Lecture#16

Time series II

Time series with explanatory variables

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \delta_1 X_{t-1} + u_t$$

- As with Y_t, can include multiple (“distributed”) lags of X_t (ADL(p,q)).
- Let’s estimate a model of the monthly euro-area unemployment rate.
- Y_t = euro-area inflation in a given month.
- X_t = euro-area UE level in a given month

```
. estimates table L_ue_1 L_pue_1 , b(%7.4f) star(0.1 0.05 0.01) stat(N r2 r2_a aic bic)
```

Variable	L_ue_1	L_pue_1
ue_euro		
L1.	1.0110***	0.9930***
pind_euro		
L1.		0.0047***
_cons	-0.0833	-0.3289***
N	167	167
r2	0.9949	0.9953
r2_a	0.9949	0.9952
aic	-3.2e+02	-3.3e+02
bic	-3.1e+02	-3.2e+02

legend: * p<.1; ** p<.05; *** p<.01

```
. estimates stat L_ue_1 L_pue_1
```

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
L_ue_1	167	-280.3921	160.7244	2	-317.4487	-311.2128
L_pue_1	167	-280.3921	167.0729	3	-328.1458	-318.7918

Note: N=Obs used in calculating BIC; see **[R] BIC note**

Stationarity

- Stationarity an important characteristic of time series.

1. Affects regression analysis.

2. Affects in particular time series analysis with multiple/explanatory variables.

Definition

A time series Y_t is ***stationary*** if its probability distribution does not change over time, that is, if the joint distribution of $(Y_{s+1}, Y_{s+2}, \ldots, Y_{s+T})$ does not depend on *s*.

Otherwise Y_t is ***nonstationary***.

Stationarity requires that in a probabilistic sense, the future is like the past.

Definition – version 2

A pair of time series $Y_t, X_t$is ***stationary*** if its probability distribution does not change over time, that is, if the joint distribution of $(Y_{s+1}, X_{s+1}, Y_{s+2}, X_{s+2}, \dots, Y_{s+T}, X_{s+T})$ does not depend on *s*.

Otherwise Y_t is ***nonstationary***.

Time series assumptions

A1: $\mathrm{E}[u_t|(Y_{t-1}, X_{t-1}, Y_{t-2}, X_{t-2}, \ldots) = 0.$

A2: i. the random variables Y_t, X_t have a stationary distribution.

ii. Y_t, X_t and Y_{t-j}, X_{t-j} become independent as j grows large.

A3: Y_t, X_t have nonzero, finite fourth moments.

A4: no perfect multicollinearity.

What's the fuzz about (non)stationarity?

1. Autoregressive coefficients biased towards zero.

- It can be shown that with a random walk,

$$E[\beta_1] \approx 1 - \frac{5.3}{T}$$

Demonstration

```
set obs 1000
gen time          = _n
tsset time
gen u             = 3 * invnorm(uniform())
gen y             = u
replace y         = y[_n - 1] + u if time > 1

regr y L.y  if time < T
```

time	u	y	u+y[_n-1]
1	-3.2819	-3.2819	
2	1.1012	-2.1807	-2.1807
3	0.4362	-1.7445	-1.7445
4	0.7973	-0.9471	-0.9471
5	1.4382	0.4911	0.4911
6	-3.7009	-3.2098	-3.2098
7	0.9043	-2.3055	-2.3055
8	-4.6377	-6.9433	-6.9433
9	0.4167	-6.5265	-6.5265
10	3.3998	-3.1267	-3.1267

Same regression with different sample sizes

```
. estimates table t_25 t_50 t_100 t_1000 , b(%7.4f) se(%7.4f) p(%7.4f) stat(N r2 r2_a aic bic)
```

Variable	t_25	t_50	t_100	t_1000
y				
L1.	0.8397	0.9685	0.9917	0.9961
	0.1170	0.0549	0.0136	0.0030
	0.0000	0.0000	0.0000	0.0000
_cons	0.9182	0.8295	-0.2251	0.1034
	0.9938	0.7253	0.2662	0.1035
	0.3651	0.2584	0.3992	0.3181
N	25	50	150	999
r2	0.6915	0.8666	0.9730	0.9909
r2_a	0.6781	0.8638	0.9728	0.9909
aic	133.3537	259.3584	780.5999	5.0e+03
bic	135.7915	263.1825	786.6211	5.0e+03

legend: b/se/p

What's the fuzz about (non)stationarity?

2. Non-normal distributions of t-statistics.

- With a stochastic trend, OLS t-statistic may have a non-normal distribution, even in large samples.

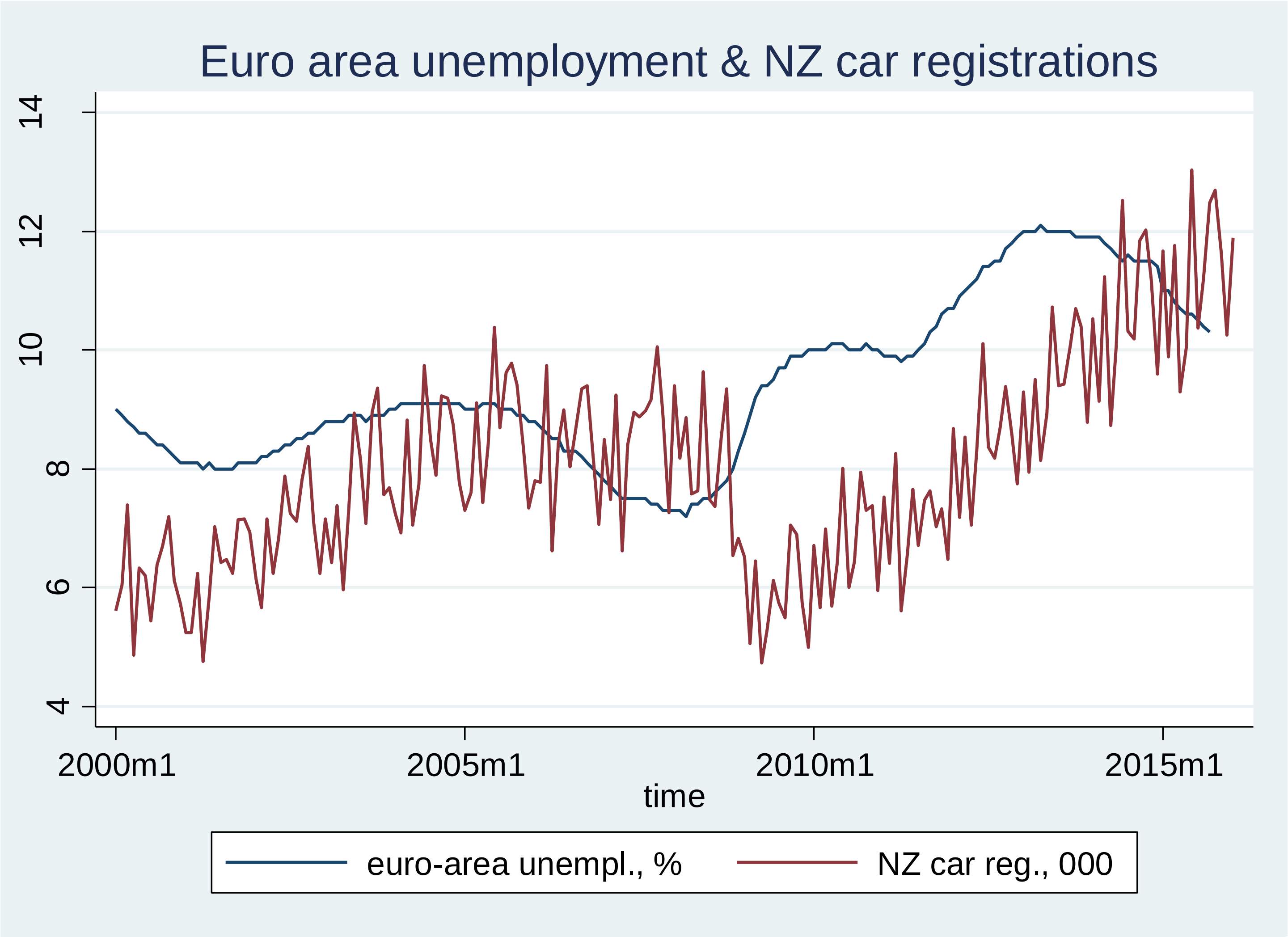
- Cannot do regular testing.

What's the fuzz about (non)stationarity?

3. Spurious regression.

- If Y has a trend and X has a trend, then you may be explaining a trend with a trend, with no real relation between Y and X.

- Demonstration: EU-unemployment and NZ car sales.



Corr(UE_euro, NZ_reg) = 0.40 (p-value 0.000).

```
. estimates table L_ue_1 L_ue_2 , b(%7.4f) star(0.1 0.05 0.01) stat(N r2 r2_a aic bic)
```

Variable	L_ue_1	L_ue_2
ue_euro		
L1.	1.0110***	1.0136***
registr		
L1.		-0.0136**
_cons	-0.0833	-0.0045
N	167	167
r2	0.9949	0.9951
r2_a	0.9949	0.9951
aic	-3.2e+02	-3.2e+02
bic	-3.1e+02	-3.1e+02

legend: * p<.1; ** p<.05; *** p<.01

```
. estimates stat L_ue_1 L_ue_2
```

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
L_ue_1	167	-280.3921	160.7244	2	-317.4487	-311.2128
L_ue_2	167	-280.3921	164.0144	3	-322.0288	-312.6748

Note: N=Obs used in calculating BIC; see **[R] BIC note**

Nonstationarity – what causes it?

- Answer: a trend.
- A trend can be deterministic, or (more often), stochastic.
- Time has a deterministic trend (outside physics, anyways).
- A deterministic trend is a nonrandom function of time.
- A stochastic trend varies over time.

Simple example – random walk

$$Y_t = Y_{t-1} + u_t$$

- The value of Y today is in expectation the same as what it actually was yesterday.

- Random walk = today's value Y_t is where you were yesterday + a (random) step u_t of unknown direction and length.

Random walk with drift

$$Y_t = \beta_0 + Y_{t-1} + u_t$$

- The value of Y today is in expectation the same as what it actually was yesterday + β_0.

- Random walk = today's value Y_t is where you were yesterday + β_0 + a (random) step u_t of unknown direction and length.

Random walk is nonstationary

$$Y_t = Y_{t-1} + u_t$$

➔

$$var(Y_t) = var(Y_{t-1}) + var(u_t)$$

Random walk is a special case of AR(1)

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$$

- If $\beta_1 = 1$, Y_t is nonstationary.
- If $\beta_1 < 1$, Y_t is stationary.
- For AR(p), p > 1, similar but more complicated statements apply.

Nonstationarity, unit root & stochastic trend

- If $\beta_1 = 1$, Y_t is nonstationary.
- If $\beta_1 = 1$, Y_t has a ***unit root***.
- If $\beta_1 = 1$, Y_t has a stochastic trend.

Is there a unit root?

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$$

- H_0: $\beta_1 = 1$ ➔ there is a unit root / Y_t is nonstationary.
- H_1: $\beta_1 < 1$ ➔ there is no unit root / Y_t is stationary.

The Dickey-Fuller test in the AR(1) model

$$\Delta Y_t = \beta_0 + \delta Y_{t-1} + u_t$$

- $\delta = \beta_1 - 1$

- $H_0: \delta = 0$ ➔ there is a unit root & stochastic trend / Y_t is nonstationary.

- $H_1: \delta < 0$ ➔ there is no unit root / Y_t is stationary.

The augmented Dickey-Fuller test in the AR(p) model

$$\Delta Y_t = \beta_0 + \delta Y_{t-1} + \gamma_1 \Delta Y_{t-1} \dots \gamma_p \Delta Y_{t-p} + u_t$$

- $H_0: \delta = 0$ ➔ there is a unit root / Y_t is nonstationary.
- $H_1: \delta < 0$ ➔ there is no unit root / Y_t is stationary.

The augmented Dickey-Fuller test in the AR(p) model, deterministic trend

$$\Delta Y_t = \beta_0 + \alpha t + \delta Y_{t-1} + \gamma_1 \Delta Y_{t-1} \ldots \gamma_p \Delta Y_{t-p} + u_t$$

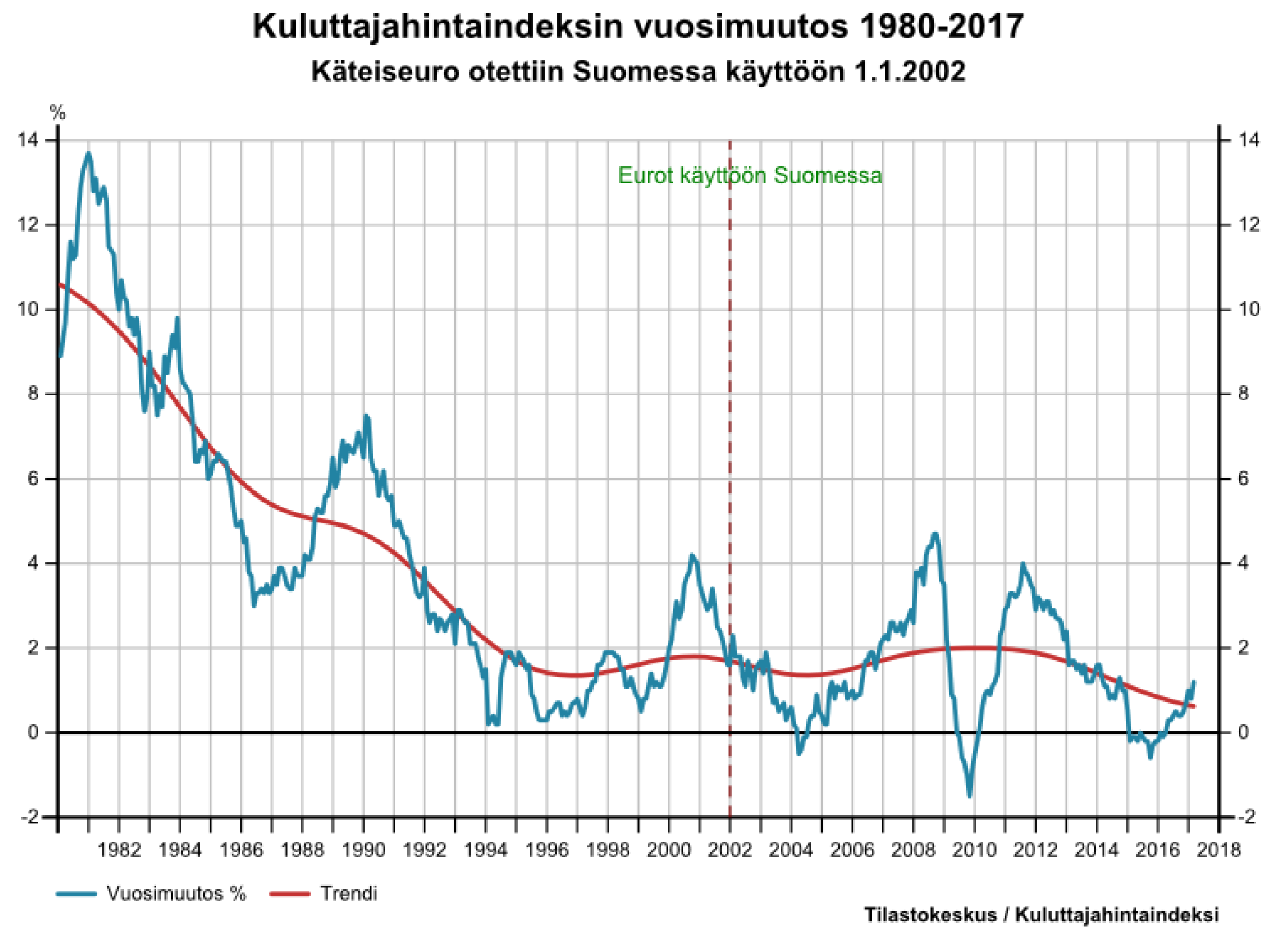
- $H_0: \delta = 0$ ➔ there is a unit root & deterministic trend / Y_t is nonstationary.
- $H_1: \delta < 0$ ➔ there is no unit root / Y_t is stationary.

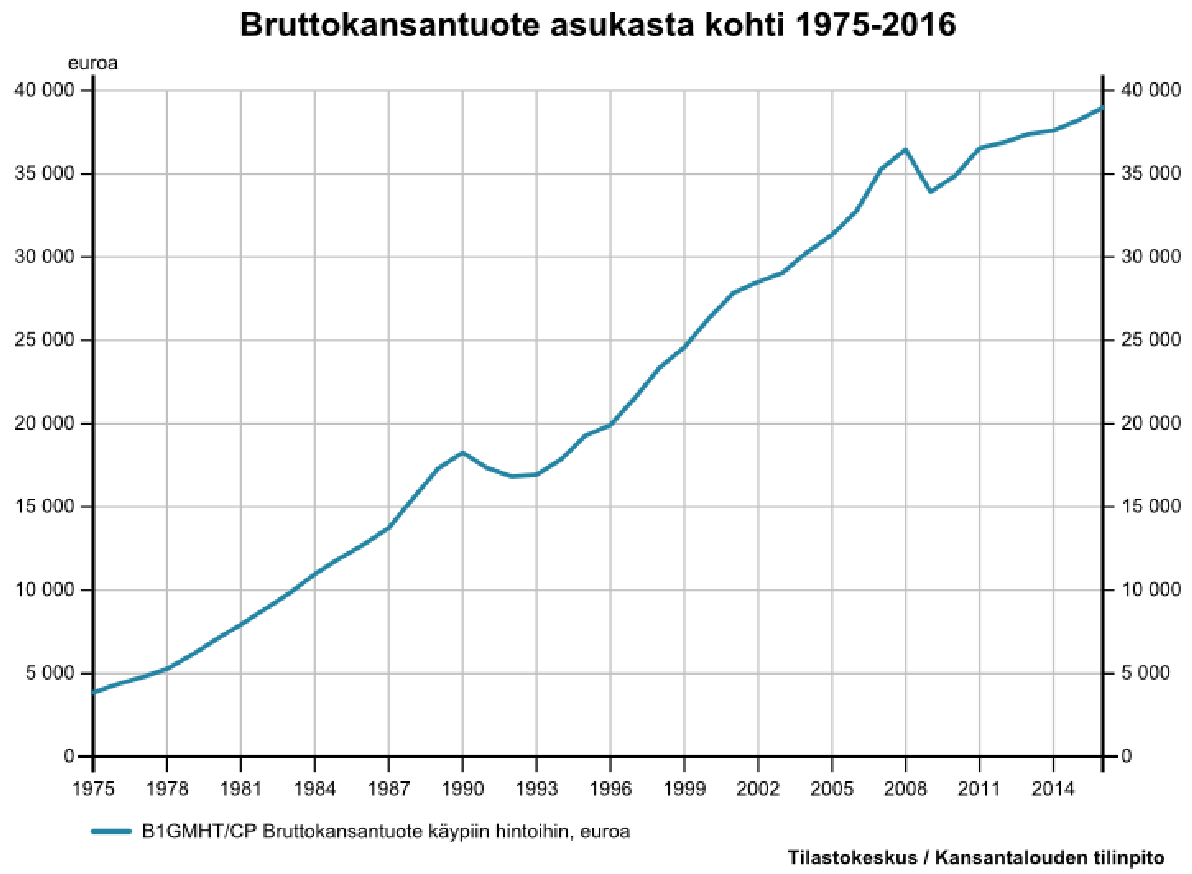
The augmented Dickey-Fuller test in the AR(p) model

- Need to use special critical values.
- Notice: (A)DF is a ***one-sided*** test!!!
- Q: why are we not worried about $\delta > 0$ $(\beta_1 > 1)$?

The augmented Dickey-Fuller test in the AR(p) model

- When stochastic, when deterministic trend?
- Stochastic trend when ”short period trends”. Think inflation.
- Deterministic trend when long run trends. Think GDP growth.


Kuluttajahintaindeksin vuosimuutos 1980-2017
Käteiseuro otettiin Suomessa käyttöön 1.1.2002
%
Eurot käyttöön Suomessa
Vuosimuutos %
Trendi
Tilastokeskus / Kuluttajahintaindeksi


Bruttokansantuote asukasta kohti 1975-2016
euroa
B1GMHT/CP Bruttokansantuote käypiin hintoihin, euroa
Tilastokeskus / Kansantalouden tilinpito

Demonstration #1

- Euro-area price index (in logs).
- No trend (with trend, larger, i.e., weaker, test results).

```
. dfuller lnpind_euro if year > 2000 ,regress

Dickey-Fuller test for unit root                   Number of obs   =       177

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -1.804            -3.484            -2.885            -2.575
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.3784

------------------------------------------------------------------------------
D.           |
 lnpind_euro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 lnpind_euro |
         L1. |  -.0067908   .0037638    -1.80   0.073    -.0142191    .0006375
             |
       _cons |   .0319235   .0169217     1.89   0.061    -.0014734    .0653205
------------------------------------------------------------------------------
```

Demonstration #1

- Rule of thumb: rather more than fewer lags.

#lags	ADF	1%	5%	10%
0	-1.804	-3.484	-2.885	-2.575
1	-1.758	-3.484	-2.885	-2.575
2	-1.998	-3.484	-2.885	-2.575
3	-2.109	-3.484	-2.885	-2.575
4	-2.577	-3.484	-2.885	-2.575
5	-2.691	-3.484	-2.885	-2.575
6	-1.933	-3.484	-2.885	-2.575
7	-2.056	-3.484	-2.885	-2.575
8	-2.203	-3.484	-2.885	-2.575

So there's a unit root...

- What to do?
- Transform the series…
- Try differencing.

Demonstration #2

- Difference in (log of) euro-area price index (= inflation).
- No unit root ➔ inflation is stationary.

#lags	ADF	1%	5%	10%
0	-12.015	-3.484	-2.885	-2.575
1	-11.006	-3.484	-2.885	-2.575
2	-8.975	-3.484	-2.885	-2.575
3	-9.556	-3.484	-2.885	-2.575
4	-7.861	-3.484	-2.885	-2.575
5	-3.548	-3.484	-2.885	-2.575
6	-3.862	-3.484	-2.885	-2.575
7	-4.149	-3.484	-2.885	-2.575
8	-3.981	-3.484	-2.885	-2.575

```
. dfuller dlnpind_euro if year > 2000, regress

Dickey-Fuller test for unit root                   Number of obs   =       177

                              ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -12.015            -3.484            -2.885            -2.575
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000

------------------------------------------------------------------------------
D.           |
dlnpind_euro |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dlnpind_euro |
         L1. |  -.9466037    .0787851   -12.02   0.000    -1.102095   -.7911124
             |
       _cons |   .0013178    .0003365     3.92   0.000     .0006537     .001982
------------------------------------------------------------------------------
```

Forecasting

$$Y_{t+1} = \beta_0 + \beta_1 Y_t + u_{t+1}$$

- one-step ahead forecast:

$$\mathrm{E}[Y_{t+1}|Y_t] = \hat{Y}_{t+1|t} = \widehat{\beta_0} + \widehat{\beta_1} Y_t$$

- forecast error:

$$Y_{t+1} - \hat{Y}_{t+1|t} = Y_{t+1} - [\widehat{\beta_0} + \widehat{\beta_1} Y_t]$$

- Mean squared forecast error = MSFE:

$$MSFE = E[(Y_{t+1} - \hat{Y}_{t+1|t})^2] = Y_t - [\widehat{\beta_0} + \widehat{\beta_1} Y_{t-1}]$$

- Root MSFE = $\sqrt{MSFE}$

Pseudo out-of-sample forecasting

1. Split your data into two:
   1. Choose #obs used for testing, P.
   2. Estimation data: t = 1, ..., s, s = T-P.

2. Estimate your model using estimation data.

3. Calculate your forecast and forecast error for the first ”out of sample” observation.

4. Repeat #1 - #3, by changing s, 1 obs at a time.

What’s the use?

1. Indicates how well your model forecasts.

2. Allows you to estimate RMSFE.

3. Allows comparison of models.