AALTO UNIVERSITY
Systems analysis laboratory

Mat-2.4136 Special Topics in Decision Making: Fuzzy Sets
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Exercise 2: Extended operations on fuzzy sets

1. Consider two t-norms $\top_1$ and $\top_2$. Is their convex linear combination, i.e. $\lambda\top_1(x,y)+(1-\lambda)\top_2(x,y)$ $\lambda \in [0,1]$, in general, a t-norm?

2. Given two t-norms $\top_1$ and $\top_2$, is it always possible to say $\top_1 > \top_2$ or $\top_1 < \top_2$ for all $(x,y) \in [0,1]^2$? Prove it or find a counterexample.

3. Consider the function $\perp_E(x,y) = \frac{x+y}{1+xy}$. Is it associative? What is its natural extension for three arguments? Namely, what is $\perp_E(x,y,z)$? Considering again, $\perp_E(x,y)$, what is its 'associated' t-norm $\top_E(x,y)$? Note that $\perp_E$ is a t-conorm.

4. You are going to have an informal dinner with friends *and* you are going to cook chicken *or* beef. So, first you choose the wine and then you decide whether to cook chicken or beef. Now you are in Alko and you want to buy a suitable wine. Suppose that there is a set of 3 wines $W = \{a, b, c\}$ and Table 1 describes some fuzzy subsets (FS) of W.

	a	b	c
FS of wines for candle light dinners	0.4	0.5	0.9
FS of wines for formal and job dinners	0.3	0.4	0.8
FS of wines for informal dinners	0.6	0.4	0.5
FS of wines suitable for fish	0.7	0.9	0.1
FS of wines suitable for chicken	0.4	0.4	0.6
FS of wines suitable for beef	0.8	0.5	0.9

Table 1: In the table there are the membership values of the wines in the different subsets.

Use the t-norm $\top_E$ and its associated t-conorm $\perp_E$ (see exercise above) to solve the problem and find the most suitable wine.

5. Consider the following three candidates evaluated by four judges

	J1	J2	J3	J4
C_1	0.9	0.8	0.3	0.5
C_2	0.2	0.7	0.7	0.7
C_3	0.6	0.6	0.6	0.7

Evaluate them by means of OWA functions with weight vectors $\mathbf{w}_1 = (0, 1/2, 1/2, 0)$ and $\mathbf{w}_2 = (1/8, 1/8, 2/8, 1/2)$.

Calculate the orness and the dispersion (entropy) for both $\mathbf{w}_1$ and $\mathbf{w}_2$.

Remember

$$disp(\mathbf{w}) = -\sum_{i=1}^{n} w_i \log w_i.$$