CS-E4800 AI Exercises 5: Symbolic Search Methods and Satisfiability February 7, 2017

1 Exercise

Translate the following actions into the propositional logic, as in the "blocks world" example in the Lecture 5 slides. These actions represent movement in a network with 6 locations a, b, c, d, e, and f.

$$\begin{aligned} \mathrm{M}ab &= \langle a, \{\neg a, b\}\rangle \\ \mathrm{M}ac &= \langle a, \{\neg a, c\}\rangle \\ \mathrm{M}cd &= \langle c, \{\neg c, d\}\rangle \\ \mathrm{M}de &= \langle d, \{\neg d, e\}\rangle \\ \mathrm{M}fe &= \langle f, \{\neg f, e\}\rangle \end{aligned}$$

Notice that you will need the formulas $\neg(\mathrm{M}ab@i \wedge \mathrm{M}ac@i)$ to prevent simultaneous move from a to both b and c.

Let the initial state be the one where the current location is a, and the goal that the current location e.

Now construct a set of clauses (CNF) from all of the above, for

1. horizon length 2 (two consecutive actions, time points 0, 1, and 2), and
2. horizon length 3 (three consecutive actions, time points 0, 1, 2, and 3).

Apply the unit propagation rule

$$\frac{l_1 \vee l_2 \vee \cdots \vee l_{n-1} \vee l_n \quad \overline{l_1} \quad \overline{l_2} \quad \ldots \quad \overline{l_{n-1}}}{l_n}$$

exhaustively (all possible applications) in both cases, to determine unsatisfiability or satisfiability.

2 Exercise

Consider actions of the form $\langle p, e\rangle$ where

- p is a set of literals, indicating the precondition, and
- e is a set of literals, indicating the effects.

Two actions $\langle p_1, e_1\rangle$ and $\langle p_2, e_2\rangle$ *interfere*, if

- $e_1 \cup e_2$ is inconsistent (that is, the effects mutually conflict), or
- there is literal l such that $l \in e_1$ and $\overline{l} \in p_2$, or $l \in e_2$ and $\overline{l} \in p_1$.

Let s be a state, and let $a_1, a_2, \ldots, a_n$ be actions such that their preconditions are true in s and they do not interfere.

1. Prove that executing the actions in *any* total ordering is possible starting in s.
2. Give also an example of interfering actions (two or more) such that execution in some orderings *is* possible (but not in all orderings).

3 Exercise

What binary relations do the following formulas correspond to? (The state variables involved are A, B and C in all cases.) List the pairs of bit-vectors.

1. $(A_0 \leftrightarrow B_1) \wedge (B_0 \leftrightarrow C_1) \wedge (C_0 \leftrightarrow A_1)$
2. $(A_0 \leftrightarrow \neg B_1) \wedge (B_0 \leftrightarrow \neg C_1) \wedge (C_0 \leftrightarrow \neg A_1)$
3. $(A_0 \rightarrow A_1) \wedge (B_0 \rightarrow B_1) \wedge (C_0 \rightarrow C_1)$
4. $A_1 \wedge B_1 \wedge C_1$