CS-E4800 AI   Exercises 9: Decision-Making Multi-Agent Settings   **March 14, 2017**

1   Exercise

Find an example where 2-Candidate Runoff with plurality rule and Single Transferable Vote with plurality rule lead to different results.

Hint: Construct rankings so that A and B go to the second round in the 2-candidate runoff but C beats them under STV after lower-ranked candidates have been eliminated.

2   Exercise

Does voting with the Borda count have the following properties? If so, give a proof, or otherwise give a counter-example.

1. The winner is the top choice for at least one voter.
2. Assume candidate X wins. If some other candidate had not been running, and the individual orderings of the candidates had otherwise remained the same, then X would still necessarily have won.

3   Exercise

Consider the Vickrey-Clarke-Groves mechanism with the Clarke pivot rule, as described in the lecture.

1. Consider a very small town that wants to decide between

   (a) building a new road,
   (b) building a bridge over a river that passes through the town.

   The three inhabitants of the town value these two projects differently, based on their need for the road and the bridge. They express their preferenes by declaring the following valuations for the two projects.

person	Bridge	Road
A	10	-5
B	3	3
C	4	6

   The collective value of the bridge – based on the public declarations – is 10+3+4=17, and the collective value of the road is -5+3+6=4. So it is the bridge that will be built.
   Calculate the payments and total utilities for all agents.
2. Show that in VCG, if all declared valuations are non-negative, all the payments will be negative.
3. Show that in VCG, an agent's value+payment (for the chosen outcome) is always non-negative.
4. Explain what can happen, if an agent declares a value of some alternative un-truthfully, A) too high, B) too low.