Digital Filters for Audio Processing

ELEC-E5620 - Audio Signal Processing, Lecture #2

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Sound check 🎧

Course Schedule in 2017 (Periods III, IV)

0. General issues (Vesa & Fabian) 13.1.2017
1. History and future of audio DSP (Vesa) 20.1.2017
2. Digital filters in audio (Vesa) 27.1.2017
3. Audio filter design (Vesa) 3.2.2017
4. Analysis of audio signals (Vesa) 10.2.2017
   No lecture (Evaluation week for Period III) 17.2.2017
5. Audio effects processing (Fabian) 24.2.2017
6. Synthesis of audio signals (Fabian) 3.3.2017
7. 3-D sound (Prof. Ville Pulkki) 10.3.2017
8. Physics-based sound synthesis (Vesa) 17.3.2017
9. Sampling rate conversion (Vesa) 24.3.2017
10. Audio coding (Vesa) 31.3.2017
Outline

- Digital filters are basic tools in audio signal processing
  - Simple FIR & IIR filters
  - Comb filters
  - Resonators
  - Allpass filters
  - Shelving filters
  - Equalizing filters
  - Fractional delay filters


What’s a Digital Filter?

- Digital filter = Computational algorithm to modify a digital signal
  - Often some frequencies may need to be attenuated or amplified
- Two main types
  1. Feedforward or FIR filters
  2. Feedback or IIR filters
FIR Filters

- **FIR = Finite Impulse Response**
  - Impulse response = Output signal of the filter when the input signal $x(n)$ is a unit impulse
  - Unit impulse = Digital signal whose first sample is 1 and the rest are 0
  - Impulse response $h(n)$ of an FIR filter consists of its coefficients

\[
x(n) \rightarrow z^{-1} \rightarrow h(0) \rightarrow h(1) \rightarrow z^{-1} \rightarrow h(2) \rightarrow \cdots \rightarrow h(N) \rightarrow z^{-1} \rightarrow y(n)
\]

First-Order FIR Filter

- Add the input signal with its delayed and scaled version
  - Real weights $h_1$ and $h_2$ for the input and the delayed input signal
- Difference equation:
  \[
y(n) = h_1x(n) + h_2x(n-1)
\]
- Impulse response: $[h_1 \ h_2]$
- Z transform:
  \[
  Y(z) = h_1X(z) + h_2X(z)z^{-1} = [h_1 + h_2z^{-1}]X(z)
  \]
- Transfer function: $H(z) = h_1 + h_2z^{-1}$
- Frequency response (set $z = e^{j\omega}$)
  \[
  H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega}) = h_1 + h_2e^{-j\omega}
  \]
- Magnitude $|H(e^{j\omega})|$ is the filter gain; angle is the phase
Magnitude Response Interpretation

- Magnitude response \(|H(\omega)|\) is the length of a vector on the z plane
  - Magnitude response can be obtained by travelling along the unit circle and measuring the distance from the zero

\[ |H(e^{j\omega})| \]


Magnitude Response Evaluation

- Magnitude response of the 1st-order FIR filter:
  \[ |H(\omega)| = |h_1^2 + h_2^2 + 2h_1h_2\cos(\omega)|^{1/2} \]

Example:
\[ h_1 = 1 \]
\[ h_2 = 0.99 \]
IIR Filters

• IIR = Infinite Impulse Response
  – Impulse-response length is infinite in theory
  – The impulse response usually decays to zero
• IIR filter structures are based on feedback
• The transfer function has poles in addition to zeros

The Simplest IIR Filter: One-Pole Filter

• Add the input signal and the delayed and scaled output signal
  – Magnitude of feedback coefficient must not be larger than 1
• Difference equation:
  \[ y(n) = x(n) + a_1y(n-1) \]
• Z transform:
  \[ Y(z) = X(z) + a_1Y(z)z^{-1} \]
• Transfer function:
  \[ H(z) = 1/(1 - a_1z^{-1}) \]
• Frequency response:
  \[ H(e^{j\omega}) = 1/(1 - a_1e^{-j\omega}) \]
  (when \( 0 < a_1 < 1 \))
**IIR Filter’s Magnitude Response**

- Magnitude response:
  \[ |H(\omega)| = \frac{1}{|1 - a_1 e^{-j\omega}|} \]
- Zero of the denominator, or the **pole**
- Geometrical interpretation
  - Mark the pole on the Z plane
  - Travel along the unit circle, 0 to \( \pi \)
  - Measure the distance from the pole
  - Compute the inverse of the distance

(Steiglitz 1996)

**Magnitude Response Evaluation**

- Magnitude response of \[ |H(\omega)| = g / |1 - a_1 e^{-j\omega}| \]
  where \( g \) controls the gain of the filter
- The choice \( g = 1 - a_1 \) maintains the **dc gain** of 0 dB

(DC gain = gain at 0 Hz)

(Steiglitz 1996)
Comb Filter (1)

- The name stems from the shape of magnitude response
- Obtained from a one-pole filter by replacing $z^{-1}$ with $z^{-L}$
- Example:
  $a_1 = 0.999, L = 8$

Comb Filter (2)

- When the unit delay is replaced with a delay line, the pole of the first-order filter is duplicated $L$ times on the z plane
- Example:
  $a_1 = 0.999, L = 8$
Comb Filter (3)

- When the feedback coefficient is negative, peaks occur at odd harmonics
- Example:
  \( a_1 = -0.999, L = 8 \)

Inverse Comb Filter (1)

- Sometimes called the FIR comb filter
- Obtained by inverting the comb filter transfer function
- Example:
  \( L = 8 \)
Inverse Comb Filter (2)

- Poles become zeros
- Digital simulation of direct sound and a single reflection (e.g. floor)

**Example:**

$L = 8$

\[ x(n) \rightarrow a_1 \rightarrow y(n) \]

Resonance

- A bump in the magnitude response
  - E.g. a formant, or a mode of a vibrating structure
- Sharpness is described by **bandwidth**:
  - Usually, the difference of points where the gain is 3 dB below the peak
- Other properties:
  - center frequency & peak value (gain)
- Quality factor:
  - \[ Q = \frac{\omega_c}{B} \]
  - Large Q value \(\rightarrow\) sharp resonance
  - Small Q value \(\rightarrow\) wide resonance
**Pole Radius**

- **Resonance is caused by a pole close to the unit circle**
- Distance can be deduced from bandwidth: \( R \approx 1 - B/2 \)
  where \( B = 2\pi f \) and \( f \) is normalized frequency
  - Alternative form: \( R \approx 1 - \pi \)
- **Example:**
  \( f_s = 44.1 \text{ kHz} \) and bandwidth is 20 Hz:
  \[ R \approx 1 - \pi \left( \frac{20}{44100} \right) = 0.998575 \]

**Digital Resonator**

- **Resonators are crucial in audio DSP**
  - Subtractive synthesis, formant filters, parallel equalizers...
- **To generate a resonance between 0 Hz and the Nyquist frequency, a pole pair is needed**
  - A pole and its complex conjugate
- **Transfer function:**
  \[ H(z) = \frac{1}{(1 - Re^{i\theta}z^{-1})(1 - Re^{-i\theta}z^{-1})} \]
  or \[ H(z) = \frac{1}{(1 - 2R\cos(\theta)z^{-1} + R^2 z^{-2})} \]
- **Difference equation:**
  \[ y(n) = x(n) + 2R\cos(\theta)y(n-1) - R^2y(n-2) \]
Digital Resonator

- Examples of magnitude responses of second-order IIR filters (with scaling)

Interactive Design With Matlab

Matlab-demo: restoola.m
How to Choose Resonator Parameters

- Sometimes it is enough to choose parameters $\theta$ ja $R$
  - $\theta$ is resonance frequency and $R = 1 - B/2$ pole radius
- However, at low frequencies, the pole and its mirror are close to each other and can bias the peak location
  - Peak is not exactly at the frequency corresponding to $\theta$!

How to Design a Digital Resonator

- An exact method to design a resonator (Steiglitz, 1996):
  1) Choose bandwidth $B$ and peak frequency $\psi$
  2) Calculate the desired pole radius $R = 1 - B/2$
  3) Calculate $\cos(\theta)$ using a correction term:
     \[ \cos(\theta) = \frac{(1 + R^2)/2R}{\cos(\psi)} \]
  4) Calculate gain factor $A_0$ that gives a max gain of 0 dB:
     \[ A_0 = (1 + R^2)\sin(\theta) \]
  5) Implement the filter:
     \[ y(n) = A_0x(n) + 2R\cos(\theta)y(n - 1) - R^2y(n - 2) \]
Correction of the Pole Frequency

- This correction is only necessary when $R < 1 - \epsilon$

\[
\frac{(1 + R^2)}{2R}
\]

Impulse Response of a Resonator

- Impulse response:
  \[ h(n) = \left(\frac{R^n}{\sin \theta}\right) \left[\sin(\theta(n + 1))\right], \]  
  where $n$ is the sample index

- Constant
- Sinewave oscillating at frequency $\theta$
- Exponentially decaying envelope (when $R < 1$)
- When $R$ is small, fast decay & wide resonance
- Digital simulation of a decaying oscillator
  - Damping caused by losses
Resonator With Zeros

- When the pole frequency is very low, the peak gets wide ("lowpass" instead of resonance)
- An improvement: Insert a zero at the 0 frequency
  - It is easy to do the same for the Nyquist frequency at the same time

\[
H(z) = \frac{1 - z^{-2}}{1 - 2R\cos(\theta)z^{-1} + R^2 z^{-2}}
\]

Proposed by Smith & Angell (1982) and Steiglitz (1994)
- The motivation was to be able to sweep of the resonance frequency while keeping bandwidth and amplitude constant
Application Examples: Popcorn Sounds

- Filter a burst of noise $x(n)$ using the resonator with zeros
  \[ y(n) = A_0 x(n) + 2R\cos(\theta)y(n - 1) - R^2 y(n - 2) \]
  - Bandwidth and center frequency are changed for every pop

Matlab demo: popcornpop.m

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Allpass Filter

- **Magnitude response is exactly 1** at all frequencies!
  - Almost 1 is not enough (not an allpass filter)
  - Does not attenuate or boost anything...

- Poles & zeros on the opposite sides of the unit circle
  - Pole radius $\rho$ and zero radius $1/\rho$: cancellation of gain effects

(Steiglitz 1996)
### Allpass Transfer Function

- Allpass transfer function:
  
  \[ A(z) = \frac{D(z^{-1})z^{-N}}{D(z)} \]

  where the denominator polynomial \( D(z) \) is

  \[ D(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N} \]

  - Numerator coefficients otherwise the same, but in reverse order

### Allpass Filter Properties

- Magnitude response is always flat:
  
  \[ |A(e^{j\omega})| = \left| \frac{D(e^{-j\omega})e^{-jN\omega}}{D(e^{j\omega})} \right| = 1 \]

- Phase response is non-trivial:
  
  \[ \angle A(e^{j\omega}) = \frac{\angle D(e^{j\omega}) - \angle D(e^{-j\omega})}{\angle D(e^{j\omega})} = 2\theta_D(e^{j\omega}) - N\omega \]

  - Twice the phase response of the denominator and delay of \( N \) samples
Allpass Filter Properties

- When coefficients $a_k$ are zero, the allpass filter is reduced to a delay line: $A(z) \rightarrow z^{-M}$
- A generalization of a delay line

First-Order Allpass Filter

- Transfer function:
  - For stability we require $-1 < a_1 < 1$
  $$A(z) = \frac{a_1 + z^{-1}}{1 + a_1 z^{-1}}$$
Applications of Digital Allpass Filters

- Artificial room reverberation
  - The first application of digital allpass filters (Schroeder and Logan, 1961)
- Digital delay compensation in filters, loudspeakers
- Phasing (J. O. Smith, 1983)
- Shelving filters and equalizers (Regalia & Mitra, 1987)
- Fractional delay filters (e.g., Thiran allpass filters) (Laakso et al., 1996)
- Warped filters (Laguerre & Kautz structures) (Härmä et al., 2000)
- Synthesis techniques, e.g. inharmonic waveguide synthesis (strings, bells) and oscillators
- Decorrelation filters in spatial audio (Kendall, 1995)
- Spring reverb modeling (Abel et al., 2006; Välimäki et al., 2010)
- Spectral delay filters (Välimäki et al., 2009)
- Reduction of peak amplitude of transients (Parker & Välimäki, 2013)

Dynamic Range Reduction using an Allpass Filter Chain

- Dispersive allpass filters (Parker & Välimäki, IEEE SPL, 2013)
- Use golden-ratio coefficients for the allpass filters \( g = \pm 0.618 \)
- Delay-line lengths of 3 allpass filters are adjusted by trial and error
Dynamic Range Reduction using an Allpass Filter Chain

- About 2.5 dB (even 5 dB) reduction in amplitude

Shelving Filters and Equalizers

- Shelving filter is a bass or treble tone control
  - Control the level of low or high frequencies
  - Preserve the rest of the frequencies

- Equalizer modifies the magnitude response
  - Originally used for flattening the response of telephone systems
  - Now wide uses particularly in audio reproduction systems and recording studios (boost/cut, effects)
Shelving Filter

- Amplifies or attenuates low (high) frequencies without affecting the high (low) ones

(Source: http://www.harmony-central.com/Effects/Articles/Equalization/)

Derivation of 1st-Order Low Shelf

- First define the gain at 3 points: $H(0) = G$, $H(1) = 1$, and geometric mean gain ($g/2$ in dB) at crossover:
  \[ |H(e^{j\omega_c})| = \sqrt{G} \]

- When $\omega_c = \pi/2$, we get the prototype filter:
  \[ H_P(z) = \sqrt{G} \frac{z + p}{z - p} \text{ with the pole at } p = \frac{G - \sqrt{G}}{G + \sqrt{G}} \]

- The crossover point is shifted using the lowpass-to-lowpass transformation (see, e.g. Reiss & McPherson, 2015):
  \[ z \rightarrow \frac{z - \beta}{1 - \beta z}, \quad \beta = \frac{1 - \tan(\omega_c/2)}{1 + \tan(\omega_c/2)} \]
Derivation of 1st-Order Low Shelf (2)

- The 1st-order shelf filter has the transfer function (Välimäki & Reiss, 2016):
  \[
  H_{LS}(z) = \frac{b'_0 + b'_1 z^{-1}}{a'_0 + a'_1 z^{-1}} = \frac{b_0 + b_1 z^{-3}}{1 + a_1 z^{-1}}
  \]

  where
  \[
  b'_0 = G \tan(\omega_c / 2) + \sqrt{G}, \quad b'_1 = G \tan(\omega_c / 2) - \sqrt{G},
  \]
  \[
  a'_0 = \tan(\omega_c / 2) + \sqrt{G}, \quad a'_1 = \tan(\omega_c / 2) - \sqrt{G}
  \]

Magnitude Response of Low Shelf

- 1st-order shelf filter
- Notice that the gain at the crossover point is 0.5 times the max gain at dc
**Magnitude Response of High Shelf**

- 1\textsuperscript{st}-order shelf filter
- Notice that the gain at the crossover point is 0.5 times the max gain at dc

![Magnitude Response of High Shelf](image)

**High Shelf Filter**

- The 1\textsuperscript{st}-order high shelf filter can be derived by modifying the 1\textsuperscript{st}-order low shelf (Välimäki & Reiss, 2016)
- They can be made complementary (mirror images)

![High Shelf Filter](image)
Higher-Order Shelf Filters

- Higher-order shelves have a steeper transition
- The 2nd-order shelf can be derived by digitizing an analog shelf filter using the bilinear transform (Välimäki & Reiss, 2016)

Practical Example of Tone Control

- Sony D-345 portable CD player has 3 options
Magnitude Response of Equalizer

- A peak (boost) or a valley (cut)
  - Adjust gain, center frequency, and bandwidth
  - Elsewhere the gain is about 0 dB

Digital Equalizing Filter

- EQ filter can be designed from gain constraints at 5 points: 0, Nyquist, peak, left and right bandwidth points
- One way is to use the prototype shelf and the lowpass-to-bandpass transformation (Välimäki & Reiss 2016)
- This leads the following transfer function:
  \[ H_{PN}(z) = \frac{\sqrt{G} + G \tan(B/2) - [2\sqrt{G \cos(\omega_c)}]z^{-1} + [\sqrt{G} - G \tan(B/2)]z^{-2}}{\sqrt{G} + \tan(B/2) - [2\sqrt{G \cos(\omega_c)}]z^{-1} + [\sqrt{G} - \tan(B/2)]z^{-2}} \]
- This filter is symmetric for G and 1/G.
Continuous Control of Center Frequency

White noise:

\[ f_0 = 300 \text{ Hz} \ldots 6.5 \text{ kHz} \]

\[ \text{…300 Hz:} \]

Special Structure for Shelfs and EQs

- Originally proposed by Regalia & Mitra (1987); improved design by Zölzer (1997)
- Suitable for both shelving and equalizing filters
  - A 1st-order allpass filter for shelves; a 2nd-order allpass for EQs
Graphic Equalizer

- A set of equalizers with fixed center freq. and Q value
  - Octave or third-octave bands are usual (10 or about 30 filters)
  - One equalizing filter running per band for each channel (e.g. 60 filters)

(Picture taken from http://www.bssaudio.com/)

Fractional Delay – Splitting the Unit Delay
What is Fractional Delay?

• Fractional delay = a delay smaller than sample interval $T$
  – For example, $d = 0.5$ samples
• Fractional delay (FD) is implemented with interpolation
  – In practice, we use digital filters that are called fractional delay filters
  – Discrete-time signals are inherently bandlimited, so the underlying continuous-time signal is known to be smooth between samples
• A common use is a long but accurate time delay
  – Consists of integral and fractional parts: $D = D_{\text{int}} + d$

Discrete vs. Continuous-Time Delay

![Diagram showing original and delayed signals with fractional delay $d = 0.3$.](image)
Audio Applications of Fractional Delay Filters

- Sampling rate conversion
  - Especially conversion between incommensurate rates, e.g., between standard audio sample rates 48 and 44.1 kHz
- Effects & music synthesis using digital waveguides
  - Comb filters using fractional-length delay lines
- Changing pitch of audio signals
  - Auto-tuning of the singing voice
  - Also, removal of wow from old recordings
- Beamforming using a microphone or loudspeaker array
- Doppler effect in virtual reality
- Wave field synthesis

Ideal Fractional Delay Filter

- FD filter = digital version of continuous time delay
- An ideal lowpass filter with a time shift: Impulse response is a sampled and shifted sinc function:

\[ h_{id}(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-jD\omega} e^{j\omega n} d\omega \]

\[ = \frac{\sin[\pi(n - D)]}{\pi(n - D)} = \text{sinc}(n - D) \]

where \( n \) is the time index and \( D \) is delay in samples
- For theoretical use only (i.e., useless)
Shifted and Sampled Sinc Function

- When $D$ integer:
  Sampled at zero-crossings
  (no fractional delay)

- When $D$ non-integer:
  Sampled between zero-crossings
  $\Rightarrow$ Infinitely long impulse response

Practical Fractional Delay Filter

- In practice, a fractional delay is approximated with a finite-length, causal FIR or IIR filter
  - These filters are called fractional delay filters
    (see Laakso et al., IEEE SP Magazine, 1996)

- FD filters approximate a “linear-phase allpass filter”
  - There is no such thing!
FIR Fractional Delay Filters

- FIR FD filters have an asymmetric impulse response, but aim at having a linear phase response
- Challenging filter design problem: complex-valued target frequency response (incl. magnitude & phase) ⇒ traditional linear-phase design methods unsuitable
- Truncated sinc and Lagrange filters are popular

Truncated Sinc Filter

- A standard FD filter design method is to truncate the ideal FD filter response
  - Shifted and truncated sinc function
  - The response suffers from the Gibbs phenomenon
- Coefficients $h(n)$ are selected from the sampled sinc function

$$h(n) = \begin{cases} \text{sinc}(n - D), & 0 \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$$

where $D = D_{\text{int}} + d$
Lagrange Fractional Delay Filter

- Lagrange interpolated refers to polynomial curve fitting
  - Also a maximally flat FD approximation
  - Good approximation at low frequencies only
- Closed-form formula for FIR filter coefficients:

\[ h(n) = \prod_{k=0, k \neq n}^{N} \frac{D-k}{n-k} \text{ for } n = 0, 1, 2, ..., N \]

where \( N \) is filter order and \( D \) is the delay parameter
- \( N \)th-order Lagrange interpolation corresponds to \( N \)th-order polynomial fitting to sample values:
  - \( N = 1 \rightarrow \) straight line, \( N = 2 \rightarrow \) parabola etc.
Linear Interpolation ($N = 1$)

Discrete time

Signal value

?
**Linear Interpolation \((N = 1)\)**

![Linear Interpolation Graph](image)

**Cubic Interpolation \((N = 3)\)**

![Cubic Interpolation Graph](image)
Cubic Interpolation ($N = 3$)

Discrete time

Signal value

-1
0
0.5
1

-1
0
0.5
1

Discrete time

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ELEC-E5620 Audio Signal Processing
Lecture #2: Digital Filters
Linear Interpolation Filter

- Lagrange interpolation with $N = 1$ yields linear interpolation
- First-order FIR filter
  
  $h(0) = 1 - d$
  
  $h(1) = d$
  
  $(0 < d < 1)$
- Easy method – try this first!

Matlab demo:
lagrtool.m

Magnitude Response of Lagrange Filters

$N = 3$

$d = 0 \ldots 0.5$

$N = 9$

$d = 0 \ldots 0.5$
Phase Delay of Lagrange Filters

\[ N = 3 \]
\[ d = 0.5 \]
\[ d = 0.4 \]
\[ d = 0.3 \]
\[ d = 0.2 \]
\[ d = 0.1 \]

\[ N = 9 \]
\[ d = 0.5 \]
\[ d = 0.4 \]
\[ d = 0.3 \]
\[ d = 0.2 \]
\[ d = 0.1 \]

Doppler Effect Simulation

- Observed frequency is changing, when the sound source is moving towards or away from the listener
  - Caused by change in the propagation delay (fractional delay!)

\[ r_{\text{doppler}} = \frac{c + v_r}{c - v_s} \]
Virtual Doppler: Example #1 Demo by Simo Broman & Heidi-Maria Lehtonen, TKK 2004

- Ambulance passing by at 60 km/h 25 meters away
  - Order of Lagrange interpolation filter: \( N = 4 \)

Virtual Doppler: Example #2 Demo by Simo Broman & Heidi-Maria Lehtonen, TKK 2004

- Ambulance passing by at 80 km/h 4 meters away
  - Order of Lagrange interpolation filter: \( N = 20 \)
Virtual Doppler: Example #3

- Ambulance passing by at 300 km/h 5 meters away
  - Order of Lagrange interpolation filter: $N = 120$

Truncated Lagrange Filter

- Impulse response of a high-order Lagrange FD filter can be truncated (Välimäki & Haghparast, 2007)
- The approximation bandwidth is widened w.r.t. Lagrange FD filter of the same order
- Two design parameters:
  1) $M$: prototype filter order (Lagrange FD filter)
  2) $N$: truncated Lagrange filter order

\[
 h(n) = \prod_{k=0, k \neq n + K_1}^{M} \frac{D-k}{n+K_1-k} \quad \text{for } n = 0, 1, 2, \ldots, N
\]

- $K_1$ is the number of deleted coefficients on each side so that $M = N + 2K_1$
Truncated Lagrange vs. Sinc & Lagrange

Figure by Azadeh Haghparast, TKK, 2007
Summary on Lagrange Interpolation

- Polynomial curve fitting
- Some advantages:
  - A simple design method
  - Good FD approximation at low frequencies (good for audio!)
  - No overshoot (filter gain \( \leq 1 \))
- Problem: Narrow approximation bandwidth even with high-order Lagrange interpolation filters
  - Fortunately, in audio, energy is concentrated on low frequencies and human hearing is sensitive to low frequencies (w.r.t. \( f_s \))
- Use other design techniques when high quality is desired
- Recent improvement: truncated Lagrange FD filter

IIR Fractional Delay Filters

- Allpass filters are well suited to fractional delay approximation
  - Magnitude response is exactly flat
- Design reduces to phase (or phase/group delay) approximation
  - FD problem is essentially a phase design problem!
Thiran Allpass FD Filter

- The simplest IIR FD filter design method is the Thiran design (Fettweis, 1972; Laakso et al. 1996)

\[ a_k = (-1)^k \binom{N}{k} \prod_{n=0}^{N} \frac{D - N + n}{D - N + k + n} \]

for \( n = 0, 1, 2, ..., N \)

- For example, \( N = 2 \):

\[ a_1 = \frac{2}{1 + D}, \quad a_2 = \frac{(1 - D)(2 - D)}{(1 + D)(2 + D)} \]

- Close relative of Lagrange interpolation

First-Order Allpass FD Filter

- Coefficient formula:

\[ a_1 = \frac{1 - D}{1 + D} \]

- Block diagram:

- One of many choices

- Difference equation:

\[ y(n) = a_1 x(n) + x(n - 1) - a_1 y(n - 1) = a_1 [x(n) - y(n - 1)] + x(n - 1) \]

Matlab demo: thirantool.m
Impulse Responses of Thiran AP Filters

\[ N = 2 \]

\[ N = 5 \]

Phase Delay of Thiran Allpass Filter

\[ N = 2 \]

\[ N = 5 \]

\[ d = -0.5 \ldots 0.5 \]

\[ d = -0.5 \ldots 0.5 \]

• Matlab tools for fractional delay filter design are available at: http://www.acoustics.hut.fi/software/fdtools

### Conclusion

• Both FIR and IIR filters are useful in audio
• Practical simple filters: leaky integrators, resonators, shelving filters, equalizers, fractional delay filters
• Two types of comb filters: IIR and FIR (inverse)
• Allpass filters are phase equalizers, which are used alone or as building blocks
• Fractional delay filters offer accurate time delays
  – Closer to analog signal processing than DSP usually
  – Often either a Lagrange FIR or a Thiran allpass filter is useful
  – Try simple things first!
References (1)


References (2)

References (3)