Optimal design of energy conversion units and envelopes for residential building retrofits using a comprehensive MILP model

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HIGHLIGHTS

- Development of a building energy model suitable for MILP.
- Verification according to ASHRAE standard and more detailed models.
- Application to retrofitting of a residential building.
- Energy system modifications appear more cost effective than envelope retrofits.

ABSTRACT

The optimal design of buildings is a complex task involving energy systems as well as construction measures. Typically, in exact optimization models, only energy systems are considered, whereas envelope components are neglected. When considering both, heuristics are commonly used, which do not guarantee optimal or close to optimal results.

Thus, this paper presents the governing equations, validation and exemplary usage of a building model suitable for exact optimization problems. The developed model simultaneously considers energy systems and building envelopes. It is based on ISO 13790 and validated according to ASHRAE 140 and further compared to a more detailed model. The findings show that the developed model largely complies with the ASHRAE requirements and is able to assess buildings’ dynamic behavior regarding indoor air temperatures as well as hourly, peak load, and annual heating loads.

The simultaneous optimization of energy system and envelope is further demonstrated analyzing retrofitting options of a residential building. We consider solely installing additional PV units, modernizing the building envelope according to governmental regulations and an optimization without constraints regarding building envelope and energy system. The results indicate that installing additional PV units can moderately reduce total costs and CO2 emissions. The envelope modernization according to governmental regulations leads to largely increased costs at lower emissions, whereas the unconstrained optimization is able to simultaneously achieve significant cost and CO2 emission advantages.

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1. Introduction

The transition towards a more energy efficient and environmentally friendly economy is a recognized objective of the European Union [1]. In Germany, this concept is known as “Energiewende” and aims at reducing greenhouse gas emissions, increasing electricity generation from renewable energy sources (RES) and achieving higher energy efficiency in general [2]. In the context of buildings, which account for approximately 40% of total energy consumption in the European Union [1], emission reductions and energy savings can for example be achieved by increasing envelope insulation as well as by installing more efficient heating devices and by improving their control strategy.

The optimal design of buildings and building energy systems (BES) can therefore make a significant contribution towards meeting these goals. However, determining such an optimal design is a complex problem that involves buildings’ electricity and heat consumption as well as generation and storage devices. According to Kaklauskas et al. [3], this problem is further complicated by the interrelationship between BES and heat consumption, which is among other factors influenced by passive construction measures.

So far, many studies have addressed specific parts of this problem with different approaches. The optimal design of building
envelopes for example is often investigated by coupling building simulation software with heuristic optimization methods. The TRNSYS\(^1\) simulation software has for example been used by Chantrelle et al. \[4\] in combination with a Genetic Algorithm \[5\]. EnergyPlus\(^2\) for instance has been used by Echenagucia et al.\[6\] together with a multi-objective Genetic Algorithm, named NSGA-II \[7\]. Lin et al.\[8\] coupled EnergyPlus with Tabu Search\[9,10\]. However, Negendahl and Nielsen \[11\] argue that for early stages of energy optimization, the aforementioned building simulation software might be too time consuming regarding the parameterization and calculation. Therefore, they propose a combination of the simplified, hourly building simulation method described in ISO 13790 and a Genetic Algorithm. All these studies conclude that optimal building envelope configurations significantly reduce the energy demand of heating, cooling and air-conditioning systems.

Most studies solely focusing on BES use exact optimization algorithms such as mixed-integer linear programming (MILP) \[12\]. MILP models have been developed for solving operation optimization of BES \[13,14\], the combined optimal design and operation of BES \[15–18\] and even city district applications \[19–22\]. In these works, buildings’ thermal demands are considered as fixed time series that are computed before the optimization and only present parameters within the optimization.

Relatively few studies have attempted to treat both, BES and envelope. Ashouri et al. \[23\] include a low-order building model into the design and operation optimization of BES. This model is able to consider building’s thermal mass as additional storage option for the BES components’ operation, hereby enabling economic advantages by increasing building temperature during periods with low tariffs or high solar availability. However, since this low-order model uses constant parameters that are computed before the optimization is executed, building envelope optimizations cannot be conducted. Asadi et al. \[24\] implement a static model based on ISO 13790 to determine optimal building envelopes for retrofit purposes. They consider windows, external wall insulation, roofs and solar collectors but their calculations are based on annual demands, therefore neglecting the BES components’ operation and buildings’ heat capacity. Stadler et al. \[25\] develop a dynamic MILP approach to simultaneously determine

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\[^1\] http://www.trnsys.com/.

\[^2\] https://energyplus.net/.

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### Nomenclature

#### Variables and parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>meaning [unit]</th>
</tr>
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<tbody>
<tr>
<td>(A)</td>
<td>area [m(^2)]</td>
</tr>
<tr>
<td>(C)</td>
<td>capacitance [kW h/K]</td>
</tr>
<tr>
<td>(E)</td>
<td>energy content [kW h]</td>
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<tr>
<td>(F)</td>
<td>form and correction factor [-]</td>
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<tr>
<td>(H)</td>
<td>heat transfer coefficient [W/K]</td>
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<tr>
<td>(I)</td>
<td>solar irradiation [kW]</td>
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<tr>
<td>(P)</td>
<td>electrical power [kW]</td>
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<tr>
<td>(R)</td>
<td>heat resistance [(m(^2) K)/W]</td>
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<tr>
<td>(R_v)</td>
<td>residual value [-]</td>
</tr>
<tr>
<td>(U)</td>
<td>thermal transmittance [W/(m(^2) K)]</td>
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<tr>
<td>(V)</td>
<td>volume [m(^3)]</td>
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<tr>
<td>(a)</td>
<td>capital recovery factor [-]</td>
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<td>(b)</td>
<td>adjustment factor [-]</td>
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<tr>
<td>(c)</td>
<td>costs [Euro]</td>
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<td>(e)</td>
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<td>(f)</td>
<td>fixed parameters [-]</td>
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<td>(g)</td>
<td>solar energy transmittance [-]</td>
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<tr>
<td>(h)</td>
<td>area specific heat transfer coefficient [W/(m(^2) K)]</td>
</tr>
<tr>
<td>(inv)</td>
<td>specific investment costs [Euro/m(^2)]</td>
</tr>
<tr>
<td>(n)</td>
<td>air exchange rate [1/h]</td>
</tr>
<tr>
<td>(p)</td>
<td>linearized product [-]</td>
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<td>(q)</td>
<td>airflow rate [m(^3)/h]</td>
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<tr>
<td>(\Delta t)</td>
<td>time interval length [h]</td>
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<tr>
<td>(w)</td>
<td>weight for each typical period [-]</td>
</tr>
<tr>
<td>(x)</td>
<td>(binary) decision of purchase [-]</td>
</tr>
<tr>
<td>(z)</td>
<td>(binary) building class [-]</td>
</tr>
</tbody>
</table>

#### Greek letters

| \(\alpha\) | absorption coefficient [-] |
| \(\theta\) | temperature [°C] |
| \(\kappa\) | heat capacity [J/(kg K)] |
| \(\rho\)  | density [kg/m\(^3\)] |
| \(\varphi\) | storage’s loss factor [-] |
| \(\phi\)  | heat flow rate [kW] |

#### Subscripts and abbreviations

ASHRAE: American Society of Heating, Refrigerating and Air-Conditioning Engineers

BAT: battery

BES: building energy system

CHP: combined heat and power

COP: Coefficient of Performance

DHL: design heat load

EH: Electrical resistance Heater

EnEV: German Energy Saving Ordinance

HC: heating and cooling

HP: heat pump

ISO: International Organization for Standardization

MILP: mixed-integer linear program

PV: photovoltaic

RES: Renewable Energy System

STC: solar thermal collector

TES: thermal energy storage

VDI: Association of German Engineers

S: set of available types

\(cl\): building class

\(e\): environment

\(d\): demand

\(f\): floor

\(g\): generator

\(int\): internal

\(inv\): investment

\(j\): component

\(lb\): lower bound

\(m\): thermal mass

\(op\): opaque

\(p\): typical period (month)

\(s\): surface

\(sol\): solar

\(t\): time step

\(tr\): transmission

\(ub\): upper bound

\(ve\): ventilation

\(w\): windows
optimal investments in envelope and BES components. This approach relies on an EnergyPlus simulation providing a basic heat demand profile from which demand reductions through investments in improved envelope components are subtracted. This approach only allows for considering heat transmission but neglects radiation. Furthermore, buildings’ heat capacity is also neglected, prohibiting the optimization from utilizing building’s thermal mass. A multi-level approach is proposed by Evins [26]. This methodology combines the usage of NSGA-II and MILP. NSGA-II varies building variables such as wall insulation and glazing, and BES component sizes. In a second step, building’s heat demand is computed with EnergyPlus and the BES operation is optimized with a MILP model. Evins’ approach therefore overcomes most simplifications of the previous works. However, since BES operation optimization and the computation of the building’s heat demand are conducted consecutively, the MILP model cannot exploit building’s heat capacity, potentially neglecting valuable storage resources [27].

In order to contribute to the field of optimal design of building envelope and BES components, this paper presents a novel MILP model for optimizing envelope and BES components as well as their operation. The main novelty of this approach is the usage of a MILP solver that can optimize envelope and BES components simultaneously, whereas previous studies rely on heuristic approaches that treat both aspects iteratively and generally lead to approximative solutions. Following Negendahl and Nielsen [11], we implement the hourly method of ISO 13790 [28] directly into a MILP model previously developed for the optimal selection and operation of BES components [17,29]. The ISO 13790 model considers ventilation as well as heat transmission through opaque components and windows, modeling radiation effects, too. The model allows for optimizing external walls, roofs, windows, as well as internal walls, ceilings and ground plates. Since the BES sizing and operation is part of the model, building’s thermal mass can be exploited for the BES operation as well. We further extend this model to a multi-objective optimization allowing for assessing trade-offs between economic and environmental aspects. The method can be used by practitioners to analyze the direction of energy solutions at early stages. Furthermore, the effects of governmental regulations on buildings like the German Energy Savings Ordinance can also be evaluated.

The rest of this paper is structured as follows. The next section presents the building model used in this study and its implementation into the optimization model. Subsequently, we describe validation results on our model based on technical standards and other, more detailed models. Next, the model is applied to analyze retrofitting options for a residential building. Finally, this paper closes with a summary and an outlook for future research.

The models and application described in this paper can be downloaded from https://github.com/RWTH-EBC/BuildingOPT.

2. Modeling

This section describes our previously developed BES optimization model [17,29] that does not consider the building envelope as well as the improvements added in this paper. The most significant additions to the existing model include the implementation of a dynamic building model into the optimization model as well as a multi-objective optimization scheme, which allows for assessing trade-offs between costs and CO₂ emissions.

2.1. BES optimization model

The BES optimization model extended in this paper is based on the authors’ previous publications [17,29]; therefore, this section only provides a brief description. The model is formulated to design, size and schedule the operation of the BES of a single building connected to an electricity and gas grid, at optimal costs. The device operation is approximated with 12 typical demand days, all consisting of 24 h that are weighted to represent one month each. Fig. 1 shows the structure of the model. Conventional heat generators such as boilers, combined heat and power (CHP) units, electrical heaters (EHs) and heat pumps (HPs) are considered. Furthermore, storage devices like electrical batteries (BATs) and thermal energy storage (TES) units are available. Solar generators like photovoltaic (PV) modules and solar thermal collectors (STCs) as well as peripheral devices like inverters are also included as possible parts of the optimal BES.

In the original model, the TES is the only link to the space heating and domestic hot water demand, summarized as building’s thermal loads. The corresponding energy balance is formulated as:

\[ E_t = (1 - \varphi) \cdot E_{t-1} + \Delta t \cdot \left( \sum \phi_e - \phi_h \right) \]  

with \( E \) as the TES’s stored energy, \( t \) the current time step, \( \varphi \) the TES loss coefficient, \( \Delta t \) the time step length and \( \phi_e \) and \( \phi_h \) as the heat flow rates of heat generation and heat demand. The heat demand is the sum of space heating demand and domestic hot water usage. In the previous model, both were computed before the optimization was carried out. With this paper’s approach, the demand for space heating is computed dynamically as part of the optimization.

2.2. Low-order building model

A building model based on ISO 13790 has been included within the BES optimization model. This standard has been developed for the calculation of energy use for space heating for residential and non-residential buildings. In this standard, a monthly and a simplified hourly calculation method are described and requirements for detailed building simulation tools are presented. Since the monthly method is not suitable for assessing the dynamics of BES operation, we implemented the simplified hourly calculation method within our optimization model.

In this model, the building’s dynamics are represented by one equivalent thermal capacity. Furthermore, five heat transfer effects are modeled. Within this model, all heat transfer effects are assumed to be linear; thus, an electrical network analogy is used to describe the model [30]. In this analogy, thermal capacitances are represented with electrical capacitances and heat transfer is modeled by electrical resistances. The connections between each
component stand for different temperatures that are being modeled.

Fig. 2 displays the low-order building model based on ISO 13790. The model lumps all building components that store thermal energy into one thermal mass with capacitance $C_m$ and area $A_m$. Heat transfer from ventilation $H_{ve}$ and transmission are considered. Heat transfer due to transmission is further split into a part resulting from windows $H_{tr,w}$ and second part through opaque components $H_{tr,op}$. Furthermore, three nodes model the indoor air temperature $\theta_{air}$, the thermal capacitance’s surface temperature $\theta_s$ and mass temperature $\theta_m$. Internal gains $\phi_{int}$ and solar gains $\phi_{sol}$ as well as the heating/cooling inputs $\phi_{HC}$ further affect these temperatures. In contrast to ISO 13790, we do not consider mechanical ventilation systems, therefore we assume that the ventilation supply temperature is equal to the environment temperature $\theta_e$.

According to this ISO standard, the entire building is modeled as a single thermal zone since set temperatures typically do not differ by more than 4 K in residential buildings and because ventilation rates do not differ drastically between different rooms and also because the building is conditioned by a central heating system. Furthermore, shading through neighboring buildings, thermal bridges and windows’ frame areas are neglected in our paper.

2.3. Building model implementation

The model requires weather inputs, such as solar irradiation on all outside surfaces and outside temperature. Additionally, a set of typical walls, windows, roofs, internal walls and ceilings as well as ground floor structures have to be provided. The optimization chooses the optimal combination from the set of provided types.

A set of binary variables $x$ are used to model this decision process. $x_{j,S}$ equals one if type $S_j$ of component $j$ is chosen and zero otherwise. Exactly one type of each component has to be chosen:

$$\sum_{S_j} x_{j,S} = 1 \quad (2)$$

2.3.1. Investment costs

The investment cost for each component $j$ result in:

$$c_{j}^{inv} = A_j \cdot f_{\text{inv}} \cdot \alpha \cdot \sum_{S_j} x_{j,S} \cdot \left(1 - R\eta_{j,S}\right) \cdot \text{inv}_{j,S} \quad (3)$$

In this equation, $A_j$ symbolizes the component’s area, $f_{\text{inv}}$ the local value added tax, $\alpha$ the capital recovery factor, $R\eta_{j,S}$ the residual value and $\text{inv}_{j,S}$ the specific investment costs.

2.3.2. Thermal mass

In this model, all thermal capacities, such as exterior and interior walls, floors and ceilings are lumped into one capacity $C_m$ with a corresponding surface area $A_m$. This representation simplifies the complex thermal behavior of multiple components by combining and treating them as a fictitious thermal storage. The capacitance is computed as the sum of the product of each existing component’s area (exterior and interior walls, etc.) and corresponding specific heat capacity:

$$C_m = \sum_{S_j} x_{j,S} \cdot A_j \cdot \kappa_{j,S} \quad (4)$$

ISO 13790 provides two methods for computing the capacitance’s surface area $A_m$ that determines the heat transfer between mass and surface nodes. First, an algebraic method that introduces many nonlinear terms and secondly a simplified, table-based method. The table-based method classifies buildings as either very light, light, medium, heavy, or very heavy, with fixed upper and lower bounds $(f_{c,d}^{ub}$ and $f_{c,d}^{lb}$) for each class. Accordingly, $A_m$ is proportional $(f_{A,d})$ to the heated floor area $A_t$. Consequently, the building’s class is not known before the optimization and the algebraic method cannot be used within a linear context, we combine both approaches.

The thermal capacitance is computed as described in Eq. (4). The building’s class $z_0$ immediately results from this capacitance and is chosen with the following two inequalities:

$$\sum_{d} z_{cl,d} \cdot f_{c,d}^{lb} \cdot A_t \leq C_m \leq \sum_{d} z_{cl,d} \cdot f_{c,d}^{ub} \cdot A_t \quad (5)$$

Afterwards, $A_m$ results in:

$$A_m = \sum_{d} z_{cl,d} \cdot f_{A,d} \cdot A_t \quad (6)$$

2.3.3. Heat transfer coefficients

$H_{tr,ms}$ describes the heat transfer between mass and surface nodes and is proportional to the capacitance’s surface area:

$$H_{tr,ms} = h_{ms} \cdot A_m \quad (7)$$

According to ISO 13790, $h_{ms}$ describes the heat transfer coefficient between nodes $m$ and $s$, and has a fixed value of 9.1 W/(m² K).

The heat transfer coefficient for all opaque components, such as exterior walls, roof and ground floor, results in:

$$H_{tr,op} = \sum_{j} \sum_{S_j} \beta_{o,j,S} \cdot x_{j,S} \cdot A_j \cdot U_{j,S} \quad (8)$$

The adjustment factor $\beta_{o,j,S}$ allows for combining all opaque components into one equivalent resistance. The factor $\beta_{o,j}$ is equal to $(\overline{\theta_{air}} - \overline{\theta_{e}})/(\overline{\theta_{set}} - \overline{\theta_{e}})$ for components with ground contact and one for the others, $\overline{\theta_{air}}$ and $\overline{\theta_{e}}$ are the averaged air and environment temperatures and $\overline{\theta_{set}}$ is the thermal zone’s set temperature. $H_{tr,op}$ is split into a serial connection of $H_{tr,ms}$ and $H_{tr,em}$ which describes the heat transfer between mass and outdoor environment. This formulation introduces a second nonlinearity that cannot be refomulated reasonably. Instead, we assume the numerical value of $H_{tr,ms}$ to be much bigger than that of $H_{tr,op}$. Consequently, $H_{tr,em}$ is approximated to be:

$^4$ The numerical values of $f_{c,d}^{ub}$ and $f_{c,d}^{lb}$ are listed in Table A.12.

$^4$ See Appendix B for a short estimate of this assumption.
Heat transfer through windows is modeled analogously to the heat transfer through opaque components:

\[ H_{t,r,w} = \sum_{w} \sum_{S_w} A_w \cdot U_{w,s} \cdot \tilde{T}_{w,s} \]  

The heat transfer between surface and air node is:

\[ H_{t,r,s} = h_s \cdot A_{tot} \]  

According to ISO 13790, \( h_s \) is the heat transfer coefficient between air and surface and equals 3.45 W/(m² K). \( A_{tot} \) describes the total internal surface area, which is the product of \( A_h \) (dimensionless ratio between internal surfaces and surface area) and \( A_s \). According to the standard, a value of 4.5 is recommended for \( A_h \).

Heat transfer through ventilation is modeled with:

\[ H_{ve} = k_{ve} \cdot \rho_{air} \cdot q_{ve, avg} \]  

According to ISO 13790, the product \( k_{ve} \cdot \rho_{air} \) describes the specific heat capacity of air and can be approximated with 1200 J/(m³ K). In Eq. (12), \( q_{ve, avg} \) stands for the time-average airflow rate of fresh air at environment temperature.

### 2.3.4 Internal and solar gains

The internal gains, for example resulting from occupancy and electrical devices are a given time-series input. Solar gains on the other hand, depend on the chosen façade and windows. The general equation for solar gains through a component \( k \) is given with:

\[ \phi_{s,k} = A_{s,k} \cdot \tilde{I}_{s,k} - F_{k} \cdot \tilde{h}_{s,k} \]  

The first term of this equation describes heat gains from irradiation onto the building and the second term radiation losses from the building to its surroundings. \( A_{s,k} \) is the effective area of component \( k \), which is computed as \( A_{s,k} = A_k \cdot F_{s,k} \cdot g_{s,k} \) for windows and \( A_{s,k} = A_k \cdot U_k \cdot x_{s,k} \cdot R_{s,k} \) for opaque components. \( F_{s,k} \) is a correction factor for non-scattering glazing and \( R_{s,k} \) the external surface's heat resistance. The solar radiation onto a tilted surface \( I_{s,k} \) is computed as the sum of direct, diffuse and reflected radiation, and depends on weather and geometric influences. The direct and reflected parts are computed as described in [31] and the influence of diffuse radiation is assessed with the approach presented in [32]. The form factor \( F_{s,k} \) between component and sky is 1 for unshaded horizontal roofs and 0.5 for unshaded vertical walls. \( \tilde{h}_{s,k} \) stands for the thermal radiation to the sky and is calculated as \( \tilde{h}_{s,k} = A_k \cdot U_k \cdot x_{s,k} \cdot \tilde{h}_{s,k} \). In this equation, \( h_{s,k} \) stands for the external radiative heat transfer coefficient. According to ISO 13790, \( h_{s,k} \) is approximated with \( \alpha_{s,k} \), where \( \alpha_s \) is the external surface's emissivity for thermal radiation. The average difference between the external air temperature and the apparent sky temperature, \( \Delta T_{enr} \), is set to 11 K according to the ISO standard.

Internal and solar gains are distributed to the three temperature nodes. The heat flow directly affecting the air node \( \phi_{ia} \) is equal to half of the internal gains.

\[ \phi_{ia} = 0.5 \cdot \phi_{int} \]  

The heat flow into the internal mass \( \phi_m \) is:

\[ \phi_m = \frac{A_m}{A_{tot}} \cdot 0.5 \cdot \phi_{int} + \frac{A_i}{A_{tot}} \sum_{j} \sum_{k} \sum_{k} \left( p_{di,j} \cdot f_{Ad,j} \cdot \phi_{sol,k} \right) \]  

In the ISO guideline, \( \phi_m \) is described using a product of \( A_m \) (capacitance’s surface area) and \( \phi_{s,k} \) (solar gains). When substituting Eq. (6) for \( A_m \) and Eq. (13) for \( \phi_{s,k} \), a nonlinear product of \( z_{dt} \) and \( \chi_{ls} \) has to be linearized. In Eq. (15), \( p_{di,j} \) represents this nonlinear product \( z_{dt} \cdot \chi_{ls} \). Since both factors are binary variables, \( p_{di,j} \) is binary, too. This expression can be linearized without loss of accuracy, by introducing the following three constraints [33]:

\[ p_{di,j} \leq z_{dt} \]  
\[ p_{di,j} \leq \chi_{ls} \]  
\[ p_{di,j} \geq z_{dt} + \chi_{ls} - 1 \]  

The heat flow onto the thermal mass’s surface \( \phi_{at} \) is:

\[ \phi_{at} = \left( 1 - \frac{1}{9.1} \cdot \frac{1}{A_{tot}} \sum_{w} \sum_{S_w} \left( p_{w,s} \cdot U_{w,s} \cdot \phi_{sol,k} \right) \right) \cdot 0.5 \cdot \phi_{int} - \frac{1}{9.1} \cdot \frac{1}{A_{tot}} \sum_{w} \sum_{S_w} \sum_{f} \sum_{j} \sum_{k} \sum_{k} \left( p_{w,s} \cdot U_{w,s} \cdot \phi_{sol,k} \right) + \sum_{k} \phi_{s,k} - \phi_{m} \]  

Similar to Eq. (15), the ISO guideline describes \( \phi_{at} \) in terms of \( H_{t,r,w} \) (heat transmission coefficient for windows) and \( \phi_{s,k} \) (solar gains), requiring further linearization. In Eq. (19), \( p_{w,s} \cdot \phi_{s,k} \) represents the product of \( x_{w,s} \) and \( \chi_{ls} \), and is constrained like \( p_{di,j} \).

### 2.3.5 Linearized building model

The energy balances of the building model shown in Fig. 2, can be formulated as three linear equations:

\[ H_{tr,m} \cdot (\theta_m - \theta_s) + H_{tr,em} \cdot (\theta_m - \theta_e) = \phi_m - C_m \cdot \frac{\partial \theta_m}{\partial t} \]  
\[ H_{tr,ms} \cdot (\theta_s - \theta_m) + H_{tr,is} \cdot (\theta_i - \theta_{air}) + H_{tr,w} \cdot (\theta_i - \theta_e) = \phi_{int} \]  

\[ H_{tr,em} \cdot (\theta_e - \theta_m) + H_{tr,in} \cdot (\theta_i - \theta_{air}) = \phi_{at} + \phi_{AC} \]  

All products of any \( H_{tr} \)-value and a node’s temperature \( \theta_{air} \) and \( \theta_{in} \) have to be further linearized. Since all \( H_{tr} \)-values can be expressed as a product of a binary decision variable and a set of given parameters, the product of \( H_{tr} \) and \( \theta \) can be linearized without loss of accuracy [33]. Exemplarily, this linearization is explained for \( H_{tr,ms} \cdot \theta_m \):
\[ \theta_{\text{air}} \geq \theta_{\text{set}} \]  

(26)

2.3.6. Design heat load

In order to appropriately size BES components, a design heat load based on [34], considering chosen building elements, is computed:

\[ e_{\text{total}}^{\text{DHL}} = \left( \theta_{\text{set}}^{\text{DHL}} - \theta_{\text{e}} \right) \cdot \left( \sum_{j} b_{h_{tj}} \cdot X_{0j} \cdot A_{j} \cdot U_{j} \cdot S_{j} \cdot V_{n} \cdot r \right) \]  

(27)

This design heat load ensures that a comfortable set point temperature can be guaranteed, even at cold winter temperatures without solar or internal gains. \( b_{h_{tj}} \) describes the adjustment factor which is 1.45 · \( b_{h_{tj}} \) for ground floors and 1 otherwise. \( V \) and \( n \) are the building’s volume and air exchange rate. \( \theta_{\text{set}}^{\text{DHL}} \) and \( \theta_{\text{e}} \) stand for the nominal indoor set temperature and nominal outdoor temperature.

2.4. Multi-objective optimization scheme

This subsection describes the inclusion of the second objective, minimizing CO2 emissions as well as the chosen multi-objective optimization algorithm, the \( \varepsilon \)-constraint method.

2.4.1. CO2 emissions

In this paper, CO2 emissions from burning natural gas and purchasing electricity from the distribution grid are considered.

The emissions from natural gas result in:

\[ e_{\text{gas}} = \int f_{\text{CO2, gas}} \cdot \Delta t \cdot \sum_{p} W_{p} \cdot \sum_{t} \left( e_{\text{CHP, p}} + e_{\text{Boiler, p}} \right) \]  

(28)

We only consider CO2 emissions from purchased electricity and do not attribute negative emissions for feed-ins. In this way, self-consumption is promoted rather than large feed-ins, which reduces the stress on the grids [35].

\[ e_{\text{grid}} = \int f_{\text{CO2, grid}} \cdot \Delta t \cdot \sum_{p} W_{p} \cdot \sum_{t} e_{\text{grid, p, t}} \]  

(29)

The numerical values of \( f_{\text{CO2, gas}} \) and \( f_{\text{CO2, grid}} \) are 250 \( \text{kg} \cdot \text{m}^{-2} \cdot \text{h}^{-1} \) [36] and 569 \( \text{kg} \cdot \text{m}^{-2} \cdot \text{h}^{-1} \). [37]

The total CO2 emissions are computed as the sum of both components:

\[ e_{\text{total}} = e_{\text{gas}} + e_{\text{grid}} \]  

(30)

2.4.2. \( \varepsilon \)-constraint method

We conduct a multi objective optimization considering annualized costs and CO2 emissions.

\[ \text{min} \left( e_{\text{total}}, e_{\text{CO2}} \right) \]  

(31)

The \( \varepsilon \)-constraint method, as explained by Mavrotas [38], is used for solving this problem by computing the set of Pareto-optimal solutions, the Pareto-frontier. In this method, the minimization of both objectives as shown in Eq. (31) is replaced by minimizing one objective and treating the other goal as a constraint as demonstrated in Eqs. (32) and (33).

\[ \text{min} \ e_{\text{total}} \]  

(32)

\[ \text{s.t.} \ e_{\text{total}} \leq e_{\text{max}} \]  

(33)

In a first step, the ranges of the Pareto-frontier are computed by individually optimizing each objective. Afterwards, the Pareto-frontier is computed by varying the limits for the CO2 emissions accordingly. In order to avoid non Pareto-optimal solutions, a lexicographic optimization is implemented, as suggested by Mavrotas [38]. Lexicographic optimization is applied by optimizing for the first objective (costs or emissions) and afterwards optimizing for the second objective (emissions or costs) while enforcing that the first objective must not decline.

3. Validation

ISO 13790 is mainly intended for estimating annual energy demands for space heating and cooling. This section illustrates that both our implementation of this guideline as well as the linearized model suitable for MILP optimization purposes, are able to accurately compute annual and hourly heating and cooling loads. The validation consists of the ASHRAE guideline 140 which provides a generic test case, and a second comparison with a more detailed low-order model for a typical residential building.

3.1. ASHRAE 140

ASHRAE standard 140 [39] provides a full set of test procedures for building energy analysis computer programs. Our validation however is restricted to the so called “Class I test procedures” for “Building Thermal Envelope and Fabric Load Basic Tests”. The Class I test procedures are detailed tests for building energy simulation tools with hourly or sub-hourly resolutions. These tests are divided into cases analyzing low mass and heavy mass constructions as well as free float tests. During free float tests, neither heating nor cooling systems are activated, only the building’s thermal response to changing outside conditions is analyzed. The other cases describe thermostatically controlled indoor environments. All test cases are derived from test case 600, which describes a single zone building located in Denver, Colorado, with two windows facing south and an adiabatic ground floor. Cases 620, 640 and 650 vary the windows’ positions, thermostat’s set points and ventilation rates. While cases 600, 620, 640, and 650 describe a building with a low thermal mass, cases 900, 920, 940, and 950 model a heavy thermal mass. Cases 610, 630, 910, 930 and 960 are omitted because they require shading effects or a second thermal zone, which is currently beyond the scope of the presented model.

3.1.1. Solar radiation

The standard also includes a method for validating the implemented solar model. Test case 600 provides results for the total annual solar irradiation on the building’s exterior walls, horizontal roof and the radiation transmitted through the southern window as well as hourly profiles for the west and south surface for March 5th and July 27th. The results of our simulation as well as the ASHRAE guideline’s reference results obtained with standard building energy simulation programs are shown in Table 1. Our simulation results largely comply with the reference values. Only for the southern orientation, the irradiation on the exterior wall as well as the transmission through the window exceed the reference results by 1 kW h and 3 kW h respectively. Our simulations differ less than 1% from the reference values; therefore the deviations are considered acceptable.

The hourly profiles shown in Figs. 3 and 4 are also in the range of the reference results for the most part. The only deviation occurs on July 27th when the computed irradiation drops below the reference results by 6.4 W m\(^{-2}\) at hour 18. Overall, the implemented solar model largely complies with the ASHRAE’s reference results and is therefore considered appropriate for this validation.
3.1.2. Light mass cases

The main differences between the original ISO 13790 model and the simplified model suitable for MILP optimization purposes are the computation of $A_m$ and $H_{tr;ms}$. Table 2 shows that for case 600, $A_m$ is 12% smaller in the simplified model than in the original one. Consequently, $H_{tr;ms}$ is also reduced by 12%. However, the effect on $H_{tr;em}$ is only 4%. As $H_{tr;em}$ is smaller in the simplified model, the building’s thermal insulation is slightly overestimated.

The annual heating and cooling demands are summarized in Fig. 5. Positive values stand for heating demands and negative values for cooling demands. The simplified and original implementations yield very similar results, which are moreover within the range of the reference results for all analyzed test cases.

Hourly heating and cooling loads for test case 600 on January 4th are shown in Fig. 7. The computed loads are slightly below the minimum reference results between hours 9 and 11 that are given in ASHRAE 140. The maximum deviation is 0.175 kW in the

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
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<td>Annual solar irradiation in kW h/(m² a).</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Reference (min)</td>
</tr>
<tr>
<td>Reference (max)</td>
</tr>
<tr>
<td>Simulation</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters ASHRAE 140, light mass cases.</td>
</tr>
<tr>
<td>$A_m$ in m²</td>
</tr>
<tr>
<td>ISO (simpl.)</td>
</tr>
<tr>
<td>ISO (orig.)</td>
</tr>
</tbody>
</table>

Fig. 3. Solar irradiation on March 5th.

Fig. 4. Solar irradiation on July 27th.
original implementation and 0.230 kW in the simplified one. The maximum deviation between both models is 0.117 kW and the average deviation is 0.063 kW.

The free float temperatures for case 600FF are shown in Fig. 8. In this case, the heating and cooling systems are shut down and the room is only affected by the outside weather conditions. The resulting room temperatures of simplified and original model are very similar and roughly follow the reference results. However, small deviations of max. 0.8 K occur during the morning hours of this day.

3.1.3. Heavy mass cases

Table 3 illustrates that the implemented simplifications affect the heavy mass building's parameters in a similar magnitude as the light mass building. However, in the light mass building, $A_m$ was 12% underestimated, whereas in the heavy mass building, $A_m$ is approx. 22% overestimated. The assumption of $H_{tr,ms}$ being much bigger than $H_{tr,op}$ proves to be justified, even if the approximation of $A_m$ is inaccurate.

Similarly, the annual and peak heating and cooling loads are shown in Figs. 9 and 10. The findings again illustrate that our implementations are within the reference results.

The hourly loads for test case 900 on January 4th are presented in Fig. 11. This plot shows that for the heavy mass building, both models comply with the reference solutions at all times. Though, the deviation between both models increases compared with the light mass cases. The maximum deviation between both models is 0.364 kW and the average deviation is 0.133 kW.

The free float temperatures for this case are depicted in Fig. 12. The simulated temperatures fluctuate around the minimum reference values with a maximum error of 1.1 K at hour 13.

3.2. Improved low-order model

Our implementations are further compared with an improved low-order model developed by Lauster et al. [40] based on the German engineering guideline VDI 6007 [41]. In contrast to the ISO 13790 standard, this guideline has been developed for assessing hourly loads. Since this model introduces more nonlinearities than the ISO model, we chose the ISO model for our optimization problem. The comparison with the improved low-order model serves for evaluating the suitability of our implementation for a more realistic example, since the modeled building is more representative than the ASHRAE building and further effects such as ground contact can be considered.

The building considered for this comparison is a two-story building with a heated floor area of 150 m$^2$ and each story has a height of 3 m. Each outside wall has an area of 34.75 m$^2$ and a win-

---

5 The building models including documentation and examples are publicly available at https://github.com/RWTH-EBC/AixLib.
The comparison again includes a building with light construction and one with heavy construction. The effects of our simplifications on $A_m$ and $H_{tr;em}$ are shown in Table 4 for the light mass building. While $A_m$ and consequently $H_{tr;ms}$ are overestimated by 11.4%, $H_{tr;em}$ is only reduced by 3.5%.

For comparing the hourly loads, January 4th has been chosen again. The hourly loads in this day are depicted in Fig. 13. The maximum difference between simplified and original ISO model is 0.3 kW and between the simplified ISO model and the VDI model 0.8 kW with an average value of 0.1 kW.

Table 5 illustrates that the simplifications again heavily affect $A_m$ and $H_{tr;em}$, leading to deviations of 19.3%. However, $H_{tr;em}$ is again estimated accurately with a deviation of only 2.2%.

The differences in the hourly heating loads of the heavy construction are more distinct as shown in Fig. 14. The maximum deviation of simplified and original ISO model is 0.6 kW. The VDI model and the simplified ISO model differ by maximum 0.5 kW and on average by 0.3 kW for this day.

The annual heating and cooling loads for this comparison are shown in Fig. 15. The annual demands for heating of all three models agree well, while the cooling demands are significantly larger in the ISO models. The simplified model leads to marginally bigger deviations from the VDI model than the original ISO implementation, but the deviation is only 3.1% for the heating and 3.7% for the cooling demands. The deviation between simplified model and VDI model is 4.9% for the heating and 40.7% for the cooling demands. This large relative deviation in the cooling demand however is due to the low absolute demands which increase the relative differences. Furthermore, these differences are due to the modeling of ground contact which is based on a predefined indoor set-temperature that is different from the actual temperature resulting in the simulations.

Fig. 16 depicts the peak loads resulting in these simulations. Again, the results of the heating loads are almost equal, whereas...
Parameters comparison with low-order model, light mass case.

<table>
<thead>
<tr>
<th>Am in m²</th>
<th>H_em in W/K</th>
<th>H_op in W/K</th>
<th>H_tr in W/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO (simpl.)</td>
<td>375.0</td>
<td>3412.5</td>
<td>108.0</td>
</tr>
<tr>
<td>ISO (orig.)</td>
<td>336.6</td>
<td>3062.6</td>
<td>108.0</td>
</tr>
</tbody>
</table>

Fig. 13. Comparison with low-order model, hourly loads on January 4th, light mass case.

The cooling loads differ by 11.7% in the light mass case to 19.3% in the heavy mass case.

3.3. Summary

The ASHRAE validation and comparison with the VDI model illustrate that the introduced simplifications barely affect the results of the model. In fact, both show that our implementations are able to accurately compute the total annual heating demands as well as the peak loads. The deviations in the cooling demands are larger than for the heating demands, but since cooling systems are currently not considered in our BES optimization, deviations in the cooling demands are negligible.

The hourly heating and cooling loads as well as free float temperatures are further predicted very accurately, proving that our model is able to represent the dynamic behavior of the building.

Overall, the simplified model has been shown to be suitable for modeling building physics in our BES optimization context.

4. Application

This section illustrates how the proposed model can be applied to analyze retrofitting options of residential buildings.

4.1. Scenario

The coupled BES and envelope optimization is applied to a residential building built during the 1970s with the same geometry as used in the comparison with the improved low-order model. Tables 6 and 7 list all available envelope components the solver can choose from. The first entry always describes existing components according to a typical residential building from the 1970s which therefore do not have any investment costs, whereas the second entry stands for a typical representative of a 1990s building standard [42]. The third type fulfills the current German Energy Saving Ordinance EnEV 2016 and the fourth entry even exceeds the requirements of EnEV 2016 [43]. The model also considers internal walls, ground floors and internal ceilings, but since this scenario describes a retrofitting example, it is assumed that these components are not modernized [44].

The main parameters of installed and available BES components are listed in Table 8. There are four to six types of each component available, from which the optimizer can choose. As with the envelope components, investment costs only describe material costs and do not include personnel costs. All TES units are considered to have an hourly loss coefficient of 0.6%, all CHP units have an electrical efficiency of 25% and a power-to-heat ratio of 0.33. The HP units are assumed to have a nominal COP of 3.8 at 35 °C flow temperature and 2 °C outside temperature. Electrical heaters, PV modules and STCs are assumed to be available in continuous sizes. For electrical heaters, an efficiency of 100% is assumed, fixed and variable investment costs are set to 245 Euro and 19 Euro/kW. PV modules cost 1615 Euro/kW, resp. 201.9 Euro/m² and have a total efficiency of 12% including inverter and transmission losses. STCs have fixed and variable investment costs of 396 Euro and 92 Euro/m² as well as optical efficiencies of 80% and loss coefficients of 3.639 W/(m² K) and 0.017 W/(m² K2). PV and STC are mounted on a 35° tilted surface with southern orientation. The maximum available roof top area for PV and STC is 50 m².

Electricity demands for non-heating devices, appliances and lighting are computed with a high-resolution, stochastic tool based on Richardson et al. [45]. Domestic hot water demand profiles are computed with a combination of users’ occupancy based on Richardson et al. [45] and daily tap water usage statistics of residential buildings developed in IEA Annex 42 [46]. Other internal heat gains from occupants are derived from ISO 13790.

We analyze the status quo setting and three different retrofit options. In the status quo setting (0), the current BES and envelope components cannot be altered and no additional device may be purchased. The existing BES consists of a low efficiency boiler with 18 kW capacity and a small TES with 0.1 m³. In the PV installation option (1), the existing BES and envelope setup may not be altered, but PV panels can be installed additionally. This setting is motivated by governmental subsidies to increase the PV capacity in Germany and by the relatively low effort of installing PV panels compared with other BES or envelope modifications. Option 2, EnEV constrained, forces the optimizer to invest in envelope components that fulfill or exceed current EnEV requirements. The third option, unconstrained does not specify any constraints on the installation of any devices or envelope components. This setting therefore serves as a true benchmark for the other settings.

Table 5
Parameters comparison with low-order model, heavy mass case.

<table>
<thead>
<tr>
<th>Am in m²</th>
<th>H_em in W/K</th>
<th>H_op in W/K</th>
<th>H_tr in W/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO (simpl.)</td>
<td>525.0</td>
<td>4777.5</td>
<td>131.0</td>
</tr>
<tr>
<td>ISO (orig.)</td>
<td>650.4</td>
<td>5918.2</td>
<td>131.0</td>
</tr>
</tbody>
</table>
All optimizations are carried out with Gurobi 6.5\textsuperscript{6} and modeled with the corresponding Python framework. We used the Branch-and-Cut algorithms \cite{47} from this solver. The optimizations are solved to an optimality gap of 1\% on a Windows 7 computer with 6 CPU cores, 12 threads and 32 GB of RAM.

4.2. Results

This section describes the results of applying the equations presented in Section 2 to the four settings defined in the previous subsection. The results are presented in tabular form and summarized graphically in Fig. C.17 in Appendix C.

4.2.1. Status quo and PV installation

In the status quo setting, the optimizer’s only degrees of freedom are the time steps in which the 18 kW boiler is activated and the corresponding modulation level. When allowing for additional PV installation, the optimizer further determines the total area of PV modules as well as the share of self-consumption of PV generated electricity and feed-ins into the grid.

The results of the status quo and PV installation settings are summarized in Table 9. Simulation “PV 0” stands for the minimum cost optimization whereas the highest number, “PV 6” represents the minimum emission calculation. In both settings, the envelope and existing heating system cannot be altered, therefore both rely on the standard envelope, a low efficiency boiler and a small TES unit. The resulting design heat load is 15.8 kW in all cases and the total heating energy is 13.3 MW h/a. The average indoor air temperature, \(\theta_{\text{air}}\) is 20.2 °C and the maximum indoor air temperature varies from 22.2 to 22.9 °C. Without changing the heating system or the envelope components, 11.3 m\(^2\) PV area, which equals approx. 1.4 kW peak power, are cost-optimal for the building under the present boundary conditions, leading to a self-consumption rate of 75\%. With forced emission reductions, the PV area is continuously expanded to 48.3 m\(^2\) at a rate of self-consumption of 24\% in the minimum emission case. Compared with the status quo setting, PV installation can reduce the overall costs by up to 4.3\% for “PV 0”. The CO\textsubscript{2} emissions can be reduced by up to 13.8\% in “PV 6” at increased costs of 3.7\%. Without increasing the total annualized costs, “PV 5” reduces the CO\textsubscript{2} emissions by 13.1\%.

4.2.2. EnEV constrained

In the EnEV constrained setting as well as in the unconstrained optimization, the optimizer has additional degrees of freedom.

<table>
<thead>
<tr>
<th>Component (j)</th>
<th>(S_j)</th>
<th>(U) in W/(m(^2) K)</th>
<th>(\kappa) in (W/(m(^2) K)</th>
<th>inv in Euro/m(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside walls 1</td>
<td>1</td>
<td>0.963</td>
<td>240.0</td>
<td>0</td>
</tr>
<tr>
<td>Outside walls 2</td>
<td>2</td>
<td>0.479</td>
<td>240.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Outside walls 3</td>
<td>3</td>
<td>0.167</td>
<td>233.1</td>
<td>50.0</td>
</tr>
<tr>
<td>Outside walls 4</td>
<td>4</td>
<td>0.122</td>
<td>233.1</td>
<td>63.6</td>
</tr>
<tr>
<td>Roof 1</td>
<td>1</td>
<td>0.494</td>
<td>240.0</td>
<td>0</td>
</tr>
<tr>
<td>Roof 2</td>
<td>2</td>
<td>0.396</td>
<td>240.0</td>
<td>50.0</td>
</tr>
<tr>
<td>Roof 3</td>
<td>3</td>
<td>0.148</td>
<td>240.0</td>
<td>67.2</td>
</tr>
<tr>
<td>Roof 4</td>
<td>4</td>
<td>0.100</td>
<td>240.0</td>
<td>100.0</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Windows</th>
</tr>
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<tbody>
<tr>
<td>(S_j)</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>1.897</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

\textsuperscript{6} http://www.gurobi.com/index.
concerning the installation of BES components, their operation and additional investments in the building envelope.

Table 8
Available BES components.

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameter</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
<th>Type 4</th>
<th>Type 5</th>
<th>Type 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAT</td>
<td>Capacity in kW h</td>
<td>5.0</td>
<td>8.1</td>
<td>9.2</td>
<td>20.5</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Investment in Euro</td>
<td>6716</td>
<td>10,220</td>
<td>12,166</td>
<td>16,790</td>
<td></td>
<td></td>
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<tr>
<td>TES</td>
<td>Volume in m³</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
<td></td>
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<tr>
<td></td>
<td>Investment in Euro</td>
<td>0</td>
<td>600</td>
<td>800</td>
<td>1300</td>
<td>1850</td>
<td></td>
</tr>
<tr>
<td>Boiler</td>
<td>Capacity in kW</td>
<td>18</td>
<td>11</td>
<td>14</td>
<td>19</td>
<td>26</td>
<td>37.5</td>
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<tr>
<td></td>
<td>Efficiency in %</td>
<td>70</td>
<td>93</td>
<td>95</td>
<td>95</td>
<td>95</td>
<td>96</td>
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<tr>
<td></td>
<td>Investment in Euro</td>
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<td>1000</td>
<td>1300</td>
<td>1600</td>
<td>2000</td>
<td>2400</td>
</tr>
<tr>
<td>CHP</td>
<td>Capacity in kW th</td>
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<td>2</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td></td>
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<tr>
<td></td>
<td>Investment in Euro</td>
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<td>15,500</td>
<td>17,685</td>
<td>21,905</td>
<td>32,140</td>
<td></td>
</tr>
<tr>
<td>HP</td>
<td>Capacity in kW th</td>
<td>5</td>
<td>8</td>
<td>10</td>
<td>13</td>
<td>19</td>
<td>25</td>
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<tr>
<td></td>
<td>Investment in Euro</td>
<td>8340</td>
<td>10,155</td>
<td>11,365</td>
<td>13,180</td>
<td>16,809</td>
<td>20,439</td>
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Table 9
Results status quo and PV installation simulations.

<table>
<thead>
<tr>
<th>Result</th>
<th>Status quo</th>
<th>PV 0</th>
<th>PV 1</th>
<th>PV 2</th>
<th>PV 3</th>
<th>PV 4</th>
<th>PV 5</th>
<th>PV 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outsides walls [type]</td>
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<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>Roof [type]</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Windows [type]</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Boiler in kW</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>TES in m³</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>PV in m²</td>
<td>0</td>
<td>11.3</td>
<td>12.9</td>
<td>15.6</td>
<td>19.3</td>
<td>23.1</td>
<td>34.2</td>
<td>48.3</td>
</tr>
<tr>
<td>$\phi_{E,T, in kW}^L$</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
<td>15.8</td>
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<tr>
<td>$\phi_{E,T, in kW}^R$</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
<td>13.3</td>
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</tr>
<tr>
<td>$\delta_{air, in °C}$</td>
<td>20.2</td>
<td>20.2</td>
<td>20.2</td>
<td>20.2</td>
<td>20.2</td>
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<tr>
<td>$\delta_{air, max, in °C}$</td>
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<td>22.7</td>
<td>22.7</td>
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<td>22.7</td>
<td>22.7</td>
<td>22.9</td>
<td>22.2</td>
</tr>
<tr>
<td>Self-consumption in %</td>
<td>0</td>
<td>75</td>
<td>69</td>
<td>60</td>
<td>51</td>
<td>42</td>
<td>32</td>
<td>24</td>
</tr>
<tr>
<td>$e_{total, in tCO2/a}$</td>
<td>7.24</td>
<td>6.50</td>
<td>6.47</td>
<td>6.42</td>
<td>6.38</td>
<td>6.33</td>
<td>6.29</td>
<td>6.24</td>
</tr>
<tr>
<td>$c_{total, in 1000 Euro/a}$</td>
<td>3.07</td>
<td>2.94</td>
<td>2.94</td>
<td>2.95</td>
<td>2.97</td>
<td>3.01</td>
<td>3.08</td>
<td>3.19</td>
</tr>
</tbody>
</table>

Table 10
Results EnEV-constrained simulations.

<table>
<thead>
<tr>
<th>Result</th>
<th>EnEV 0</th>
<th>EnEV 1</th>
<th>EnEV 2</th>
<th>EnEV 3</th>
<th>EnEV 4</th>
<th>EnEV 5</th>
<th>EnEV 6</th>
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<tr>
<td>CHP in kWth</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>EH in kWth</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>HP in kWth</td>
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<td>0</td>
<td>0</td>
<td>5</td>
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<td>STC in m²</td>
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<td>3.4</td>
<td>20.7</td>
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<td>Battery in kW h</td>
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<td>0</td>
<td>5</td>
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<td>0.1</td>
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<td>7.2</td>
<td>6.8</td>
<td>6.8</td>
<td>7.2</td>
<td>7.2</td>
<td>6.3</td>
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<tr>
<td>$\phi_{E,T, in kW}^R$</td>
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<td>7.7</td>
<td>12.9</td>
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<td>7.0</td>
<td>7.0</td>
<td>6.3</td>
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<td>20.2</td>
<td>20.7</td>
<td>20.3</td>
<td>20.1</td>
<td>20.1</td>
<td>20.1</td>
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<td>20.9</td>
<td>22.9</td>
<td>29.6</td>
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<td>20.9</td>
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<td>Coverage boiler in %</td>
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<td>60</td>
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<td>Coverage CHP in %</td>
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<td>0</td>
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<td>0</td>
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<tr>
<td>Coverage HP in %</td>
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<td>0</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>91</td>
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<tr>
<td>Coverage STC in %</td>
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<td>23</td>
<td>61</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>$e_{total, in tCO2/a}$</td>
<td>3.48</td>
<td>3.07</td>
<td>2.60</td>
<td>2.13</td>
<td>1.66</td>
<td>1.19</td>
<td>0.72</td>
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<tr>
<td>$c_{total, in 1000 Euro/a}$</td>
<td>3.82</td>
<td>3.87</td>
<td>4.11</td>
<td>4.72</td>
<td>4.96</td>
<td>5.49</td>
<td>10.18</td>
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</table>
fill the EnEV requirements, which is true for types 3 and 4 of the available components. In the minimum cost optimization. “EnEV 0”, all envelope components are of type 3 and the BES consists of a smaller, high efficiency boiler with 11 kW, a small TES unit and 0", all envelope components are of type 3 and the BES consists of available components. In the minimum cost optimization, "EnEV fill the EnEV requirements, which is true for types 3 and 4 of the grid. Overall, the CO2 emissions result in 3.48 t/a and the total heating energy consumption is 6.9 MW h/a. The average and maximum indoor temperature are 20.1 and 20.9 °C. The entire heat demand is covered with the boiler and 75% of the generated electricity is used within the building, only 25% are exported to the grid. Overall, the CO2 emissions result in 3.48 t/a and the total costs are 3820 Euro/a. Compared with the status quo, the minimum costs calculation with EnEV constraints is 24.3% more expensive than the status quo setting, however the costs are increased by 231.6%.

When forcing higher CO2 reductions, first STCs are installed that heavily use the building’s thermal mass, leading to increased peak and average indoor temperatures. For very low CO2 emissions, the heating system is transformed to a small sized HP with a backup EH. When this switch to the HP occurs, the TES unit is also enlarged. Furthermore, a battery system is installed to reduce grid-dependency and the inherent CO2 emissions from grid purchases. Only in the minimum CO2 emissions optimization, the solver chooses to upgrade the building envelope beyond the minimum requirements of EnEV. In this manner, the CO2 emissions can be reduced to 0.72 t/a, which is 90.0% less than in the status quo setting, however the costs are increased by 231.6%.

4.2.3. Unconstrained optimization

The results of the unconstrained calculations are shown in Table 11. The most economical simulation, “UC 0” chooses not to upgrade the building envelope but rather to improve the boiler and install 11.3 m² of PV. Like in the EnEV constrained scenario, the solver chooses STC and PV modules to reduce CO2 emissions, whereas the building envelope is barely modernized. At “UC 5” the heating system is restructured to a HP with backup EH and the TES unit is increased to 0.3 m³. In “UC 7” a battery storage system is installed to reduce grid dependency. The only result that complies with current EnEV standards is “UC 9” that is identical to “EnEV 6”. Calculations “UC 0” to “UC 3” reduce the total costs by 3.7–13.4% compared with the status quo setting while simultaneously lowering CO2 emissions by 32.4–51.1%. “UC 4” to “UC 9” offer the same CO2 emissions as “EnEV 1” to “EnEV 6”, however in the unconstrained setting, total costs can be reduced by 14.6% on average.

5. Conclusions

In this paper, a dynamic building model based on ISO 13790 has been integrated within a mixed-integer linear optimization problem for optimizing building energy systems’ structure, sizing, and operation. In contrast to previous studies, this approach allows for using an exact optimization algorithm instead of heuristics. Additionally, the developed model accounts for dynamic effects such as solar radiation and building’s thermal mass that have previously not been considered. We have validated this model based on ASHRAE 140 and compared it to a more detailed low-order model. The validation illustrates that the developed model accurately computes annual heating demands and peak loads. Furthermore, hourly demand profiles as well as free float temperatures are also predicted accurately, proving that our model is able to assess buildings’ dynamic behaviors.

The building model has been included into an optimization model used to design and operate building energy systems. We have applied this extended model to analyze retrofitting options for a representative building from the 1970s. These options include a status quo scenario, only adding photovoltaic modules, an EnEV conform modernization of the building envelope as well as an unconstrained optimization in which no governmental restrictions onto the building envelope and BES components are enforced.

Solely adding PV modules to the existing building reduces overall costs by up to 4.3% while simultaneously cutting CO2 emissions by 10.1%. The CO2 emissions can even be reduced by 13.1% without monetary losses compared with the status quo scenario when installing more PV units. The EnEV-conform simulations suggest that current German regulatory restrictions are not economically worthwhile as the minimum costs in this setting exceed the status quo by 24.3%, however reducing CO2 emissions by 51.9%. The unconstrained scenario shows that the costs of the status quo setting can be reduced by up to 13.4% while simultaneously lowering...
CO₂ emissions by 32.4%. At smaller cost improvements (3.7%), even further CO₂ reductions of 51.1% are possible. Compared with the EnEV-conform modernization, the unconstrained optimization is able to provide the same emissions reductions at 14.6% lower costs on average. These cost reductions are achieved mostly without cost-intensive modernizations of the building envelope but rather through improved BES components.

In future works the presented model could be applied to analyze different regulatory restrictions besides the German EnEV for new buildings and retrofits. Furthermore, the model can be extended to multiple thermal zones taking into account different usage and boundary conditions for separate parts of the investigated building. Additionally, the presented model could be validated using measurement data for indoor temperatures and heating loads.

Acknowledgment

This work was supported by the Helmholtz Association under the Joint Initiative “Energy System 2050 - A Contribution of the Research Field Energy”.

Appendix A

Additional parameters.

**Table A.12** Default values for dynamic parameters, based on [28, p.68].

<table>
<thead>
<tr>
<th>Class</th>
<th>( f_{\text{h,m}}^0 ) in ( \text{kJ}/(\text{m}^2 \text{K}) )</th>
<th>( f_{\text{h,s}}^0 ) in ( \text{kJ}/(\text{m}^2 \text{K}) )</th>
<th>( f_{\text{A,cl}} ) in w/o unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very light</td>
<td>0.0</td>
<td>95.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Light</td>
<td>95.0</td>
<td>137.5</td>
<td>2.5</td>
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<tr>
<td>Medium</td>
<td>137.5</td>
<td>212.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Heavy</td>
<td>212.5</td>
<td>313.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Very heavy</td>
<td>313.5</td>
<td>10000.0</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Estimate \( H_{\text{rms}} \gg H_{\text{rop}} \).

When assuming a rectangular ground floor with edge lengths \( a \) and \( b \), as well as \( n \)-stories of height \( c \), \( H_{\text{rop}} \) results in:

\[
H_{\text{rop}} = 2 \cdot (a + b) \cdot n \cdot c \cdot U_{\text{Wall}} + a \cdot b \cdot (U_{\text{roof}} + U_{\text{floor}})
\]

(B.1)

The heat transfer coefficient between \( m \) and \( s \) is defined as:

\[
H_{\text{rms}} = 9.1 \ \frac{W}{m^2 K} \cdot f_{\text{A,cl}} \cdot A_l
\]

(B.2)

Since \( f_{\text{A,cl}} \) is always larger than 2.5 (cf. Table A.12) and the floor area is the product of \( a \), \( b \), and \( n \), \( H_{\text{rms}} \) is estimated to be:

\[
H_{\text{rms}} \geq 9.1 \ \frac{W}{m^2 K} \cdot 2.5 \cdot a \cdot b \cdot n
\]

(B.3)

The original assumption, \( H_{\text{rms}} \gg H_{\text{rop}} \) can be interpreted as a difference in at least one order of magnitude:

\[
H_{\text{rms}} \geq 10 \cdot H_{\text{rop}}
\]

(B.4)

Furthermore, we assume that buildings have a compact shape, leading to \( a \approx b \). This results in the following condition for the building’s edge length:

\[
a \geq \frac{40 \cdot c \cdot n \cdot U_{\text{Wall}}}{9.1 \ \frac{W}{m^2 K} \cdot 2.5 \cdot n - 10 \cdot (U_{\text{roof}} + U_{\text{floor}})}
\]

(B.5)

For standard 2-story houses with 2.8 m height between floors and 1970 insulation standards [42], an edge length of \( a \geq 7.3 \) m fulfills the initial requirements. For buildings meeting current (2016) German insulation requirements [43], an edge length of 0.9 m would be sufficient. As 7.3 m can already be considered relatively small for buildings, the initial assumption is justified.

Appendix C

Additional figures. See Fig. C.17.

References


