Lecture 3: Vector-Controlled Induction Motor Drive
ELEC-E8402 Control of Electric Drives and Power Converters (5 ECTS)

Marko Hinkkanen
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Learning Outcomes

After this lecture and exercises you will be able to:

- Explain the principle of rotor-flux orientation
- Derive the rotor-flux orientation equations (torque, flux dynamics, slip relation) using the inverse-$\Gamma$ model
- Draw block diagrams for the most typical control schemes and explain them
- Derive the current model and explain its properties
Vector Control Methods

- Based on the dynamic motor model
- Rotor-flux-oriented vector control, direct torque control (DTC)
- Torque can be controlled
- High accuracy and fast dynamics
- Speed measurement can be replaced with speed estimation in most applications

DC-link voltage is typically measured. However, it will be omitted in the following block diagrams (or constant $U_{dc}$ is assumed).

\[
\begin{align*}
\omega_{M,ref} & \rightarrow \text{Speed controller} \\
T_{M,ref} & \rightarrow \text{Vector controller} \\
U_{dc} & \rightarrow M \\
i_a, i_b, i_c & \rightarrow M \\
\omega_M & \rightarrow M
\end{align*}
\]
Outline

State-Space Representation

Principle of Rotor-Flux Orientation

Flux Estimation With the Current Model
Review: Model in Synchronous Coordinates

- **Voltage equations**

  \[
  u_s = R_s i_s + \frac{d}{dt} \psi_s + j \omega_s \psi_s
  \]

  \[
  u_R = R_R i_R + \frac{d}{dt} \psi_R + j \omega_r \psi_R = 0
  \]

- **Flux linkages**

  \[
  \psi_s = L_\sigma i_s + \psi_R
  \]

  \[
  \psi_R = L_M (i_s + i_R)
  \]

- **Steady state:** \( \frac{d}{dt} = 0 \)
State-Space Representation

- Stator current $i_s$ and rotor flux $\psi_R$ are selected as state variables.
- Derivation: rotor current $i_R$ and stator flux $\psi_s$ are eliminated from the voltage equations by means of the flux equations:

$$L_\sigma \frac{d i_s}{d t} = u_s - (R_s + R_R + j_\omega_s L_\sigma) i_s + \left( \frac{R_R}{L_M} - j_\omega_m \right) \psi_R$$

$$\frac{d \psi_R}{d t} = R_R i_s - \left( \frac{R_R}{L_M} + j_\omega_r \right) \psi_R$$

- Dynamics of the stator current are governed by current control.
- Dynamics of the rotor flux are taken into account by rotor-flux orientation.

Go through the derivation of these equations (see the compendium).
Outline

State-Space Representation

Principle of Rotor-Flux Orientation

Flux Estimation With the Current Model
Rotor-Flux Dynamics

- Fast closed-loop stator-current controller is used
- Stator current is the input from the point of view of the rotor-flux dynamics
- Rotor equations in synchronous coordinates rotating at $\omega_s$

$$\begin{align*}
\frac{d\psi_R}{dt} &= -R_R i_R - j(\omega_s - \omega_m)\psi_R \\
\psi_R &= L_M(i_s + i_R) \quad \Rightarrow \quad i_R = \psi_R / L_M - i_s
\end{align*}$$

- Rotor current can be eliminated

$$\frac{d\psi_R}{dt} = -\left(\frac{R_R}{L_M} + j\omega_r\right)\psi_R + R_R i_s$$
Rotor-Flux Orientation

- d-axis of coordinate system is fixed to the rotor flux

\[ \psi_R = \psi_{Rd} + j\psi_{Rq} = \psi_R + j \cdot 0, \quad i_s = i_d + j i_q \]

- Real and imaginary parts of the rotor-flux dynamics

\[ \frac{d\psi_R}{dt} = - \frac{R_R}{L_M} \psi_R + R_R i_d \quad \text{(in the steady state } \psi_R = L_M i_d) \]
\[ 0 = -\omega_r \psi_R + R_R i_q \]

- Rotor-flux magnitude \( \psi_R \) follows \( i_d \) slowly,

\[ \psi_R(s) = \frac{L_M}{1 + s\tau_r} i_d(s) \quad \text{(in the Laplace domain)} \]

due to the rotor time constant \( \tau_r = L_M / R_R \) (typically 0.1…1.5 s)
Rotor-Flux Orientation

- d axis of coordinate system is fixed to the rotor flux:

\[ \psi_R = \psi_R + \textbf{j} \cdot \textbf{0}, \quad i_s = i_d + \textbf{j}i_q \]

- Electromagnetic torque

\[ T_M = \frac{3p}{2} \text{Im} \left\{ i_s \psi_R^* \right\} = \frac{3p}{2} \psi_R i_q \]

- If \( \psi_R \) is constant, the torque can be controlled using \( i_q \) (without delays)

The coordinate system could be fixed to the stator flux \( \psi_s \) instead of the rotor flux. This stator-flux orientation would simplify the field weakening, but other parts of the control system would become more complicated.
Steady-State Equivalent Circuit in Rotor-Flux Coordinates

\[ i_s = R_s i_d + j \omega_s L_{\sigma} i_d + j\omega_s L_M i_d + j\omega_s \psi_R \]

\[ j i_q = \frac{R_R}{\omega_r/\omega_s} \]

\[ i_R = -j i_q \]
Stator Coordinates \((\alpha\beta)\)

- Vectors are rotating (in the steady state \(\vartheta_s = \omega_s t\))
- Controlling the torque

\[
T_M = \frac{3p}{2} \text{Im} \left\{ i_s^s (\psi_R^s)^* \right\} \\
= \frac{3p}{2} \left( i_\beta \psi_{R\alpha} - i_\alpha \psi_{R\beta} \right)
\]

would be difficult
Rotor-Flux Coordinates (dq)

- Variables are constant in the steady state
- Torque

\[ T_M = \frac{3p}{2} \text{Im} \left\{ i_s \psi_R^* \right\} = \frac{3p}{2} \psi_R i_q \]

easily controllable via \( i_q \)
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Example Measured Waveforms: 45-kW Induction Motor Drive
Rotor-Flux-Oriented Vector Control

Estimated rotor-flux coordinates

Stator coordinates

Calculated in estimated rotor-flux coordinates

Very fast current-control loop

Rotor-flux angle is estimated

\( \psi_{R,\text{ref}} \)

\( i_{s,\text{ref}} \)

\( u_{s,\text{ref}} \)

\( u_{a,\text{ref}}, u_{b,\text{ref}}, u_{c,\text{ref}} \)

\( i_a, i_b, i_c \)

\( \omega_m \)
Space-Vector and Coordinate Transformations

- **Space-vector transformation (abc/αβ)**
  
  \[ i_s^s = \frac{2}{3} \left( i_a + i_b e^{j2\pi/3} + i_c e^{j4\pi/3} \right) \]

- **Transformation to rotor coordinates (αβ/dq)**
  
  \[ i_s = i_s^s e^{-j\hat{\vartheta}_s} \]

- **Combination of these two transformations is often referred to as an abc/dq transformation**

- **Similarly, the inverse transformation is referred to as a dq/abc transformation**
1. Flux-producing current reference

\[ i_{d,\text{ref}} = \frac{\psi_{R,\text{ref}}}{\hat{L}_M} \]  

(where the hat refers to estimates)

- Integral term based on \( u_{\text{max}} - |u_{s,\text{ref}}| \) can be used for field weakening
- Flux controller could be used for improving the flux dynamics in the case of varying \( \psi_{R,\text{ref}} \)

2. Torque-producing current reference

\[ i_{q,\text{ref}} = \frac{2T_{M,\text{ref}}}{3p\psi_{R,\text{ref}}} \]

- Flux reference \( \psi_{R,\text{ref}} \) is often replaced with the estimate \( \hat{\psi}_R \)

Outline

State-Space Representation

Principle of Rotor-Flux Orientation

Flux Estimation With the Current Model
Current-Model Flux Estimator in Stator Coordinates

Current model is based on the rotor voltage equation
\[
\frac{d\hat{\psi}_R^s}{dt} = -\left(\frac{\hat{R}_R}{\hat{L}_M} - j\omega_m\right)\hat{\psi}_R^s + \hat{R}_R i_s^s
\]

Corresponding forward Euler approximation
\[
\hat{\psi}_R^s(k + 1) = \hat{\psi}_R^s(k) + T_s \left\{-\left[\frac{\hat{R}_R}{\hat{L}_M} - j\omega_m(k)\right] \hat{\psi}_R^s(k) + \hat{R}_R i_s^s(k)\right\}
\]

where \(T_s\) is the sampling period and \(k\) is the discrete-time index.

At each time step, the angle of the flux estimate \(\hat{\psi}_R^s = \hat{\psi}_{R\alpha} + j\hat{\psi}_{R\beta}\) is
\[
\hat{\theta}_s = \text{atan2} \left(\hat{\psi}_{R\beta}, \hat{\psi}_{R\alpha}\right)
\]

In practice, the forward Euler approximation should not be used in stator coordinates due to its poor accuracy and limited stability.
Current Model in Estimated Rotor Flux Coordinates

- Signals fed to the flux estimator are DC in the steady state
- Discrete-time implementation becomes easier
Current-Model Flux Estimator in Estimated Rotor-Flux Coordinates

\[
\frac{d\hat{\psi}_R}{dt} = - \left( \frac{\hat{R}_R}{\hat{L}_M} + j\hat{\omega}_r \right) \hat{\psi}_R + \hat{R}_R i_s \\
\hat{\psi}_R = \hat{\psi}_R + j \cdot 0
\]

- Real and imaginary parts in estimated flux coordinates

\[
\frac{d\hat{\psi}_R}{dt} = - \frac{\hat{R}_R}{\hat{L}_M} \hat{\psi}_R + \hat{R}_R i_d \\
\hat{\omega}_r = \frac{\hat{R}_R i_q}{\hat{\psi}_R}
\]

- Flux-angle estimation

\[
\hat{\vartheta}_s = \int \hat{\omega}_s dt = \int (\omega_m + \hat{\omega}_r) dt
\]
Indirect Field Orientation (IFO)

- Current reference is used as an input of the flux estimator
- Flux estimator is also simplified (see the following slide)
Flux-magnitude dynamics are omitted in the slip relation

\[ \hat{\omega}_r = \frac{R_R i_{q,\text{ref}}}{\psi_{R,\text{ref}}} \]

Flux-angle estimation

\[ \hat{\psi}_s = \int (\omega_m + \hat{\omega}_r) \, dt \]

Poor performance if the flux reference \( \psi_{R,\text{ref}} \) is not constant or if the current controller does not work as intended
Properties of the Current Model and IFO

Disadvantages:

▶ Rotor speed measurement is needed
▶ Converges slowly (with the rotor time constant), which can be a problem if the flux reference $\psi_{R,\text{ref}}$ is varied
▶ Inaccurate model parameters $\hat{R}_R$ and $\hat{L}_M$ cause errors in field orientation ⇒ degraded control performance

Advantages:

▶ Simplicity
▶ Robustness
Reasons for Parameter Detuning: Actual Motor Parameters Vary

- Resistances depend on:
  - Temperature (about 0.4%/K)
  - Frequency due to the skin effect (especially the resistances of the rotor bars)
- Inductances depend on the magnetic state
  - Stator (magnetizing) inductance increases as the stator (rotor) flux decreases in the field-weakening region
  - Torque may also have an effect on the inductances
- Identification of the motor parameters is never perfect
- Some phenomena are omitted in the model but exist in the actual machine (e.g. core losses)
Magnetic Saturation Characteristics: 2.2-kW Motor as an Example

- Stator inductance $L_s = L_\sigma + L_M$ depends on the stator-flux magnitude $\psi_s$
- Effect should be taken into account in control, if field weakening is used

No-load saturation characteristics

Rated operating point $\psi_s = \psi_{sN}$

Field weakening at $\psi_s = 0.5\psi_{sN}$

Corresponding stator inductance

$\psi_s$ (p.u.)

$\psi_s$ (p.u.)

$L_s$ (p.u.)

$L_s$ (p.u.)
Summary: Rotor-Flux Orientation

- Decoupled control of the flux and the torque, as in the DC machines
- d-axis of the coordinate system is fixed to the rotor flux vector (or its estimate in practice)
- Rotor-flux magnitude is controlled using the d-component of the stator current
- Torque is controlled using the q-component of the current

Similar control structure can also be used in sensorless methods, but a different flux (and speed) estimator is needed.
Further Reading

Modeling


Flux control, field weakening, loss minimization:


Sensorless methods:
