Lecture 6: Appendix

Notes on WACC vs. APV

Notes on WACC vs. APV methods

- WACC-method ($WACC = \frac{D}{D+E} r_D (1 - T_c) + \frac{E}{D+E} r_E$) assumes:
  - Constant debt ratio (D/D+E), i.e., rebalancing
  - Tax shields have the same risk as the company
- APV doesn't require rebalancing
  - In the specific case of constant debt level (D):

    $$PV(\text{tax shield}) = (\text{Interest} \cdot T_c)/r_D = (D \cdot r_D \cdot T_c)/r_{D=}\ D \cdot T_c$$

    Because tax shields have the same risk as the debt payments!
  - If you use $PV(\text{tax shield}) = (D \cdot T_c \cdot r_D)/WACC_{\text{All-equity}}$, you implicitly assume rebalancing which means tax shields have the same risk as the company
    - Same assumptions è same results from WACC and APV (see following slides for proof)

Bottom line: Use the discount rate that is based on the riskiness of tax shields!

Simple demonstration of WACC-APV consistency

- Assume rebalancing (i.e., risk of tax shields is same as risk of the firm)
- APV= PV (base case) + PV (tax shield)
- Suppose we are in year T and valuing a firm with one-year life (over T+1):

$$\text{APV}: V_T = \frac{CF_{T+1}}{1+WACC_{All\_equity}} + \frac{T_C \cdot r_D \cdot D_T}{1+WACC_{All\_equity}}$$

$$V_T \cdot (1+WACC_{All\_equity}) = CF_{T+1} + (T_C \cdot r_D \cdot D_T)$$

$$V_T \cdot (1+WACC_{All\_equity}) - (T_C \cdot r_D \cdot D_T) = CF_{T+1}$$

$$V_T \cdot (1+r_D \cdot \frac{D_T}{V_T} + r_E \cdot \frac{E_T}{V_T}) - (T_C \cdot r_D \cdot D_T) = CF_{T+1}$$

Simple demonstration of WACC-APV consistency

$$(V_T + r_D \cdot D_T + r_E \cdot E_T) - (T_C \cdot r_D \cdot D_T) = CF_{T+1}$$

$$V_T + (r_D \cdot D_T - T_C \cdot r_D \cdot D_T) + r_E \cdot E_T = CF_{T+1}$$

$$V_T + [r_D \cdot D_T \cdot (1 - T_C)] + r_E \cdot E_T = CF_{T+1}$$

$$V_T [\frac{V_T}{V_T} + \frac{r_D \cdot D_T \cdot (1 - T_C)}{V_T} + \frac{r_E \cdot E_T}{V_T}] = CF_{T+1}$$

$$V_T [1 + r_D \cdot (1 - T_C) \cdot \frac{D_T}{V_T} + r_E \cdot \frac{E_T}{V_T}] = CF_{T+1}$$

Simple demonstration of WACC-APV consistency

$$V_T = \frac{CF_{T+1}}{1 + r_D \cdot (1 - T_C) \cdot \frac{D_T}{V_T} + r_E \cdot \frac{E_T}{V_T}}$$

- The denominator is nothing but 1 + WACC that is adjusted for interest tax shields:

$$V_T = \frac{CF_{T+1}}{1 + WACC}$$

- This is a simple version of the full proof for a one-year firm
- Generalization is available in the textbook – Ch. 19, pg. 483, footnote 3 (Note: Debt ratio $L=D/V$ is assumed constant, not D)