Lecture 13
Inventory control

Basics of inventory control
Inventory models
- continuous review
- periodic review
- other models

Inventory control is boring but important

Operations try to meet customer requirements!

Inventory management decisions

What?
How much?
When?
Sales influences inventory management - ABCD - classification -

- There are differences between products on how important they are to the company
  - sales, number of customers, profit potential, invested capital, stock-out cost, criticality etc. → products should be managed differently
  - ABCD-classification divides products in 4 categories based on sales

- A and B - products objective high turnover and good service levels
  - strict control, continuous review (A) and periodic review (B), regular replenishment (variable lot size) and small delivery batches

- C and D - products objective to minimize economic burden
  - periodic review and 2-bin system, reducing number of products, minimizing fulfillment costs, safety stocks

- Classification only considers sales
  - no life-cycles, criticality, strategic importance etc. considered

Not all products are equal

Objectives influence inventory management

- **Swatch**
  - Low price
  - High discounts
  - Medium margin
  - Medium turns

- **Citizen**
  - Medium price
  - No discounts
  - High margin
  - High turns

- **Rolex**
  - High price
  - No discounts
  - Medium margin
  - Low turns

- **Minimize extras**
  - Do market testing and research at beginning
  - Use also central warehousing
  - Use early sales data to reorder / cut back

- **Maximize volume**
  - Set high service levels
  - Invest in inventory
  - Improve forecasting

- **Minimize inventory costs**
  - Set lower service levels
  - Invest in “central depot” stock location
  - Replenish/transfer among locations
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Inventory models

There are varied inventory control models

- Number and nature of products
  - one vs. many products
  - non-perishable vs. perishable
- Type of demand
  - constant, random, unknown demand
  - stationary vs.
  - non-stationary model
  - back-order vs. losing orders
- Inventory control model
  - continuous vs. periodic review
- Nature of deliveries
  - immediate, delayed, gradual, occasional replenishment
- Time horizon
  - one period, several periods, infinite time horizon
- Number of warehouses
  - one, parallel, network of warehouses
- Nature of costs/expenses
  - average cost, present value of costs etc.

Basic inventory control models

- Continuous review (Q) system
  - time between orders varies, lot size is fixed
  - economic order quantity
  - volume discounts
  - economic production lot size
  - requires continuous inventory control!
  - became more popular lately due to improved computerized solutions and lower prices (e.g. bar code, point-of-sale, voice recognition)
- Periodic review (P) system
  - time between orders is fixed, lot size varies
  - is based on periodic inventory control
  - still the more used control method
- Other systems
  - e.g. bin systems

Inventory control simple in theory

One product, level demand, fixed delivery time etc.
**Basic models are opposites of each other**

<table>
<thead>
<tr>
<th></th>
<th>Continuous review system</th>
<th>Periodic review system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time between orders</td>
<td>Varies (when inventory reaches reorder point)</td>
<td>Fixed (schedule)</td>
</tr>
<tr>
<td>Order quantity</td>
<td>Fixed</td>
<td>Varies</td>
</tr>
<tr>
<td>Operative costs</td>
<td>Higher</td>
<td>Lower</td>
</tr>
<tr>
<td>Size of inventory</td>
<td>Smaller</td>
<td>Larger</td>
</tr>
<tr>
<td>Products</td>
<td>Critical, popular and expensive products</td>
<td>Less critical products</td>
</tr>
</tbody>
</table>

**Continuous review system**

- In continuous review orders of fixed size are made after periods with variable length
  - Central questions: order quantity, timing of the order, pursued service level, size of safety stock
  - Requires a lot especially from inventory IT systems as balances have to be correct all the time
  - Instructing and motivating employees very important

**Trade-off between costs**

- Economic order quantity

**Shape of cost functions and common sense**

- Smaller order quantity means more orders
- Larger order quantity means more products to be inventoried
EOQ depends on the size of cost components

Calculating EOQ

1. Determine ordering costs (not necessarily easy)
   
   \[ \text{Ordering costs} = \frac{D}{Q} (S) = \frac{\text{Demand}}{\text{Lot size}} \times \text{Order cost per one order} \]

2. Determine holding costs (not necessarily easy)
   
   \[ \text{Holding costs} = \frac{Q}{2} (H) = \frac{\text{Lot size}}{2} \times \text{Holding cost per unit} \]

3. Calculate EOQ
   
   \[ \text{EOQ} = \sqrt{\frac{2DS}{H}} \]
   \[ = \sqrt{\frac{2 \times \text{Demand} \times \text{Order cost per one order}}{\text{Holding cost per unit}}} \]

Determining reorder point in EOQ-model

\[ \text{R} = dL \]

EOQ example

Sam’s Cat Hotel needs a lot of kitty litter to operate. Hotel entrepreneur purchases litter at the price of $11.70/bag and average demand is 90 bags per week. Ordering cost has been estimated to be $54 per order and annual holding cost 27% from purchasing costs. Delivery lead time is currently 3 weeks (18 work days). Hotel uses continuous review inventory system and is open around the year (52 weeks, 6 days a week). Calculate economic order quantity, time between orders, reorder point and total annual costs.

Order quantity:

\[ \text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 400 \times 90 \times 52}{11,70 \times 27\%}} = 400 \text{ units} \]

Time between orders:

\[ \text{D} = \frac{400}{90 \times 52} = 0,08547 \text{ years} \approx 4.44 \text{ weeks} \]

Reorder point:

\[ R = dL = 90 \times 3 = 270 \text{ units} \]

Total costs:

\[ \text{TC} = \frac{400}{2} \times (11,70 \times 27\%) + \frac{4680}{400} \times 54 = \$1263,60 \]
**EOQ example**

**EOQ’s sensitivity analysis**

- Close to EOQ volume the total costs function is rather flat
  - impact of wrongly estimating the cost variables rather small
  - especially to the right from EOQ (larger lot size) the total costs increase only slowly

- Impact of different cost variables’ change to total costs can be seen from the formula
  - increase in demand increases lot size
  - increase in ordering costs increases lot size
  - increase in holding costs decreases lot size
  - increase in interest rate decreases lot size
  - increase in unit price decreases lot size

\[
EOQ = \sqrt{\frac{2DS}{H}}
\]

**EOQ -models main assumptions**

- Demand is constant and known
  - demand is fulfilled from inventory; no stock-outs, no back orders and no uncertainty what so ever

- Lead time is constant and known

- Products’ unit price is fixed
  - no volume discounts

- Deliveries are complete lots
  - single delivery, no constraints on size of each lot

- Limited cost functions
  - only cost are ordering and holding; ordering assumed to be fixed and holding is based on average inventory

- Products independent from each other

**EOQ -models extensions**

- Variability in demand
  - Distribution of demand during lead time
  - Lower order quantity often leads to larger safety stock!

<table>
<thead>
<tr>
<th>Demand</th>
<th>Service level (Z₀L)</th>
<th>Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Probability</td>
<td>P(Stockout)</td>
<td>Lot size</td>
</tr>
<tr>
<td>Time</td>
<td>order moment</td>
<td>delivery moment</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Probability</th>
<th>Manager’s decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S</th>
<th>Safety stock (S)</th>
</tr>
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</table>

**How realistic are these?**

- Demand is seldom stable and stock-out cost can be the highest cost variable
  - service level thinking eases optimization
  - higher service level means higher safety stock

\[
R = dL + z \sigma_L
\]
Standard deviation of delivery lead time’s demand

\[
\sigma_t = 15 \\
90 \text{ units} \\
\sigma_r = 15 \\
90 \text{ units} \\
\sigma_t = 15 \\
90 \text{ units} \\
\sigma_t = 26 \\
270 \text{ units}
\]

Safety stock example

Use of kitty litter in Sam’s Cat Hotel is not totally steady. Due to liter’s criticality, the entrepreneur wants to be prepared also for higher consumption levels. Desired service level has been estimated to be 80%. Standard deviation of weekly demand has been estimated from historical data to be 15 bags per week. How do safety stocks change key inventory management numbers?

Order quantity and time between orders:

\[
\text{400 units and 4.44 weeks (stays the same)}
\]

Reorder point:

\[
R = dL + z\sigma_L = dL + z\sigma_t\sqrt{L} = 90 \times 3 + 0.84 \times 15 \times \sqrt{3} = 292
\]

from normal distribution

Total costs:

\[
TC = \left(\frac{400}{2} + 22\right)(11.70 \times 27\%) + \frac{4680}{400} \times 54 = $1333.10 + 69.50 \text{ per year}
\]

EOQ - models extensions

- variability in delivery lead times -

- Unfortunately there is variability also in delivery lead times

Decreasing the variability in lead times can be more advantageous than cutting the lead times themselves.
A baseball team is trying to decide the size of an order from the manufacturer. To make an analytical decision, the team's purchaser has been going around the organization and collected information he needs to make the order quantity decision. The total demand is 208 bats per year, the order cost is $70 per order, and the annual holding cost per bat per year is 38% of the purchase price. The bat selling company has priced its product in the following way: order 1-11 at $54.00 per bat, order 12-143 at $51.00 per bat, and in larger orders the price is $48.50 per bat. How many bats should purchaser order?

**Volume discount example**

**Price 54.00:**\[ \text{EOQ} = \sqrt{\frac{2 \times 208 \times 70}{54.00 \times 38\%}} = 37.7 \] order \( \text{EOQ} \) - 38

\[ \text{TC} = \frac{38}{2} * (54.00 \times 38\%) + \frac{208}{38} * 70 + 54.00 \times 208 = 12005 \]

**Price 51.00:**\[ \text{EOQ} = \sqrt{\frac{2 \times 208 \times 70}{51.00 \times 38\%}} = 38.7 \] order \( \text{EOQ} \) - 39

\[ \text{TC} = \frac{39}{2} * (51.00 \times 38\%) + \frac{208}{39} * 70 + 51.00 \times 208 = 11359 \] **cheapest**

**Price 48.50:**\[ \text{EOQ} = \sqrt{\frac{2 \times 208 \times 70}{48.50 \times 38\%}} = 39.7 \] order at least 144

\[ \text{TC} = \frac{144}{2} * (48.50 \times 38\%) + \frac{208}{144} * 70 + 48.50 \times 208 = 11516 \]
**Volume discount example**

- Inventory can also be replenished gradually during some period (not everything at the same time)
- Very practical in production environments
- E.g., consecutive steps in the production process or vertical integrated company with its own sales outlets (so both producer and reseller)
- Practical also in some other situations
- E.g., order is sent in portions immediately at the rate fulfillment (Amazon)

**EOQ - model extensions**

\[
\text{build up rate} = \frac{(p-d)}{p}
\]

**Economic Lot Sizing (ELS) example**

Vertically integrated carpet company produces popular Super Shag model. Management accounting shows that SS models holding costs are about 0.75 pounds per meter per year and ordering costs are 150 pounds (=set up cost). SS's demand has been forecasted to be 10 kilometers per year. Production factory is operating six days a week (just as stores) (311 days a year), deliveries are daily and SS's production speed is 150 meters per day.

Calculate Super Shag carpet's economic lot size, number of orders per year, how long it takes to produce each batch, maximum inventory level, and total inventory costs.

**ELS example**

Number of orders per year:

\[
D = \frac{10,000\text{m}}{2.257\text{m}} = 4.43 \text{ times/year} \approx \text{in every 70 working days}
\]

Production time:

\[
Q = \frac{2.257\text{m}}{150\text{m/day}} = \frac{15}{0.05} \text{ days}
\]

Maximum inventory:

\[
Q \left( \frac{p-d}{p} \right) = \frac{2.257 \times 150 - 32.15}{150} = 1.773\text{m}
\]

Total costs:

\[
TC = \frac{Q \times (p-d)}{2} + \frac{D}{Q} = \frac{2.257 \times 150 - 32.15}{2} \times 0.75 + \frac{10,000}{2257} \times 150 = £1329.6
\]
If inventory would be refilled with instantaneous replenishment

EOQ - model extensions
- noninstantaneous replenishment -
- if $p$ is much larger than $d$, ELS and EOQ are almost equal
  - due to slow usage rate the inventory filling resembles EOQ
- if $p$ and $d$ are nearly equal, production is less like batch production and more like a production line
  - product usage rate is same as production rate, and production is almost continuous
- lowering set up costs lowers the optimal production lot size
  - reduced holding costs will also lead to savings
- cooperation between companies and standardization of ordering costs can dramatically decrease the order size (JIT-production)

Periodic review system
- Periodic review is used because continuous review is not always economically feasible and takes too long time
  - part of the orders can be done only with fixed intervals
    - e.g. in grocery stores fixed schedules and routes
  - method is also used when several orders to one supplier are combined
- Periodic review increases stock-out risk
  - requires higher safety stock to guarantee same service level
- Demand influences on how much is ordered
  - e.g. season has to be taken into account

In periodic review models orders of variable size are made after regular time intervals
- central questions are the length of review interval, order quantity, pursued service level and size of safety stock
Periodic review systems formulas

Review interval / time between orders:
\[ P = \frac{EOQ}{D} \]
stable demand assumed
\[ D \] is the demand per period.
\[ P \] is the time between orders.

Target inventory level:
\[ T = d(P + L) + z\sigma_{P+L} \]
where:
- \( P \) is the demand during the protection interval.
- \( L \) is the lead time.
- \( d \) is the demand per period.
- \( z \) is the safety stock multiplier.
- \( \sigma_{P+L} \) is the standard deviation of demand during the protection interval.

Standard deviation of demand during the protection interval:
\[ \sigma_{P+L} = \sigma_d \sqrt{P + L} \]
Be accurate about time units!

Order quantity:
\[ Q = T - (IP + \text{inventory position} + \text{scheduled receipts} - \text{backorders}) \]

Total costs:
\[ TC = \left( \frac{dP}{2} + z\sigma_{P+L} \right) H + \frac{D}{dP} S \]

Periodic review example

Due to constant hurry Sam’s Cat Hotel is moving to periodic review system. Calculate the inventory review interval, target inventory level, amount to be ordered if there are 330 bags in the inventory right now, and annual total inventory costs.

Review interval / time between orders:
\[ P = \frac{EOQ}{D} = \frac{400}{4680} \]
\[ P = 0.086 \]
\[ 52 \times 6 = 26,67 \approx 27 \text{ days} \]

Target inventory level:
\[ T = d(P + L) + z\sigma_{P+L} \]
\[ T = 15 \times (27 + 18) + 0.84 \times 6,124 \times 27 + 18 \approx 710 \text{ units} \]

Order quantity:
\[ Q = T - IP = 710 - 330 = 380 \text{ units} \]

Total costs:
\[ TC = \left( \frac{dP}{2} + z\sigma_{P+L} \right) H + \frac{D}{dP} S \approx 1373 \]

Other inventory models - bin systems -

Two bin system
- Order one box to inventory
  - e.g. reminder if checkbook, "notify salespeople" in hardware store, bottom of label in a bar, line in the wall
- Full
- Empty

One bin system
- Order enough to fill up the box again