

**Problem 1: Characteristics of an interior-permanent-magnet motor**

The data of an interior-permanent-magnet motor is:

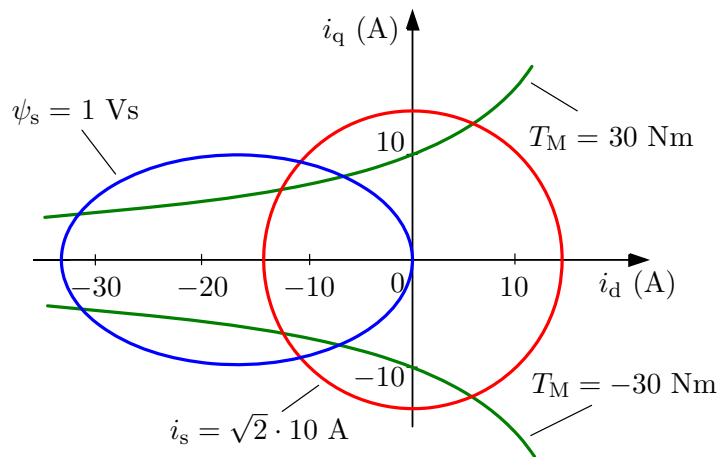
rated voltage	$U_N = 400 \text{ V}$	direct-axis inductance	$L_d = 0.06 \text{ H}$
rated current	$I_N = 10 \text{ A}$	quadrature-axis inductance	$L_q = 0.10 \text{ H}$
pole pairs	$p = 2$	permanent-magnet flux	$\psi_f = 1 \text{ Vs}$

The stator resistance is omitted. Draw the following characteristics in the  $i_d$ - $i_q$  plane:

- (a) constant current  $i_s = \sqrt{2}I_N$  (rated value);
- (b) constant torque of 30 Nm;
- (c) constant stator flux  $\psi_s = 1 \text{ Vs}$ .

**Solution**

The figure below shows the loci in the  $i_d$ - $i_q$  plane corresponding to given constant values.



- (a) The square of the stator current can be expressed as

$$i_s^2 = i_d^2 + i_q^2 \quad \text{or} \quad |i_q| = \sqrt{i_s^2 - i_d^2}$$

Hence, the constant stator current in the  $i_d$ - $i_q$  plane corresponds to the circle (having the radius of  $i_s = \sqrt{2} \cdot 10 \text{ A}$  in this case). The operating point should be located inside the circle corresponding to the maximum current.

- (b) The electromagnetic torque is

$$\begin{aligned} T_M &= \frac{3p}{2} \text{Im} \{ i_s \psi_s^* \} = \frac{3p}{2} \text{Im} \{ (i_d + j i_q)(\psi_f + L_d i_d - j L_q i_q) \} \\ &= \frac{3p}{2} [\psi_f i_q + (L_d - L_q) i_d i_q] \end{aligned}$$

The q-component of the current can be solved as

$$i_q = \frac{2T_M}{3p [\psi_f + (L_d - L_q) i_d]}$$

The current locus corresponding to the constant torque is a hyperbola. The point of the hyperbola being closest to the origin is an optimal operating point if the stator current is to be minimized.

- (c) The square of the stator flux is

$$\psi_s^2 = \psi_d^2 + \psi_q^2 = (\psi_f + L_d i_d)^2 + (L_q i_q)^2$$

from which the q-component of the stator current can be solved as

$$|i_q| = \frac{\sqrt{\psi_s^2 - (\psi_f + L_d i_d)^2}}{L_q}$$

The current locus is now an ellipse. The stator-flux magnitude is limited due to the limited voltage ( $u_s \approx \omega_m \psi_s$ ) and the operating point has to be located inside the ellipse. Why does the ellipse touch the origin in this case ( $\psi_s = 1$  Vs)?

### Problem 2: Current-minimizing control characteristics

Consider the interior-permanent-magnet motor in the preceding problem.

- Derive expressions for the current components  $i_d$  and  $i_q$ , when the stator current is constant and the torque is maximized.
- Calculate the maximum torque at the rated current.
- Calculate the rotational base speed corresponding to the rated voltage for the current and torque obtained above.
- Calculate the displacement power factor  $\cos \varphi$  and draw a vector diagram for the operating point obtained above.

### Solution

- (a) Maximization of the torque at constant current corresponds to minimization of the current at constant torque. The current component  $i_q$  is

$$i_q = \sqrt{i_s^2 - i_d^2} \quad (1)$$

where positive  $i_q$  has been assumed. Hence, the torque can be expressed using  $i_d$  as

$$T_M = \frac{3p}{2} [\psi_f + (L_d - L_q) i_d] i_q = \frac{3p}{2} [\psi_f + (L_d - L_q) i_d] \sqrt{i_s^2 - i_d^2}$$

where  $i_s$  is constant. Differentiation of the torque with respect to  $i_d$  leads to

$$\begin{aligned} \frac{2}{3p} \frac{dT_M}{di_d} &= \frac{d}{di_d} \{ [\psi_f + (L_d - L_q) i_d] (i_s^2 - i_d^2)^{1/2} \} \\ &= (L_d - L_q) (i_s^2 - i_d^2)^{-1/2} + [\psi_f + (L_d - L_q) i_d] \cdot \frac{1}{2} \cdot (i_s^2 - i_d^2)^{-1/2} \cdot (-2i_d) \\ &= (L_d - L_q) \sqrt{i_s^2 - i_d^2} - \frac{\psi_f i_d + (L_d - L_q) i_d^2}{\sqrt{i_s^2 - i_d^2}} \end{aligned}$$

The maximum torque is achieved at  $dT_M/di_d = 0$ , leading to

$$i_d^2 + \frac{\psi_f}{2(L_d - L_q)}i_d - \frac{i_s^2}{2} = 0 \quad (2)$$

The solutions of this quadratic equation are

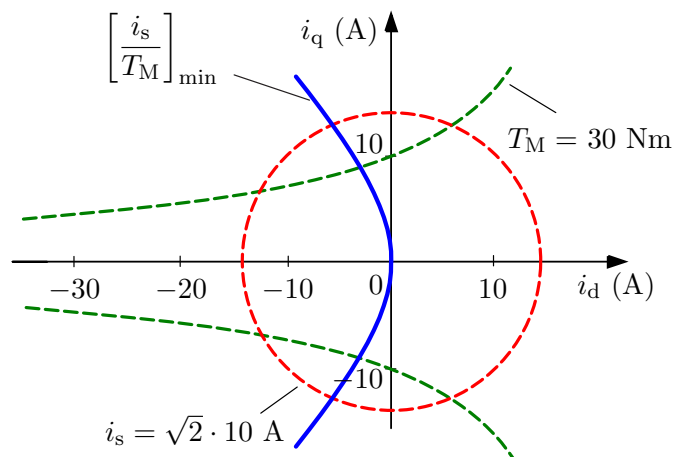
$$i_d = \frac{\psi_f}{4(L_q - L_d)} \pm \sqrt{\frac{\psi_f^2}{16(L_q - L_d)^2} + \frac{i_s^2}{2}} \quad (3)$$

The plus sign in front of the square root can be omitted (at least if  $L_q > L_d$  as in this problem).

**Remark:** For plotting the MTPA locus, (2) can be rewritten using (1) as

$$i_q^2 = i_d^2 + \frac{\psi_f}{L_d - L_q}i_d \quad \text{or} \quad i_q = \pm \sqrt{i_d^2 + \frac{\psi_f}{L_d - L_q}i_d}$$

The figure below shows the locus in the  $i_d$ - $i_q$  plane.



(b) Based on (3) and (1), the current components are

$$\begin{aligned} i_d &= \frac{\psi_f}{4(L_q - L_d)} - \sqrt{\frac{\psi_f^2}{16(L_q - L_d)^2} + \frac{i_s^2}{2}} \\ &= \frac{1 \text{ Vs}}{4 \cdot (0.1 \text{ H} - 0.06 \text{ H})} - \sqrt{\frac{(1 \text{ Vs})^2}{16 \cdot (0.1 \text{ H} - 0.06 \text{ H})^2} + \frac{(\sqrt{2} \cdot 10 \text{ A})^2}{2}} = -5.54 \text{ A} \\ i_q &= \sqrt{i_s^2 - i_d^2} = \sqrt{(\sqrt{2} \cdot 10)^2 - 5.54^2} \text{ A} = 13.0 \text{ A} \end{aligned}$$

The maximum torque at the given current is

$$\begin{aligned} T_M &= \frac{3p}{2} [\psi_f + (L_d - L_q)i_d] i_q \\ &= \frac{3}{2} \cdot 2 \cdot [1 \text{ Vs} + (0.06 \text{ H} - 0.1 \text{ H}) \cdot (-5.54 \text{ A})] \cdot 13.0 \text{ A} = 47.7 \text{ Nm} \end{aligned}$$

- (c) In the steady state, the magnitude of the stator voltage is  $u_s = \omega_m \psi_s$ , where the stator resistance is omitted. The stator flux components are

$$\begin{aligned} \psi_d &= \psi_f + L_d i_d = 1 \text{ Vs} + 0.06 \text{ H} \cdot (-5.54 \text{ A}) = 0.67 \text{ Vs} \\ \psi_q &= L_q i_q = 0.1 \text{ H} \cdot 13.0 \text{ A} = 1.30 \text{ Vs} \end{aligned}$$

and the flux magnitude is

$$\psi_s = \sqrt{\psi_d^2 + \psi_q^2} = \sqrt{0.67^2 + 1.30^2} \text{ Vs} = 1.46 \text{ Vs}$$

At the base speed, the stator voltage is  $u_s = \sqrt{2/3} \cdot 400 \text{ V} = 326.7 \text{ V}$  (peak value of phase-to-neutral voltage). Hence, the base speed as electrical radians is

$$\omega_m = \frac{u_s}{\psi_s} = \frac{326.7 \text{ V}}{1.46 \text{ Vs}} = 223.3 \text{ rad/s}$$

corresponding to the rotational speed

$$n = \frac{\omega_m}{2\pi p} = \frac{223.3 \text{ rad/s}}{2\pi \cdot 2} \cdot 60 \text{ s/min} = 1066 \text{ r/min}$$

- (d) The output power of the machine is

$$P_M = T_M \omega_M = T_M \frac{\omega_m}{p}$$

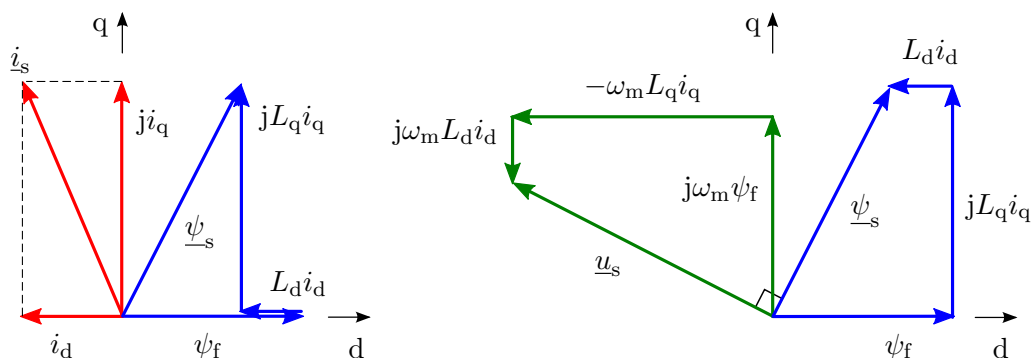
Since the losses are omitted, the output power equals the input power

$$P_s = \frac{3}{2} \text{Re} \{ \underline{u}_s \underline{i}_s^* \} = 3 \frac{U_N}{\sqrt{3}} I_N \cos \varphi$$

The power factor can be solved

$$\cos \varphi = \frac{T_M \omega_m}{\sqrt{3} p U_N I_N} = \frac{47.7 \text{ Nm} \cdot 223.3 \text{ rad/s}}{\sqrt{3} \cdot 2 \cdot 400 \text{ V} \cdot 10 \text{ A}} = 0.77$$

The vector diagrams are illustrated in the figure below.



Current and flux diagram

Flux and voltage diagram

The current, flux, and voltage components used in the diagrams are:

$$\begin{aligned} i_d &= -5.54 \text{ A} & i_q &= 13.0 \text{ A} \\ \psi_d &= 0.67 \text{ Vs} & \psi_q &= 1.30 \text{ Vs} \\ u_d &= -291.5 \text{ V} & u_q &= 149.1 \text{ V} \end{aligned}$$

and the corresponding vectors are

$$\underline{i}_s = 14.1/113.1^\circ \text{ A} \quad \underline{\psi}_s = 1.46/62.8^\circ \text{ Vs} \quad \underline{u}_s = 326.7/152.8^\circ \text{ V}$$

The power factor can also be obtained from the vector diagram:

$$\cos \varphi = \cos (152.8^\circ - 113.1^\circ) = 0.77$$

