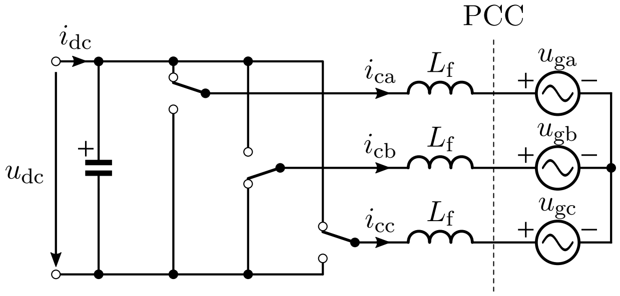
Problem 1: An operating point of a grid converter

The figure below shows a grid converter, whose DC voltage is $u_{\mathrm{dc}} = 600$ V, DC current is $i_{\mathrm{dc}} = 10$ A, and filter inductance is $L_{\mathrm{f}} = 10$ mH. The electric grid is assumed to be a balanced three-phase voltage source with frequency of 50 Hz and phase-to-phase rms voltage of 400 V. The displacement power factor at the PCC is controlled to unity. The converter can be assumed to be lossless and switching-cycle-averaged quantities are considered.

(a) Calculate the converter current vector in grid-voltage coordinates.

(b) Calculate the magnitude of the converter output voltage vector.



Solution

(a) In grid-voltage coordinates,

$$\underline{u}_{\mathrm{g}} = u_{\mathrm{gd}} + \mathrm{j}u_{\mathrm{gq}} = u_{\mathrm{g}} + \mathrm{j}0 = \sqrt{2/3} \cdot 400 \text{ V} = 326.6 \text{ V}$$

holds. The converter output current is $\underline{i}_{\mathrm{c}} = i_{\mathrm{cd}} + \mathrm{j}i_{\mathrm{cq}}$. Hence, the real power and reactive power at the PCC are

$$p_{\mathrm{g}} = \frac{3}{2}\operatorname{Re}\left\{\underline{u}_{\mathrm{g}}\underline{i}_{\mathrm{c}}^*\right\} = \frac{3}{2}u_{\mathrm{g}}i_{\mathrm{cd}} \qquad q_{\mathrm{g}} = \frac{3}{2}\operatorname{Im}\left\{\underline{u}_{\mathrm{g}}\underline{i}_{\mathrm{c}}^*\right\} = -\frac{3}{2}u_{\mathrm{g}}i_{\mathrm{cq}}$$

respectively. The real power in the steady state is

$$p_{\mathrm{g}} = u_{\mathrm{dc}}i_{\mathrm{dc}} = 600 \text{ V} \cdot 10 \text{ A} = 6 \text{ kW}$$

since the converter is assumed to be lossless. The displacement power factor is unity, corresponding to the reactive power $q_{\mathrm{g}} = 0$. The d and q currents are

$$i_{\mathrm{cd}} = \frac{2p_{\mathrm{g}}}{3u_{\mathrm{g}}} = \frac{2 \cdot 6 \text{ kW}}{3 \cdot 326.6 \text{ V}} = 12.2 \text{ A} \qquad i_{\mathrm{cq}} = -\frac{2q_{\mathrm{g}}}{3u_{\mathrm{g}}} = 0$$

Remark: The power flows from the DC link to the grid, i.e., the converter operates as an inverter in this operating point. Due to the inductance L_{f}, the reactive power at the output of the converter, $\frac{3}{2}\operatorname{Im}\{\underline{u}_{\mathrm{c}}\underline{i}_{\mathrm{c}}^*\}$, is nonzero, where $\underline{u}_{\mathrm{c}}$ is the converter output voltage, see Part (b).

(b) The converter output voltage in synchronous coordinates is

$$\underline{u}_{\mathrm{c}} = L_{\mathrm{f}}\frac{\mathrm{d}\underline{i}_{\mathrm{c}}}{\mathrm{d}t} + \mathrm{j}\omega_{\mathrm{g}}L_{\mathrm{f}}\underline{i}_{\mathrm{c}} + \underline{u}_{\mathrm{g}}$$

In grid-voltage coordinates, the steady-state voltage is

$$\underline{u}_\mathrm{c} = \mathrm{j}\omega_\mathrm{g} L_\mathrm{f} \underline{i}_\mathrm{c} + u_\mathrm{g} = u_\mathrm{g} - \omega_\mathrm{g} L_\mathrm{f} i_\mathrm{cq} + \mathrm{j}\omega_\mathrm{g} L_\mathrm{f} i_\mathrm{cd}$$

Hence, the components are

$$\begin{aligned} u_\mathrm{cd} &= u_\mathrm{g} - \omega_\mathrm{g} L_\mathrm{f} i_\mathrm{cq} = 326.6\ \mathrm{V} \\ u_\mathrm{cq} &= \omega_\mathrm{g} L_\mathrm{f} i_\mathrm{cd} = 2\pi \cdot 50\ \mathrm{rad/s} \cdot 0.01\ \mathrm{H} \cdot 12.2\ \mathrm{A} = 38.4\ \mathrm{V} \end{aligned}$$

The magnitude of the converter output voltage is

$$u_\mathrm{c} = \sqrt{u_\mathrm{cd}^2 + u_\mathrm{cq}^2} = \sqrt{(326.6\ \mathrm{V})^2 + (38.4\ \mathrm{V})^2} = 329\ \mathrm{V}$$

Remark: For comparison, the maximum voltage in the linear modulation region is $u_\mathrm{dc}/\sqrt{3} = 600\ \mathrm{V}/\sqrt{3} = 346\ \mathrm{V}$.

Problem 2: DC-link voltage controller

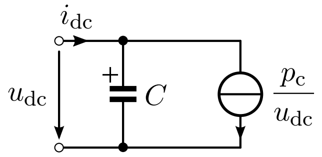
A PI controller is used to regulate the DC-link voltage of a power converter,

$$p_\mathrm{c,ref} = -k_\mathrm{p}(W_\mathrm{dc,ref} - W_\mathrm{dc}) - k_\mathrm{i} \int (W_\mathrm{dc,ref} - W_\mathrm{dc})\mathrm{d}t$$

where $p_\mathrm{c,ref}$ is the reference of the converter output power, $W_\mathrm{dc} = (C/2)u_\mathrm{dc}^2$ is the energy of the DC-link capacitor, and $W_\mathrm{dc,ref} = (C/2)u_\mathrm{dc,ref}^2$ is its reference. Power control is assumed to be ideal, i.e. $p_\mathrm{c} = p_\mathrm{c,ref}$. The input power $p_\mathrm{dc} = u_\mathrm{dc} i_\mathrm{dc}$ is an unknown disturbance.

(a) Calculate the closed-loop transfer functions $W_\mathrm{dc}(s)/W_\mathrm{dc,ref}(s)$ and $W_\mathrm{dc}(s)/p_\mathrm{dc}(s)$.

(b) Express the controller gains k_p and k_i as functions of the damping ratio ζ and the undamped natural frequency ω_0 of the closed-loop characteristic polynomial.



Solution

(a) The dynamics of the DC link are governed by

$$\frac{\mathrm{d}W_\mathrm{dc}}{\mathrm{d}t} = p_\mathrm{dc} - p_\mathrm{c} \tag{1}$$

where $W_\mathrm{dc} = (C/2)u_\mathrm{dc}^2$ is the energy of the DC capacitor and the power $p_\mathrm{dc} = u_\mathrm{dc} i_\mathrm{dc}$ is an unknown load disturbance. The converter output power is

$$p_\mathrm{c} = -k_\mathrm{p}(W_\mathrm{dc,ref} - W_\mathrm{dc}) - k_\mathrm{i} \int (W_\mathrm{dc,ref} - W_\mathrm{dc})\mathrm{d}t \tag{2}$$

since ideal power control is assumed. Substituting the control law (2) in (1) and Laplace transforming yields

$$W_{\mathrm{dc}}(s) = \frac{sk_{\mathrm{p}} + k_{\mathrm{i}}}{s^2 + sk_{\mathrm{p}} + k_{\mathrm{i}}} W_{\mathrm{dc,ref}}(s) + \frac{s}{s^2 + sk_{\mathrm{p}} + k_{\mathrm{i}}} p_{\mathrm{dc}}(s)$$

It can be seen that $W_{\mathrm{dc}} = W_{\mathrm{dc,ref}}$ or $u_{\mathrm{dc}} = u_{\mathrm{dc,ref}}$ in the steady state ($s = 0$) due to the integral action of the control law. The transient response depends on the controller gains.

(b) The characteristic polynomial of the second-order system can be expressed as $s^2 + 2\zeta\omega_0 s + \omega_0^2$. Hence, the gains are

$$k_{\mathrm{p}} = 2\zeta\omega_0 \qquad k_{\mathrm{i}} = \omega_0^2$$

Typically, it is desirable to select the damping ratio in a range $\zeta = 0.7\ldots1$. A critically damped system is obtained if $\zeta = 1$, i.e. there is a double pole at $s = -\omega_0$. The assumption on the ideal power control holds well if the undamped natural frequency ω_0 is much lower than the current-control bandwidth.