AALTO UNIVERSITY
Systems analysis laboratory

Mat-2.4136 Special Topics in Decision Making: Fuzzy Sets
Matteo Brunelli

Exercise 3: Extension principle

1. A function $f : X \to Y$ can be seen as a subset of $X \times Y$. Consider the function defined by the pairs $\{(1,2),(2,3),(3,2),(4,1),(5,3),(6,2)\}$ where, for instance, $(1,2)$ means that 1 is mapped to 2. Now, considering the crisp set

$$A = \{1,2,3,4\} \subseteq X$$

what is its image $B = f(A)$? And if A is the following fuzzy set?

$$A = \{(1,0.3),(2,0.9),(3,1),(4,0.1),(5,0.2),(6,1)\}$$

2. Consider the function $f(x) = x^2$ as a $f : \mathbb{R} \to \mathbb{R}$. If A is the following symmetric fuzzy number in the parameters a and α,

$$\mu_A(x) = \begin{cases} 1 - \frac{|a-x|}{\alpha} & \text{if } |a-x| \leq \alpha \\ 0, & \text{otherwise} \end{cases}$$

then what is B? Find the analytic expression of B.

3. Consider $f(x_1, x_2) = x_1 + x_2$ as a mapping $f : X \times X \to Y$ and the following two fuzzy sets on X

$$A = \{(1,0.6),(2,0.8),(3,1),(4,0.6)\} \quad B = \{(0,0.5),(1,1),(2,0.9)\}$$

Use the extension principle to find the fuzzy subset C of Y obtained by applying f to A and B.

4. Calculate

$$[-1,2] + [1,3];$$
$$[-2,4] - [3,6];$$
$$[-3,4] \cdot [-3,4];$$
$$[-4,6]/[1,2]$$

5. Consider the two fuzzy numbers

$$\mu_A(x) = \begin{cases} (x+1)/2, & if -1 \leq x \leq 1 \\ (3-x)/2, & if 1 < x \leq 3 \\ 0, & otherwise \end{cases} \qquad \mu_B(x) = \begin{cases} (x-1)/2, & if 1 \leq x \leq 3 \\ (5-x)/2, & if 3 < x \leq 5 \\ 0, & otherwise \end{cases}$$

Use the interval arithmetic to find their product. (Klir & Yuan, sections 4.3-4-4).