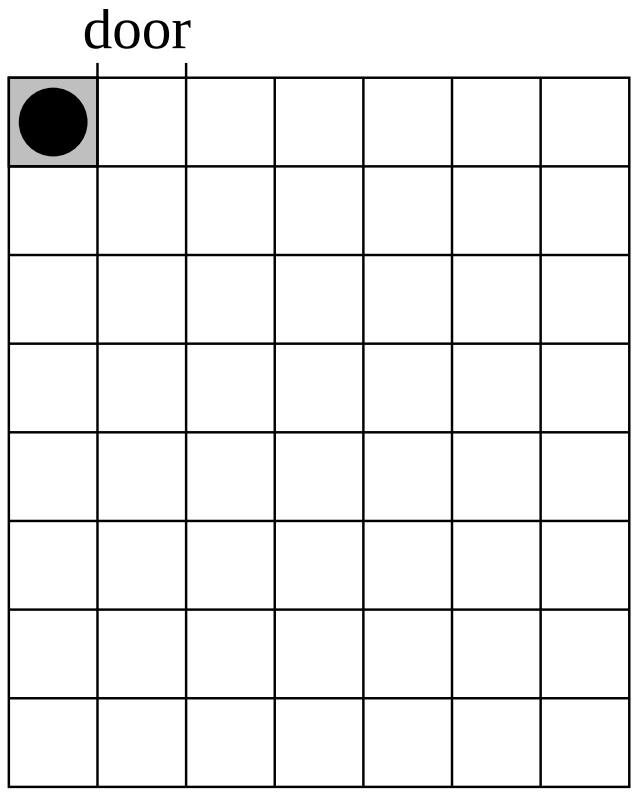
Example: WWWWWWNNNNN



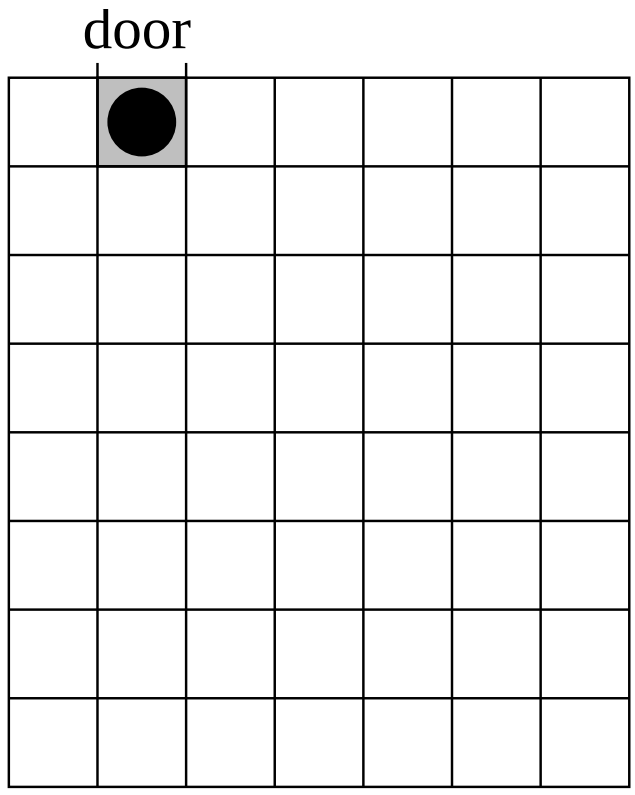
Example: WWWWWWNNNNNN



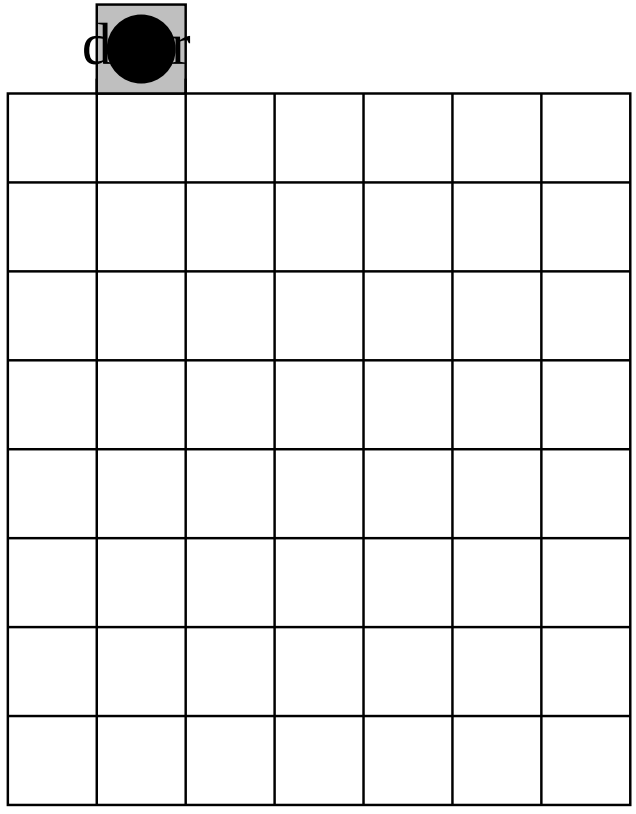
Example: WWWWWWWNNNNNNN



Example: WWWWWWNNNNNNNE



Example: WWWWWWNNNNNNNEN



The belief space

Example

Example

Belief space over state space $\{00, 01, 10, 11\}$

state variables	states	belief states
$n = 2$	$2^n = 4$	$2^{2^n} = 16$

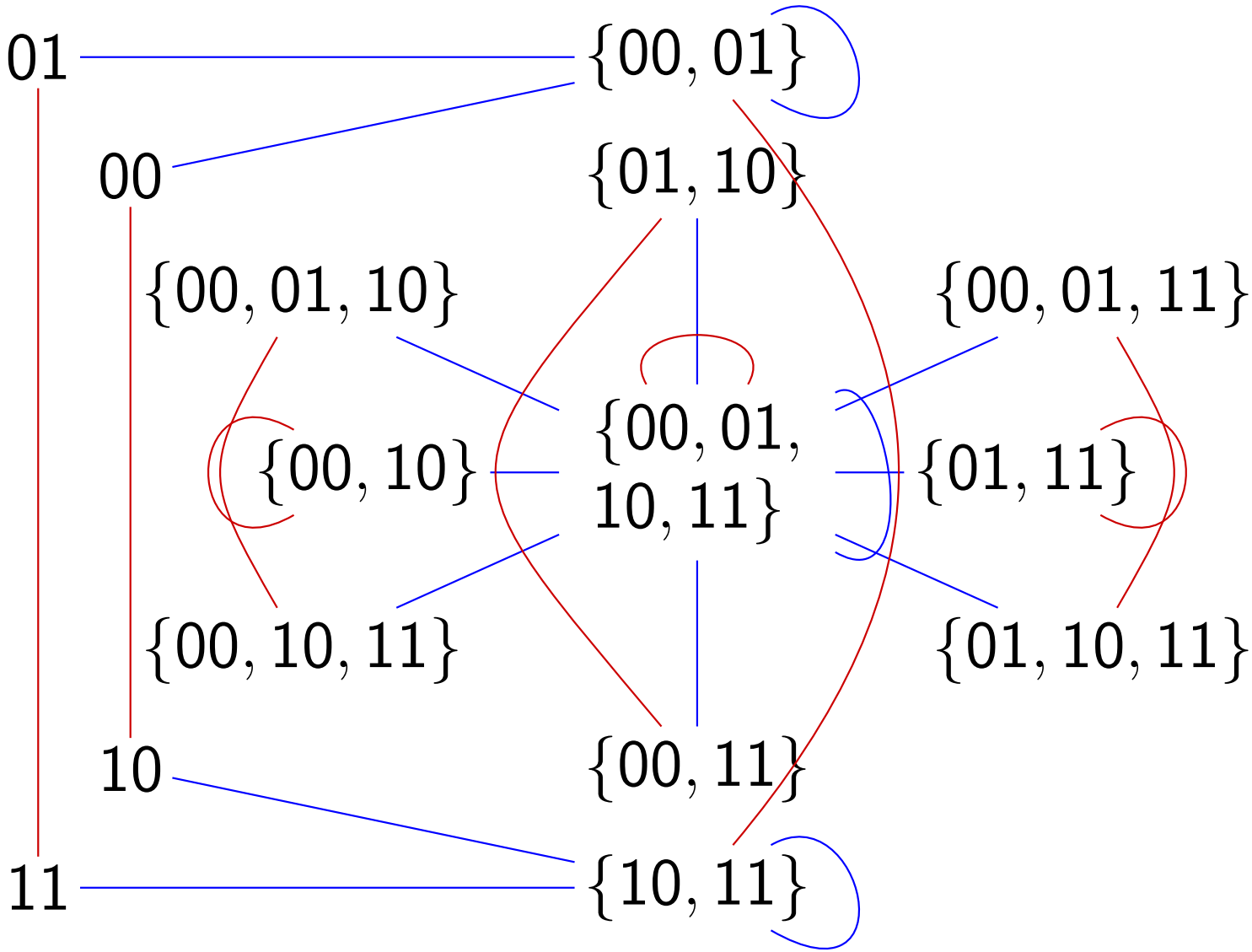
Next slide:

red action: the complement 1st bit

blue action: assign a random value to the 2nd bit

The belief space

Example



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Algorithms

Without observations, plans are sequences of actions from the initial belief state to a belief state included in the goals

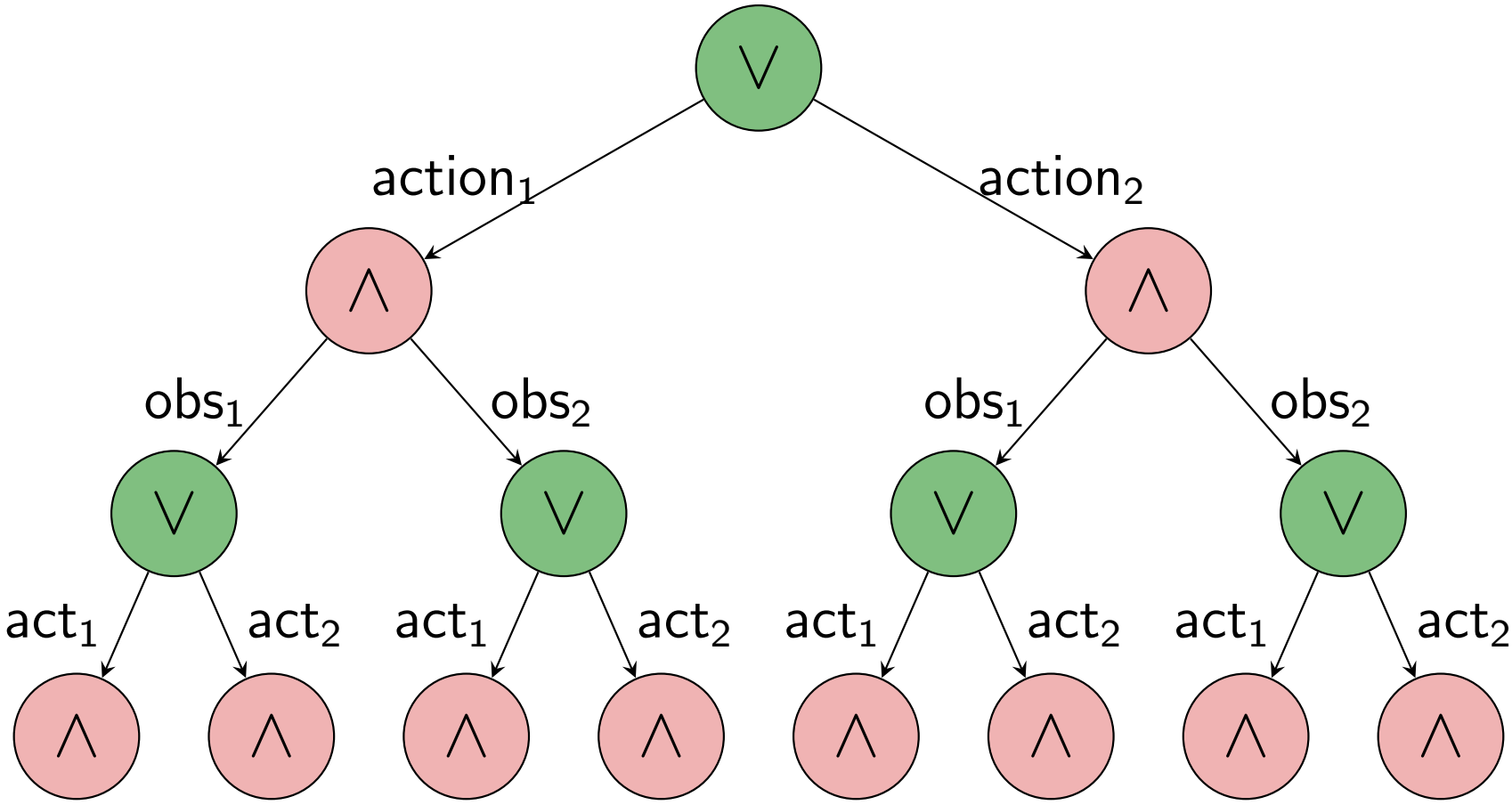
- Belief states represented as formulas, BDDs, ...
- Standard search algorithms (A$*$, Greedy BFS, ...)
- Belief space very large; limits applicability of these algorithms with belief states

(Special case: deterministic actions, one initial state. This is the case covered in the first lectures!)

Conditional Plans and AND-OR Trees

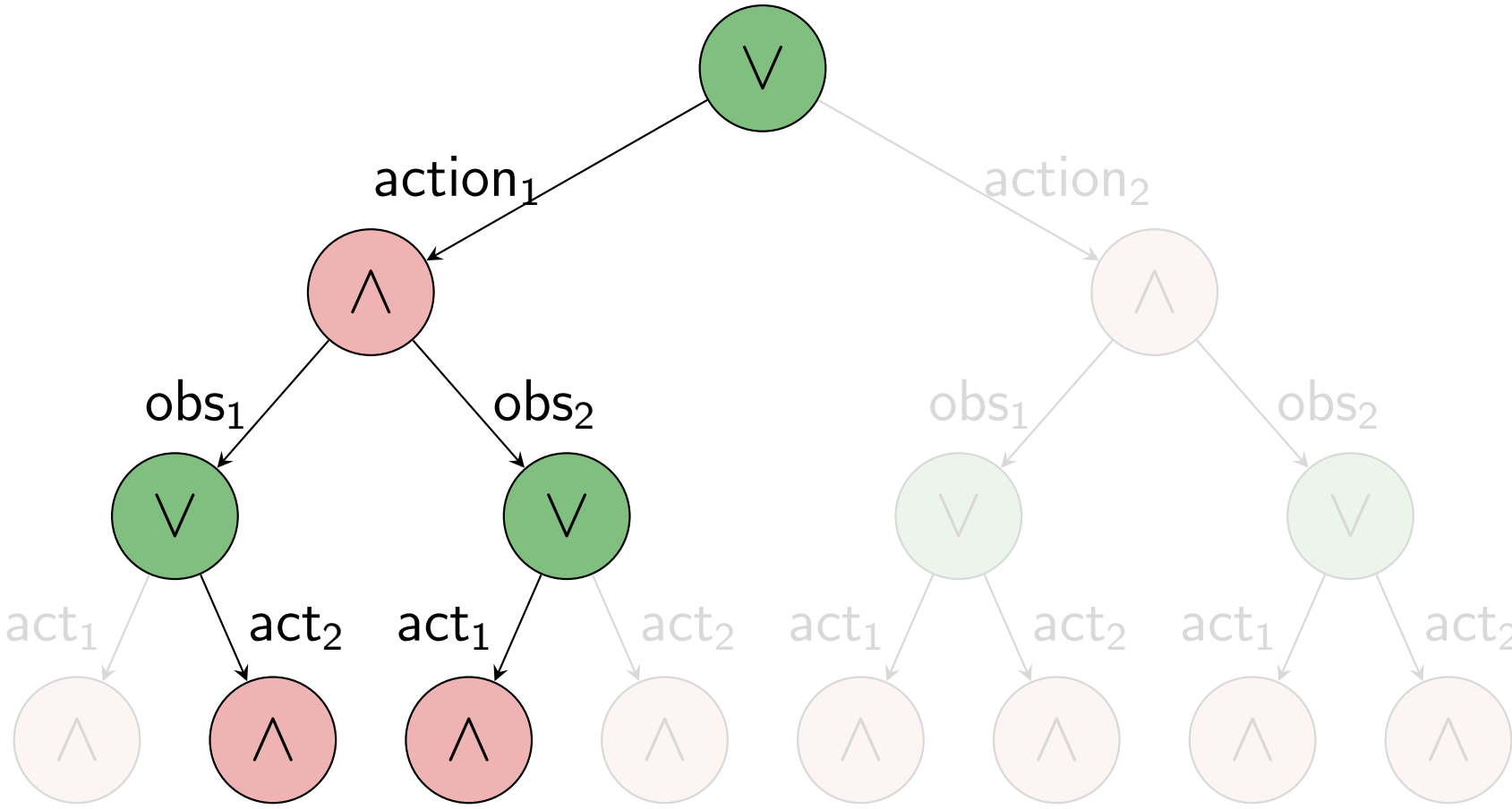
- Possible behaviors under nondeterminism and sensing/observations
- AND-node: Uncontrollable events (nondeterminism, observation)
- OR-node: Controllable events (actions)
- A conditional plan represent one possible action at each (reachable) OR-node
- AND-OR tree $\sim$ game tree (next lecture)

AND-OR Trees



action$_1$; IF obs$_1$ THEN (action$_2$;...) ELSE (action$_1$; ...)

AND-OR Trees and Conditional Plans



action$_1$; IF obs$_1$ THEN (action$_2$;...) ELSE (action$_1$; ...)

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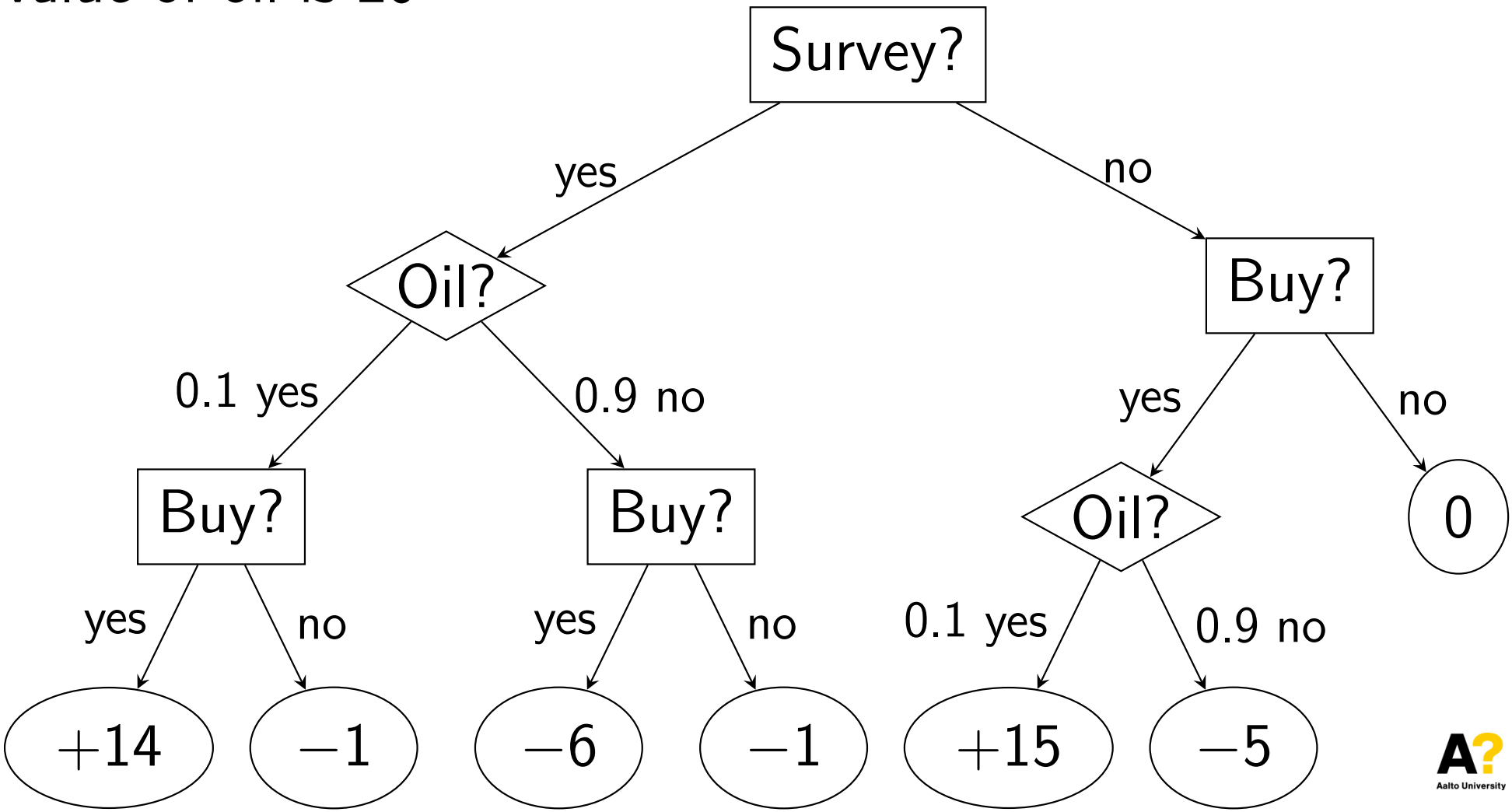
Mastermind

- Code: sequence of 4 colors
- The code is hidden from the player
- Player tries to guess the code
- Response to each guess: 0 to 4 of
  - correct color, correct position
  - correct color, wrong position
- Solution by AND-OR search in discrete belief space

- Critical: heuristics for choosing the guesses
- Can always be solved with max. 5 moves (but is difficult)

Expectimax Trees

survey action: cost 1
block: cost 5
value of oil is 20



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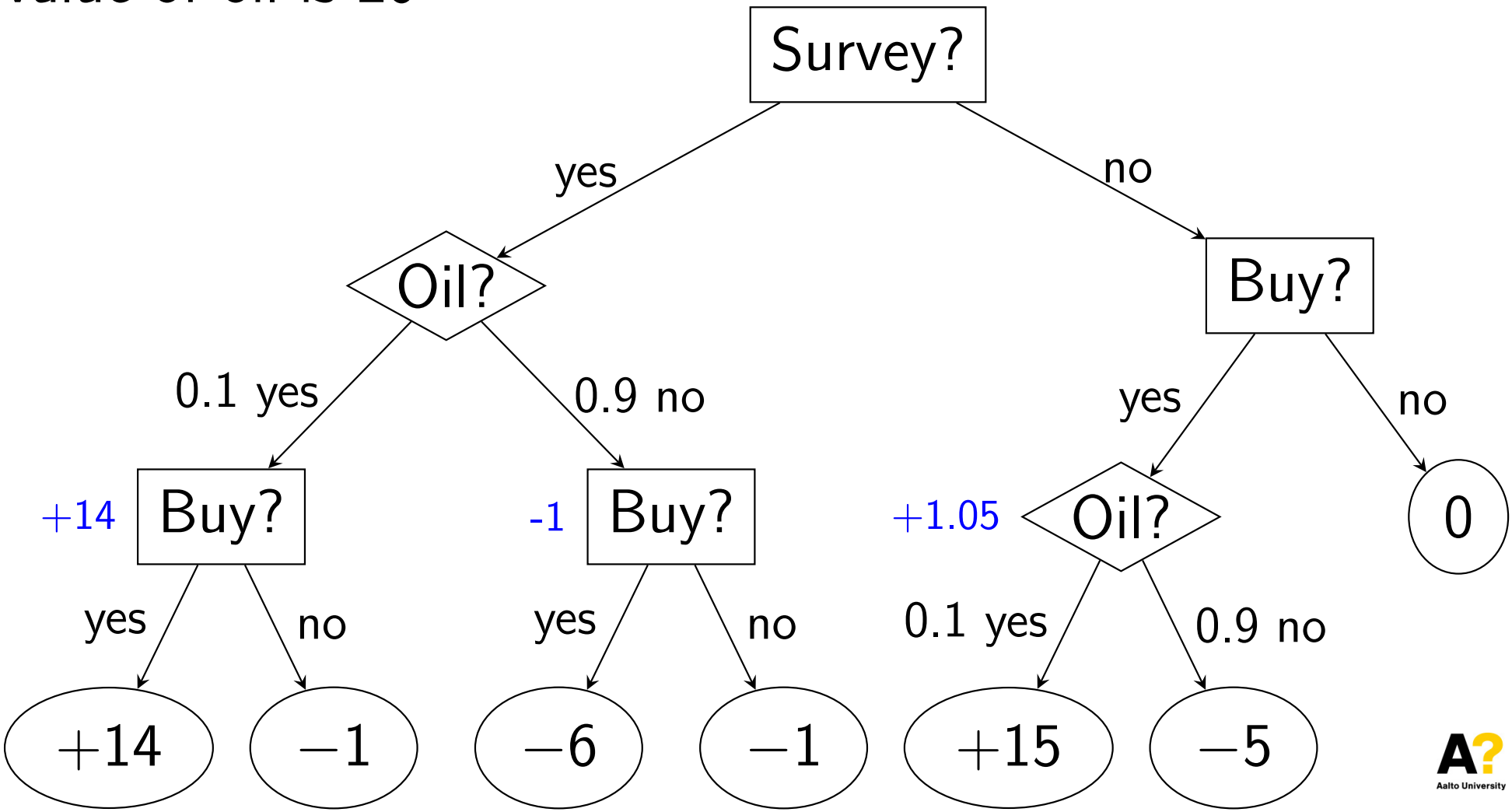

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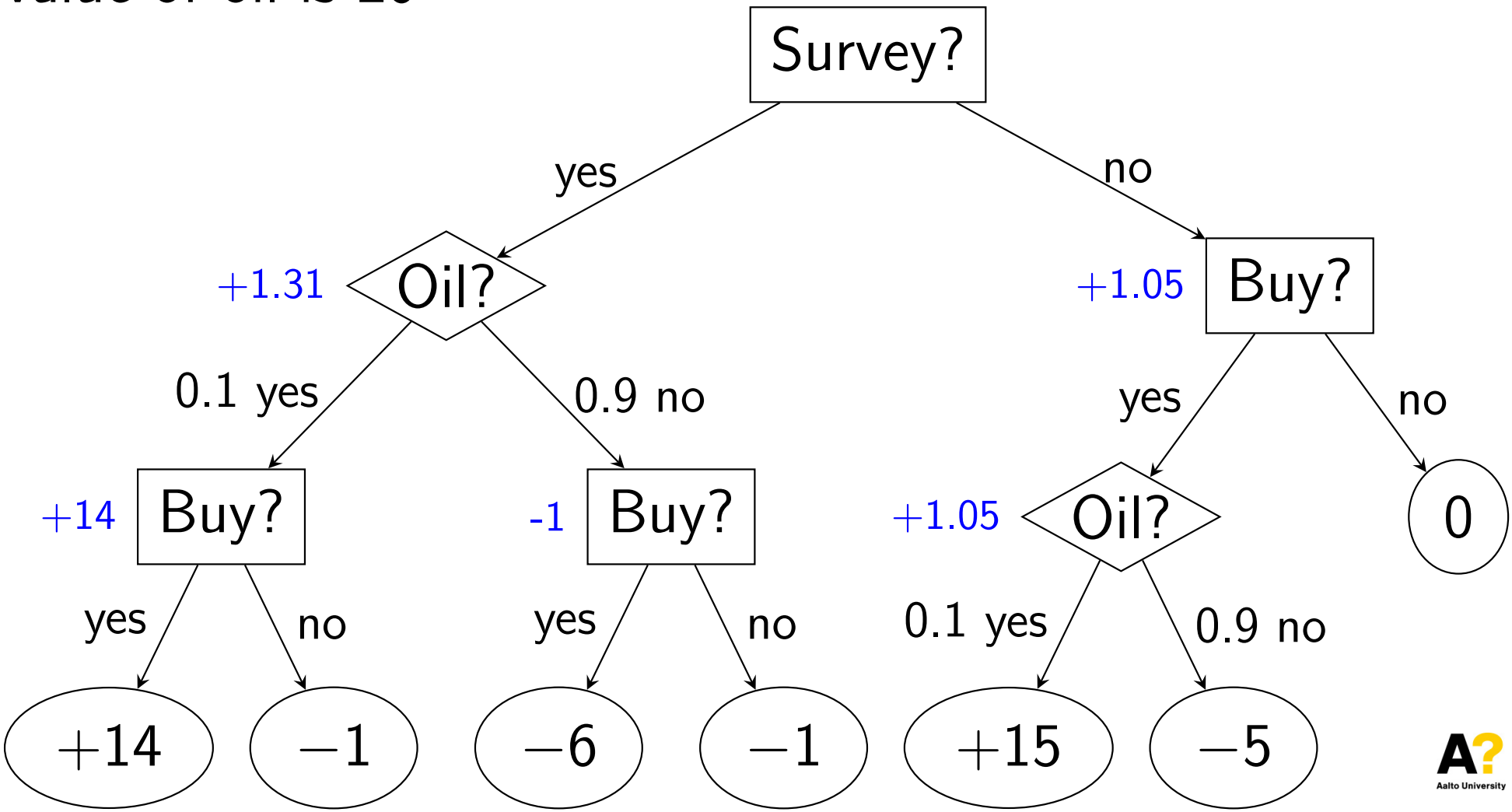

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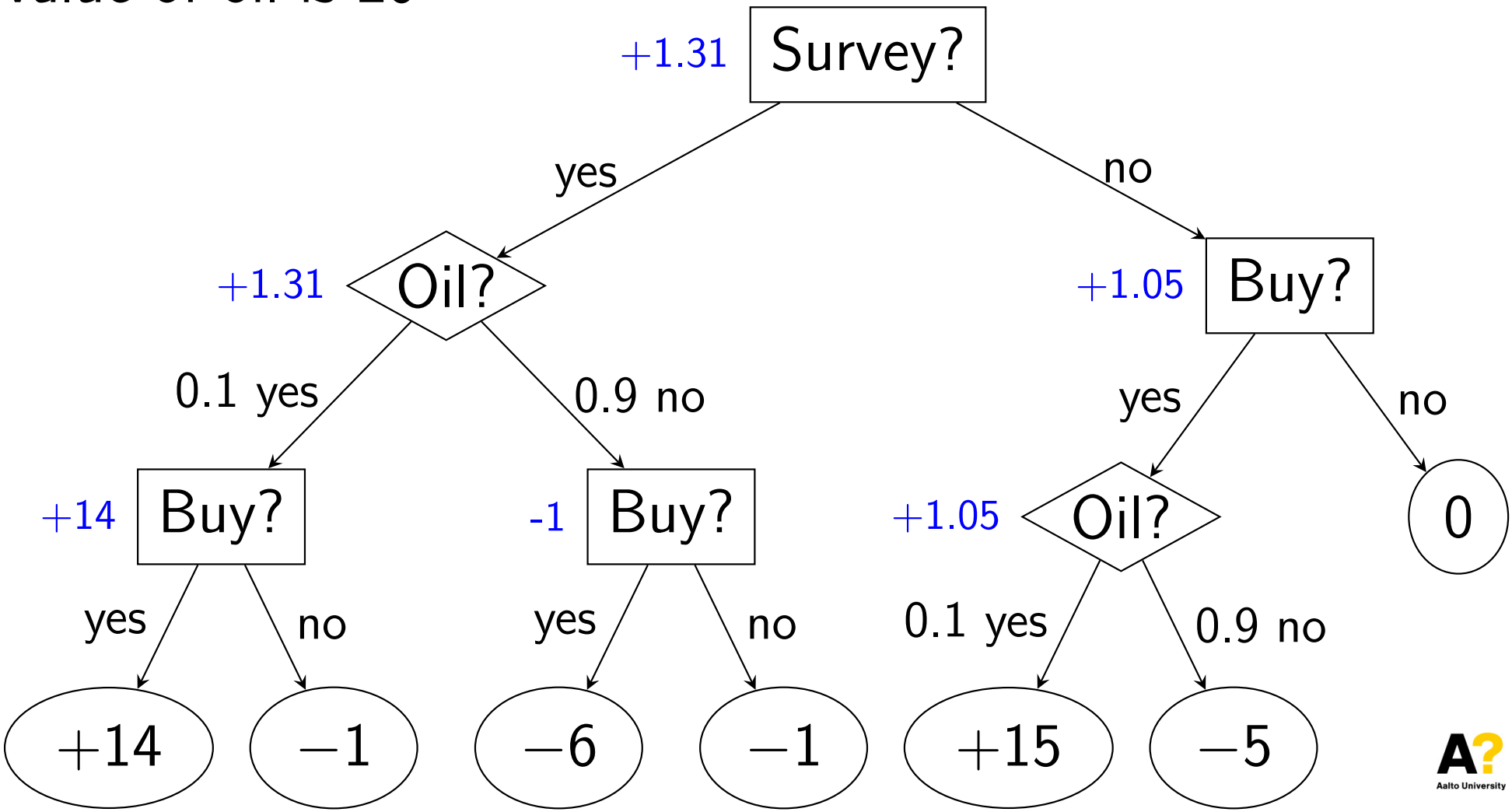

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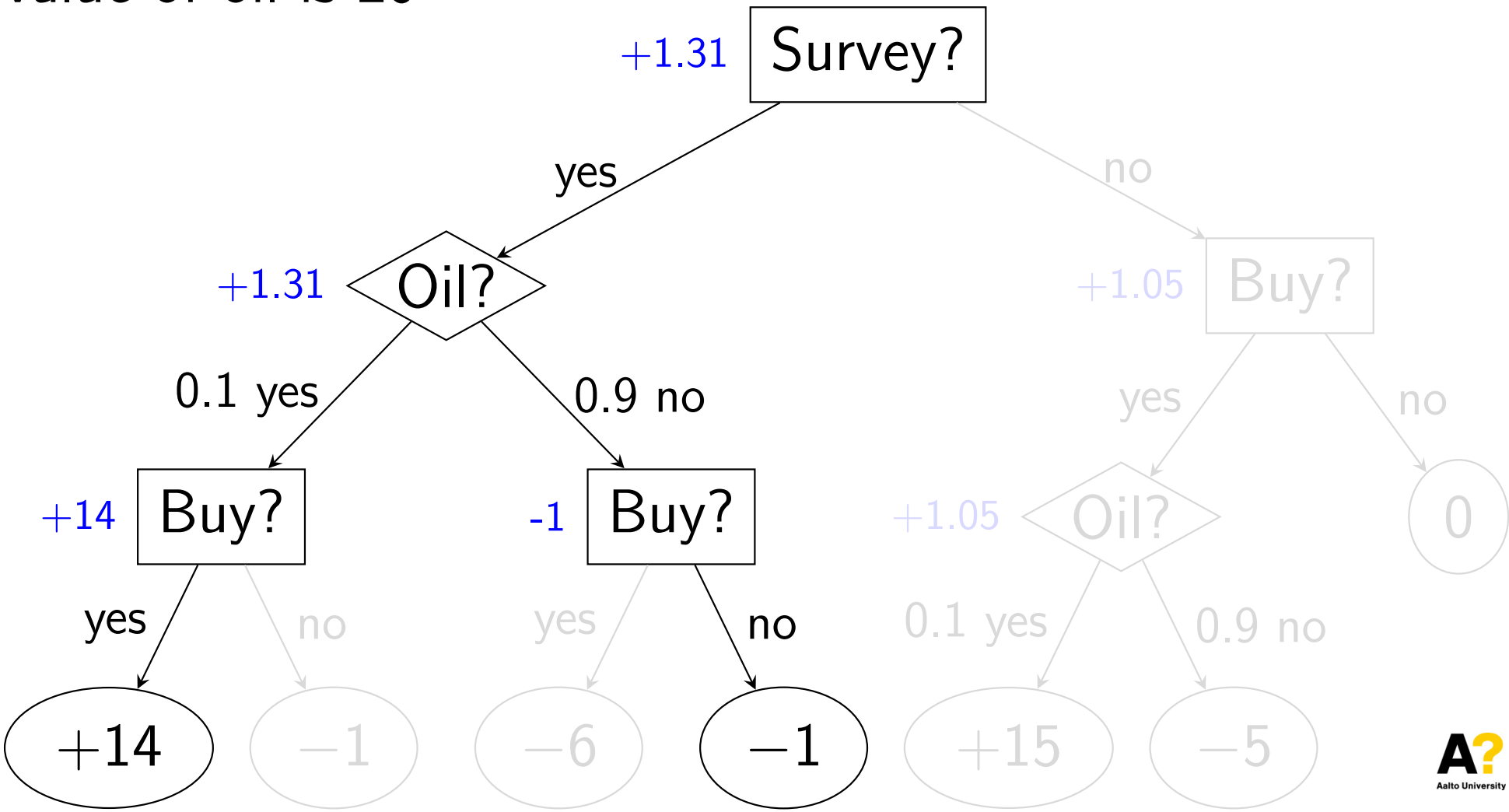

Expectimax Trees

- decision/action node: maximum

$$\max_{c \in children} v(c)$$

- chance/observation node: expected value

$$\sum_{c \in children} P(c) \times v(c)$$

- solution tree = prune all but best child of action nodes

Partial Observability and Sensing

- Most applications include sensing / observations
- Sensing (observations) allows reducing size of belief state
  - sensing $\rightarrow$ alternative belief states
- In many application, probabilities of states critical
  $\Rightarrow$ extensions of probabilistic methods like MDPs

Observations + Discrete Belief States

Deterministic observations corresponds to sets of states (those states where the observation can be made.)

Belief State update with an Observation

An observation $O \subseteq S$ updates a belief state $B \subseteq S$ to

$$B' = B \cap O$$

Example

For belief B $= \{$ Sunday, Monday, Tuesday $\}$ the observation "weekend" $O = \{$ Saturday, Sunday $\}$ yields

$$B' = B \cap O = \{\text{Sunday}\}$$

Observations + Probabilistic Belief States

Instead of set of possible states, a belief is probability distribution $B(S)$ over states.

Belief State update with an Observation

An observation O updates $B(S)$ to $B'(S)$ according to Bayes Rule:

$$B'(s) = P(s|O) = \frac{P(O|s)B(s)}{\sum_{s_2 \in S} P(O|s_2)B(s_2)} \quad (1)$$

The denominator represents, for belief state B, and before any observations are made, the probability that observation O will be made.

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Observations + Probabilistic Belief States

Example

$$B(s_0) = 0.01$$
$$B(s_1) = 0.99$$
$$P(O|s_0) = 0.1$$
$$P(O|s_1) = 0.0$$
$$P(s_0|O) = \frac{0.1 \cdot 0.01}{0.1 \cdot 0.01 + 0.0 * 0.99} = 1.0$$
$$P(s_1|O) = \frac{0.0 \cdot 0.99}{0.1 \cdot 0.01 + 0.0 * 0.99} = 0.0$$

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Partially Observable MDPs (POMDP)

Definition (POMDP $\langle S, A, P, R, E, O \rangle$)

- S is a (finite) set of states
- A is a (finite) set of actions
- $P : S \times A \times S \to \mathbb{R}$ gives transition probabilities
- $R : S \times A \times S \to \mathbb{R}$ is a reward function
- E is a (finite) set of observations (evidence)
- $O : A \times S \times E \to \mathbb{R}$ gives observation probabilities

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Belief Update for POMDPs

Belief update: action $a \in A$

$$B'(s') = \sum_{s \in S} (P(s, a, s')B(s))$$

Belief update: action $a \in A$, observation $e \in E$

$$B'(s') = \frac{O(a, s', e) \sum_{s \in S} (P(s, a, s')B(s))}{\sum_{s_2 \in S} (O(a, s_2, e) \sum_{s_1 \in S} (P(s_1, a, s_2)B(s_1)))}$$

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Value Iteration for POMDPs

- Problem: Belief space infinite $\Rightarrow$ Size of value function representation unbounded
- Solution:
  - Iteration i: generate conditional plans π of depth i
  - Evaluate value of each plan π for every state $(v_\pi(s_1), \ldots, v_\pi(s_n))$
  - The collection of all these vectors $(v_\pi(s_1), \ldots, v_\pi(s_n))$ represent value function for all possible belief states: The maximum value obtainable for a given belief state by one of the conditional plans

Value Iteration for POMDPS

Example

- Two states s_0 and s_1
- Two actions Stay and Go
- Two observations s_0 and s_1
- $R(s_0) = R(s_0, \cdot, \cdot) = 0$ and $R(s_1) = R(s_1, \cdot, \cdot) = 1$
- $P(s_i, Stay, s_i) = 0.9$ and $P(s_i, Stay, s_{1-i}) = 0.1$
- $P(s_i, Go, s_{1-i}) = 0.9$ and $P(s_i, Go, s_i) = 0.1$
- $O(\cdot, s_i, s_i) = 0.6$ and $O(\cdot, s_i, s_{1-i}) = 0.4$

Values of states under 1-step plans

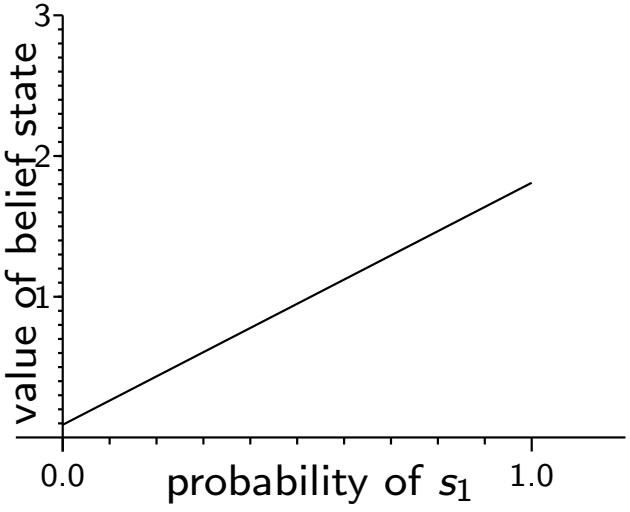
$$
\begin{aligned}
v_{Go}(s_0) &= R(s_0) + \gamma(0.9R(s_1) + 0.1R(s_0)) = 0.9\gamma \\
v_{Go}(s_1) &= R(s_1) + \gamma(0.9R(s_0) + 0.1R(s_1)) = 1.0 + 0.1\gamma \\
v_{Stay}(s_0) &= R(s_0) + \gamma(0.9R(s_0) + 0.1R(s_1)) = 0.1\gamma \\
v_{Stay}(s_1) &= R(s_1) + \gamma(0.9R(s_1) + 0.1R(s_0)) = 1.0 + 0.9\gamma
\end{aligned}
$$

(Reward from starting state + reward from successor state!)

With $\gamma = 0.9$ these are:

$$
\begin{aligned}
v_{Go}(s_0) &= 0.81 \\
v_{Go}(s_1) &= 1.09 \\
v_{Stay}(s_0) &= 0.09 \\
v_{Stay}(s_1) &= 1.81
\end{aligned}
$$

Values under Stay:



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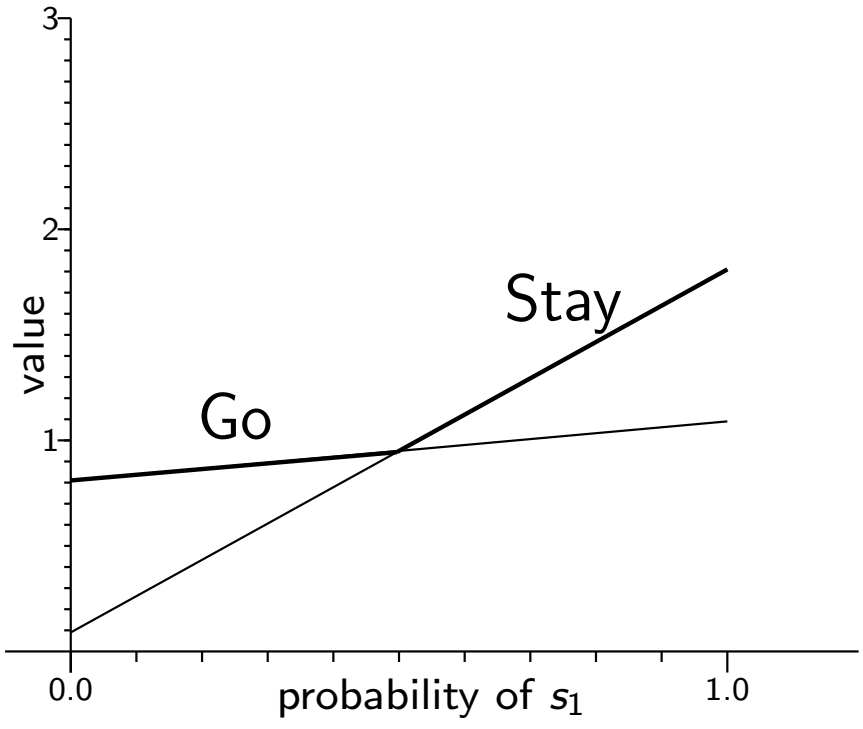
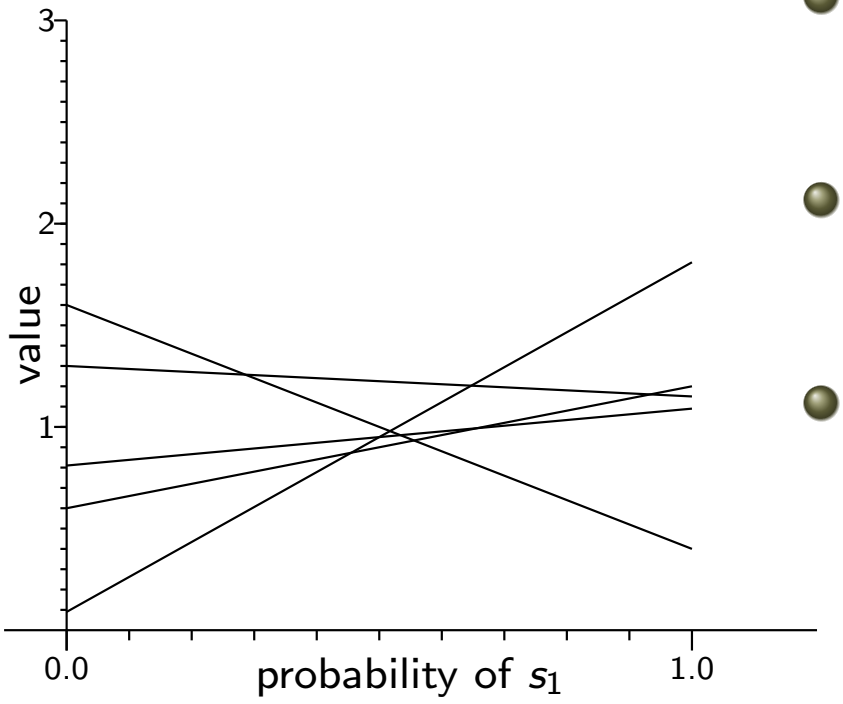
Values under optimal 1-step plans



For every belief state (probability distribution), use the plan that has the highest value.

Values under optimal plans



- Next iterations generate 2, 3, 4, ... step plans.
- Dominated plans (not best anywhere) ignored
- Optimal plan has a piecewise-linear value function consisting of non-dominated segments (top three in the diagram left)

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POMDP Value Iteration Algorithm

1. $i := 1$, $U' := \emptyset$
2. $U := U'$
3. $U_0 :=$ all policies of depth i
4. $U' :=$ policies in U_0 not dominated in U_0
5. $i := i + 1$
6. If improvement from U and U' "small", go to 2.

Dominated policy

π is dominated in U_0 if for every belief state B there is $\pi' \in U_0$ such that $v_{\pi'}(B) > v_\pi(B)$, where

$$v_\pi(B) = \sum_{s \in S} B(s) \cdot v_\pi(s)$$

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Domination test

The following has a solution iff $(w_1, \ldots, w_n)$ not dominated in $\{(w_1, \ldots, w_n^1), \ldots, (w_1^N, \ldots, w_n^N)\}$.
(Vectors are values of π and members of U_0 in all states.)

$$
\begin{aligned}
p_1 + \cdots + p_n &= 1 \quad (p_1, \ldots, p_n \text{ represent a belief state}) \\
p_1 &\geq 0 \\
&\vdots \\
p_n &\geq 0 \\
w_1 p_1 + \cdots + w_n p_n &\geq w_1^1 p_1 + \cdots + w_n^1 p_n \\
&\vdots \\
w_1 p_1 + \cdots + w_n p_n &\geq w_1^N p_1 + \cdots + w_n^N p_n
\end{aligned}
$$

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Execution of a Policy

1. Belief state initially is some $B = (p_1, \ldots, p_n)$
2. Find best plan π for belief state $B = (p_1, \ldots, p_n)$ from value function
3. Execute first action a of π (rest of π ignored!)
4. Get observation e
5. Update B to B' according to a and e
6. Go to 2.

Auxiliary concept: Markov Chains

Definition (Markov chain $\langle S, P, R \rangle$)

- S is a (finite) set of states,
- $P : S \times S \to \mathbb{R}$ gives transition probabilities
- $R : S \times S \to \mathbb{R}$ is a reward function

Value function of a Markov chain can be evaluated by solving a set of linear inequalities.

(This is what we did when evaluation a policy in Policy Iteration: the Markov chain $\langle S, P, R \rangle$ with $P(s, s') = P_0(s, \pi(s), s')$ and $R(s, s') = R_0(s, \pi(s), s')$ was obtained from MDP $\langle S, A, P_0, R_0 \rangle$ and a policy π.)

Bounded-Memory Policies for POMDPs

- Bounded-memory policies with memory/belief states $M = \{m_1, \ldots, m_N\}$ instead of the infinite and continuous space of probability distributions over S
- Algorithms: Search in the (finite) space of bounded-memory policies

Bounded-Memory Policies for POMDPs

Definition

A bounded memory policy with memory state M, for a POMDP $\langle S, A, P, R, E, O \rangle$ is a mapping $\pi : M \times E \to A \times M$.

The execution of π is a Markov chain $\langle S_x, P_x, R_x \rangle$ with

- $S_x = S \times M \times E$ (state, memory, last observation)
- $P_x((s, m, e), (s', m', e')) = P(s, a, s')O(a, s', e')$ where $\pi(m, e) = (a, m')$
- $R_x((s, m), (s', m')) = R(s, a, s')$ where $\pi(m, e) = (a, m')$

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Bounded-Memory Policies for POMDPs

General scheme for finding bounded-memory policies:

1. Generate policies π
2. Evaluate the Markov chain corresponding to π
3. Choose the best policy

For example, choose policy π and initial memory state m to maximize

$$\sum_{s \in S} v_\pi(s, m, obs(s))B(s)$$

where B represents the initial *belief state* (unrelated to the initial memory state m) and $obs(s)$ is observations with initial states.

Execution of a Bounded-Memory Policy

1. Memory state initially some $m \in M$ and initial observation $e = obs(e)$
2. $(a, m') := \pi(m, e)$
3. Execute a, obtaining new observation e'
4. Update belief state $m := m'$ and $e := e'$
5. Go to 2.

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Special Case: Memory-Free Policies

- Memory-free policy = Bounded-memory policy with 1 memory state
- Policies are mappings from observations to actions