Lecture#11
Difference – in – difference regression
Causal parameters part II

Difference estimator

- Let's assume a discrete treatment:

X_i = 1 if individual i gets the treatment

X_i = 0 if individual i does not get the treatment

Randomized Control Trial (RCT)

- In economics:

1. The researcher decides on what the experiment is.
2. The researcher decides what the population of interest is.
3. The researcher draws a random sample.
4. Individuals in the random sample are randomly allocated into control and treatment groups.

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

- Q1: what is the interpretation of β_1?

- Q2: is there an omitted variable problem?

- Q3: what if individuals truly randomized and the researcher observes other characteristics besides Y_i, X_i?

- Q4: is there any reason to include control variables (W_i)?

1. Efficiency - $cor(X_i, W_i)$=0 by design.

2. Control for randomization: if β_1 without controls $\neq \beta_1$ with controls, then RCT has failed.

RCT and Difference estimator

Control: $Y_i = \beta_0 + u_i$

Treatment: $Y_i = \beta_0 + \beta_1 + u_i$

Difference: $\Delta Y_i = \beta_1$

So what is DiD?

- Imagine the researcher has 2 consecutive observations / individual:

1. Before experiment

2. After experiment

Examples of DiD setups

- WFH:

1. performance of workers before and after WFH allowed.

2. difference in change of performance between WFH and those who stay in office.

Examples of DiD setups

- VI:

1. Ticket prices in theatres before and after VI is allowed (notice difference to Gil data).

2. difference in change of ticket prices between VI and non-VI theatres.

Regression framework

Write generally

$$Y_{it} = \beta_0 + \beta_{group}X_i + \beta_{period}D_{after,it} + \beta_1 X_i D_{after,it} + u_i + u_{it}$$

X_i = 1 if in control group, 0 otherwise.

$D_{after,it}$ = 1 if period after, 0 otherwise.

u_i = observation unit unobservables (”fixed effect”).

Regression framework – control group

For control group, X_i=0.

Therefore can rewrite

$$Y_{it} = \beta_0 + \beta_{group}X_i + \beta_{period}D_{after,it} + \beta_1 X_i D_{after,it} + u_i + u_{it}$$

as

$$Y_{it} = \beta_0 + \beta_{period}D_{after,it} + u_i + u_{it}$$

Regression framework – control group

$$Y_{it} = \beta_0 + \beta_{period} D_{after,it} + u_i + u_{it}$$

1. After $Y_{i2} = \beta_0 + \beta_{period} + u_i + u_{i2}$

2. Before $Y_{i1} = \beta_{0i} + u_i + u_{i1}$

3. Diff $\Delta Y_i = \beta_{period} + u_{i2} - u_{i1}$

Regression framework – treatment group

For treatment group, X_i=1.

Therefore can rewrite

$$Y_{it} = \beta_0 + \beta_{group}X_i + \beta_{period}D_{after,it} + \beta_1 X_i D_{after,it} + u_i + u_{it}$$

as

$$Y_{it} = \beta_0 + \beta_{group} + \beta_{period}D_{after,it} + \beta_1 D_{after,it} + u_i + u_{it}$$

Regression framework – treatment group

$$Y_{it} = \beta_0 + \beta_{group} + \beta_{period} D_{after,it} + \beta_1 D_{after,it} + u_i + u_{it}$$

1. After $\quad Y_{i2} = \beta_0 + \beta_{group} + \beta_{period} + \beta_1 + u_i + u_{i2}$

2. Before $\quad Y_{i1} = \beta_0 + \beta_{group} + u_i + u_{i1}$

3. Diff $\quad \Delta Y_i = \beta_{period} + \beta_1 + u_{i2} - u_{i1}$

DiD

1. Treatment group: $\Delta Y_i = \beta_{period} + \beta_1$

2. Control group: $\Delta Y_i = \beta_{period}$

3. DiD: $\Delta\Delta Y_i = \beta_1$

Diff vs. DiD

1. DiD needs data over at least 2 periods.

2. DiD allows for individual-specific constants.

3. ➔ DiD doesn't necessitate randomization.

4. Identifying assumption #1: ”common trends”.

5. Identifying assumption #2: $E[u_{it}|X_{it}, D_{after,it}, u_i] = 0$

Diff vs. DiD

- Identifying assumption #1: common trends.

- When would this be violated?

- Technically, $\beta_{period|control} \neq \beta_{period|treatment}$.

- Call $\beta_{period|treatment} - \beta_{period|control} = \Delta\beta_{period}$

DiD

1. Treatment group: $\Delta Y_{it} = \beta_1 + \beta_{period|treatment}$

2. Control group: $\Delta Y_{it} = \beta_{period|control}$

3. DiD: $\Delta\Delta Y_{it} = \beta_1 + \Delta\beta_{period}$

Diff vs. DiD

- Substantively?

- Example #1: WFH: those that know their productivity is (permanently) declining decide to work from home (or office).

- Example #2: Think of the effect of hiring a new CEO on firm performance.

- Firm observes performance is (permanently) declining compared to peers, and therefore hires a new CEO.

Diff vs. DiD

- Identifying assumption #2: $E[u_{it}|X_{it}, D_{after,it}, u_i]$ = 0.

- ➔ selection into treatment can depend on individual-specific ”things” that are constant over the periods.

- Even 2-period DiD allows control variables.

- Controls may be more important than in an RCT.

Diff vs. DiD

- Identifying assumption #2.

- When would this be violated?

- Technically, the ”shock” in the 1st period leads somebody to (not) choose the treatment.

Diff vs. DiD

- Substantively?

- Example #1: WFH: those that know their productivity was (temporarily) lower decide to work from home (or office).

- Example #2: Think of the effect of hiring a new CEO on firm performance.

- Firm observes a shock to performance compared to peers, and therefore hires a new CEO.

What data for more than 2 periods?

- More data always a plus.
- Makes distinction between (differential) trends and temporary shocks clearer.
- Can allow for more flexible models (e.g. treatment of time; testing of common trends).
- BUT: notice that one stretches what u_i captures.
- Remember: Even 2-period DiD allows control variables.

A complication on all causal estimators

- What have we assumed about the effect of treatment on the control group?

- That there is none.

- = ”no general equilibrium effects”

- = ”**S**table **U**nit **T**reatment **V**alue **A**ssumption (SUTVA)”

A complication on all causal estimators

- When is this an issue?

- In a lab, think of infectious deceases.

- Regarding human behavior, think of interactions (markets).

- Important but difficult topic. We will neglect it.