CS-E4800 AI Exercises 8: Decision-Making in Adversial and Multi-Agent Settings March 7, 2017

1 Exercise: Second Price Auctions

A limited, but commonly occurring multi-agent coordination problem is that of deciding, which of a number of agents gets to do something, or obtain something, so that a collective utility measure is maximized. For example, a certain task has to be performed, and the agents offer to do it, by expressing their personal cost (including a "reward") of doing it, and the payment they are given depends on this personal cost. To maximize the collective utility, the payment has to be minimized, so the task should be allocated to the agent who can perform the task with the lowest cost. This types of problems in multi-agent systems, including non-A.I. ones, are often solved with *auctions*. Auctions are commonly also used for *selling* an object (in which case the agent who obtains an object will pay for it, and the highest (not the lowest) bidder will win). The basic features of auctions are the same in both settings, of course.

A *second-price auction* proceeds as follows. All bidders write their bid (the sum they are willing to pay for the object being auctioned) on a sheet of paper and place it in a sealed envelope (so that nobody else can see the bid). Once the auctioner has all the envelopes, she opens them to see the bids. The winner of the auction is the bidder with the highest bid. The amount he pays is, however, the *second highest* bid.

- Argue that every bidder is best off bidding exactly the sum which is the value of the object to them. Why would it be bad to bid *more* than one's own value? Why would it be bad to bid *less* than one's own value? Why cannot one benefit from bidding *less* than one's own value?
- Explain why and how is the situation different for *first-price auctions*, where the winner pays the sum stated in his own bid.

2 Exercise: Iterated Strict Dominance

Does the following normal form game have a unique Nash equilibrium? Try repeatedly identifying strategies that are strictly dominated by another strategy, and remove them from the game.

	A	B	C
D	3,2	0,4	4,0
E	4,4	2,0	3,2
F	6,2	1,1	2,1

3 Exercise: Finding Mixed Strategies

Derive the mixed-strategy equilibrium for the following game in normal form.

	L	R
U	10,5	2,0
D	0,2	5,10

For this, you need to find out how each player has to randomize so that *the opponent is indifferent between his two strategies, in terms of the expected pay-off for him.*

1. The expected pay-off of player 1 with strategy U as a function of player 2 playing L with probability p and R with probability $1-p$:
$$v_U(p) = 10p + 2(1-p)$$
Similarly,
$$v_D(p) = 0p + 5(1-p)$$
2. Plot a diagram with the expected pay-off of player 1 with strategy U when player 2 varies the probability of playing L from 1 to 0.
Draw a curve for strategy D in the same diagram.
With what probability of L is player 1 indifferent between playing U and D? This is the probability player 2 must choose to make player 1 randomize between U and D (otherwise one of them would be a better response than the other.)
3. Do the same with the roles of the two players reversed: plot the values of strategies R and L against the probabilities of player 1 mixing between strategies U and D.

State what the mixed-strategy Nash equilibrium is for this game.