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1 Exercise

Translate the following actions into the propositional logic, as in the "blocks world" example in the Lecture 5 slides. These actions represent movement in a network with 6 locations a, b, c, d, e, and f.

$$\begin{aligned} \mathrm{M}ab &= \langle a, \{\neg a, b\}\rangle \\ \mathrm{M}ac &= \langle a, \{\neg a, c\}\rangle \\ \mathrm{M}cd &= \langle c, \{\neg c, d\}\rangle \\ \mathrm{M}de &= \langle d, \{\neg d, e\}\rangle \\ \mathrm{M}fe &= \langle f, \{\neg f, e\}\rangle \end{aligned}$$

Notice that you will need the formulas $\neg(\mathrm{M}ab@i \wedge \mathrm{M}ac@i)$ to prevent simultaneous move from a to both b and c.

Let the initial state be the one where the current location is a, and the goal that the current location e.

Now construct a set of clauses (CNF) from all of the above, for

1. horizon length 2 (two consecutive actions, time points 0, 1, and 2), and
2. horizon length 3 (three consecutive actions, time points 0, 1, 2, and 3).

Apply the unit propagation rule

$$\frac{l_1 \vee l_2 \vee \cdots \vee l_{n-1} \vee l_n \quad \overline{l_1} \quad \overline{l_2} \quad \ldots \quad \overline{l_{n-1}}}{l_n}$$

exhaustively (all possible applications) in both cases, to determine unsatisfiability or satisfiability.

Solution

Preconditions:

$$\begin{aligned} &\mathrm{M}ab@i \rightarrow a@i && \mathrm{M}ac@i \rightarrow a@i \\ &\mathrm{M}cd@i \rightarrow c@i && \mathrm{M}de@i \rightarrow d@i \\ &\mathrm{M}fe@i \rightarrow f@i \end{aligned}$$

Effects:

$$\begin{aligned} &\mathrm{M}ab@i \rightarrow \neg a@(i+1) && \mathrm{M}ac@i \rightarrow \neg a@(i+1) \\ &\mathrm{M}cd@i \rightarrow \neg c@(i+1) && \mathrm{M}de@i \rightarrow \neg d@(i+1) \\ &\mathrm{M}fe@i \rightarrow \neg f@(i+1) \end{aligned}$$

$$\begin{aligned} &\mathrm{M}ab@i \rightarrow b@(i+1) && \mathrm{M}ac@i \rightarrow c@(i+1) \\ &\mathrm{M}cd@i \rightarrow d@(i+1) && \mathrm{M}de@i \rightarrow e@(i+1) \\ &\mathrm{M}fe@i \rightarrow e@(i+1) \end{aligned}$$

Frame axioms:

$$\begin{aligned} &a@i \wedge \neg a@(i+1) \rightarrow \mathrm{M}ab@i \vee \mathrm{M}ac@i && b@i \wedge \neg b@(i+1) \rightarrow \bot \\ &c@i \wedge \neg c@(i+1) \rightarrow \mathrm{M}cd@i && d@i \wedge \neg d@(i+1) \rightarrow \mathrm{M}de@i \\ &e@i \wedge \neg e@(i+1) \rightarrow \bot && f@i \wedge \neg f@(i+1) \rightarrow \mathrm{M}fe@i \\ &\neg a@i \wedge a@(i+1) \rightarrow \bot && \neg b@i \wedge b@(i+1) \rightarrow \mathrm{M}ab@i \\ &\neg c@i \wedge c@(i+1) \rightarrow \mathrm{M}ac@i && \neg d@i \wedge d@(i+1) \rightarrow \mathrm{M}cd@i \\ &\neg e@i \wedge e@(i+1) \rightarrow \mathrm{M}de@i \vee \mathrm{M}fe@i && \neg f@i \wedge f@(i+1) \rightarrow \bot \end{aligned}$$

Prevent simultaneous moves:

$$\neg(\mathrm{M}ab@i \quad \wedge \quad \mathrm{M}ac@i)$$

Initial state:

$$a@0 \quad \neg b@0 \quad \neg c@0 \quad \neg d@0 \quad \neg e@0 \quad \neg f@0$$

Above in conjunctive normal form

Preconditions:

$$\neg\mathrm{M}ab@i \vee a@i \qquad \neg\mathrm{M}ac@i \vee a@i$$
$$\neg\mathrm{M}cd@i \vee c@i \qquad \neg\mathrm{M}de@i \vee d@i$$
$$\neg\mathrm{M}fe@i \vee f@i$$

Effects:

$$\neg\mathrm{M}ab@i \vee \neg a@(i+1) \qquad \neg\mathrm{M}ac@i \vee \neg a@(i+1)$$
$$\neg\mathrm{M}cd@i \vee \neg c@(i+1) \qquad \neg\mathrm{M}de@i \vee \neg d@(i+1)$$
$$\neg\mathrm{M}fe@i \vee \neg f@(i+1) \qquad \neg\mathrm{M}ab@i \vee b@(i+1)$$
$$\neg\mathrm{M}ac@i \vee c@(i+1) \qquad \neg\mathrm{M}cd@i \vee d@(i+1)$$
$$\neg\mathrm{M}de@i \vee e@(i+1) \qquad \neg\mathrm{M}fe@i \vee e@(i+1)$$

Frame axioms:

$$\neg a@i \vee a@(i+1) \vee \mathrm{M}ab@i \vee \mathrm{M}ac@i \qquad \neg b@i \vee b@(i+1)$$
$$\neg c@i \vee c@(i+1) \vee \mathrm{M}cd@i \qquad \neg d@i \vee d@(i+1) \vee \mathrm{M}de@i$$
$$\neg e@i \vee e@(i+1) \qquad \neg f@i \vee f@(i+1) \vee \mathrm{M}fe@i$$
$$a@i \vee \neg a@(i+1) \qquad b@i \vee \neg b@(i+1) \vee \mathrm{M}ab@i$$
$$c@i \vee \neg c@(i+1) \vee \mathrm{M}ac@i \qquad d@i \vee \neg d@(i+1) \vee \mathrm{M}cd@i$$
$$e@i \vee \neg e@(i+1) \vee \mathrm{M}de@i \vee \mathrm{M}fe@i \qquad f@i \vee \neg f@(i+1)$$

Prevent simultaneous moves:

$$\neg\mathrm{M}ab@i \vee \neg\mathrm{M}ac@i$$

Initial state:

$$a@0 \quad \neg b@0 \quad \neg c@0 \quad \neg d@0 \quad \neg e@0 \quad \neg f@0$$

The Pattern of Unit Propagations

The interesting pattern in unit propagations (UP) for this problem is the following. Consider a starting node x and a node y that to which there is *no* path of length one from x, i.e. there is no action $\mathrm{M}xy$. Consider the frame axiom for y, of the form

$$y@i \vee \neg y@(i+1) \vee \mathrm{M}z_1y@i \vee \cdots \vee \mathrm{M}z_ny@i \tag{1}$$

We have unit clauses $x@0$ and $\neg y@0$.

From $\neg y@0$ and the precondition axioms $\neg\mathrm{M}z_jy@0 \vee y@0$ for $\mathrm{M}z_jy$ we can infer $\neg\mathrm{M}z_jy@0$.

Further, from $\neg y@0$, the frame axiom (1) and all those $\neg\mathrm{M}z_jy@0$ we can infer $\neg y@1$.

If there is no path to y from x of length 2 or less, then there is no path of length 1 from x to any of y's possible predecessor z, and therefore we can infer $\neg z@1$ with UP. And so on.

Thus, more generally, if any location w is not reachable from the initial location in k steps or less, we can infer $\neg w@k$ with UP.

This pattern, that actions are inferred to be not possible in a given number of steps, by propagation from the initial state, is typical. It closely resembles the computation of the Bonet & Geffner *max*-heuristic discussed in the lecture. However, it typically only allows inferring the impossibility of reaching goals when the horizon length is quite short.

Horizon length 2

Let $i \in \{0, 1, 2\}$.

Goal state: $e@2$

By unit propagation rule:

$$\frac{\neg \mathrm{M}cd@0 \vee c@0 \quad \neg c@0}{\neg \mathrm{M}cd@0}$$

$$\frac{d@0 \vee \neg d@1 \vee \mathrm{M}cd@0 \quad \neg d@0 \quad \neg \mathrm{M}cd@0}{\neg d@1}$$

$$\frac{\neg \mathrm{M}de@0 \vee d@0 \quad \neg d@0}{\neg \mathrm{M}de@0}$$

$$\frac{\neg \mathrm{M}de@1 \vee d@1 \quad \neg d@1}{\neg \mathrm{M}de@1}$$

$$\frac{f@0 \vee \neg f@1 \quad \neg f@0}{\neg f@1}$$

$$\frac{\neg \mathrm{M}fe@0 \vee f@0 \quad \neg f@0}{\neg \mathrm{M}fe@0}$$

$$\frac{\neg \mathrm{M}fe@1 \vee f@1 \quad \neg f@1}{\neg \mathrm{M}fe@1}$$

$$\frac{e@0 \vee \neg e@1 \vee \mathrm{M}de@0 \vee \mathrm{M}fe@0 \quad \neg e@0 \quad \neg \mathrm{M}de@0 \quad \neg \mathrm{M}fe@0}{\neg e@1}$$

$$\frac{e@1 \vee \neg e@2 \vee \mathrm{M}de@1 \vee \mathrm{M}fe@1 \quad \neg e@1 \quad \neg \mathrm{M}de@1 \quad \neg \mathrm{M}fe@1}{\neg e@2}$$

which is in contradiction with the goal state.

Horizon length 3

Since we have inferred $\neg e@2$ already, and there is no path to f at all, yielding $\neg f@2$, the frame axiom for e gives $\mathrm{M}de@2$. Similarly, since we already had $\neg d@1$ and we now have the precondition $d@2$ of $\mathrm{M}de@2$, we can infer $\mathrm{M}cd@1$. So the path between a and e is inferred step by step backwards, as there is always only one way of reached e, d, and c from a.

All clauses will be satisfied, because preconditions and effects of $\mathrm{M}ac@0$, $\mathrm{M}cd@1$, and $\mathrm{M}de@2$ are true, and all other action variables $\mathrm{M} \cdot \cdot @ \cdot$ have been inferred to be false.

That the plan is found with UP only this way is not typical, and shows up only in simple problems.

Let $i \in \{0, 1, 2, 3\}$.

Goal state: $e@3$

Continuing the above:

$$\frac{f@1 \vee \neg f@2 \quad \neg f@1}{\neg f@2}$$

$$\frac{\neg \mathrm{M}fe@2 \vee f@2 \quad \neg f@2}{\neg \mathrm{M}fe@2}$$

$$\frac{e@2 \vee \neg e@3 \vee \mathrm{M}de@2 \vee \mathrm{M}fe@2 \quad \neg e@2 \quad e@3 \quad \neg \mathrm{M}fe@2}{\mathrm{M}de@2}$$

$$\frac{\neg \mathrm{M}de@2 \vee d@2 \quad \mathrm{M}de@2}{d@2}$$

$$\frac{d@1 \vee \neg d@2 \vee \mathrm{M}cd@1 \quad \neg d@1 \quad d@2}{\mathrm{M}cd@1}$$

$$\frac{\neg \mathrm{M}cd@1 \vee c@1 \quad \mathrm{M}cd@1}{c@1}$$

$$\frac{c@0 \vee \neg c@1 \vee \mathrm{M}ac@0 \quad \neg c@0 \quad c@1}{\mathrm{M}ac@0}$$

Continuing this further would allow us to infer the truth value of each variable without getting a contradiction. (Obviously all other state and action variables, except the ones that we inferred above to be true, will be false.) This means that the corresponding truth value assignment satisfies all the clauses.

2 Exercise

Consider actions of the form $\langle p, e \rangle$ where

- p is a set of literals, indicating the precondition, and
- e is a set of literals, indicating the effects.

Two actions $\langle p_1, e_1 \rangle$ and $\langle p_2, e_2 \rangle$ *interfere*, if

- $e_1 \cup e_2$ is inconsistent (that is, the effects mutually conflict), or
- there is literal l such that $l \in e_1$ and $\bar{l} \in p_2$, or $l \in e_2$ and $\bar{l} \in p_1$.

Let s be a state, and let $a_1, a_2, \ldots, a_n$ be actions such that their preconditions are true in s and they do not interfere.

1. Prove that executing the actions in *any* total ordering is possible starting in s.
2. Give also an example of interfering actions (two or more) such that execution in some orderings *is* possible (but not in all orderings).

Solution

1. Assume there is an ordering $a_{i_1}, a_{i_2}, \ldots, a_{i_n}$ that cannot be executed. That means that there is an action a_{i_j} whose preconditions are not met. But this is not possible since the preconditions of a_{i_j} are true in s and by the second bullet point none of the preceding actions $a_{i_1}, a_{i_2}, \ldots, a_{i_{j-1}}$ change the state of any of the preconditions of a_{i_j}. Thus, no such ordering exists.
2. $\langle a, \{b\} \rangle$ and $\langle b, \{\neg b, \neg a\} \rangle$

3 Exercise

What binary relations do the following formulas correspond to? (The state variables involved are A, B and C in all cases.) List the pairs of bit-vectors.

1. $(A_0 \leftrightarrow B_1) \wedge (B_0 \leftrightarrow C_1) \wedge (C_0 \leftrightarrow A_1)$
2. $(A_0 \leftrightarrow \neg B_1) \wedge (B_0 \leftrightarrow \neg C_1) \wedge (C_0 \leftrightarrow \neg A_1)$
3. $(A_0 \rightarrow A_1) \wedge (B_0 \rightarrow B_1) \wedge (C_0 \rightarrow C_1)$
4. $A_1 \wedge B_1 \wedge C_1$

Solution

1. $\{(000, 000), (100, 010), (010, 001), (110, 011), (001, 100), (101, 110), (011, 101), (111, 111)\}$
   (rotate right)

2. $\{(000, 111), (100, 101), (010, 110), (110, 100), (001, 011), (101, 001), (011, 010), (111, 000)\}$
   (rotate right and take complement)

3. $\{(000, 000), (000, 100), (000, 010), (000, 110), (000, 001), (000, 101), (000, 011), (000, 111),$
   $(100, 100), (100, 110), (100, 101), (100, 111),$
   $(010, 010), (010, 110), (010, 011), (010, 111),$
   $(110, 110), (110, 111),$
   $(001, 001), (001, 101), (001, 011), (001, 111),$
   $(101, 101), (101, 111),$
   $(011, 011), (011, 111),$
   $(111, 111)\}$
   (elementwise keep or set to 1)

4. $\{(000, 111), (100, 111), (010, 111), (110, 111), (001, 111), (101, 111), (011, 111), (111, 111)\}$
   (set every element to 1)