Practise Exam Questions

1. Order the complexity classes **P**, **EXP**, Σ_1^p, **RP**, **PH**, **NC**, **AP**, **NL**, and **ZPP** by set inclusion (that is, write enough set inclusion statements of the form

$$X \subseteq Y$$

where X and Y are complexity classes given above such that all known set inclusions follow from the statements).

2. (a) Given a complexity class $\mathcal{C}$, define what is a $\mathcal{C}$-complete language.

   (b) Show that if there is a language L that is $\mathcal{C}_1$-complete for a complexity class $\mathcal{C}_1$, $L \in \mathcal{C}_2$ for another complexity class $\mathcal{C}_2$, and both of the classes are closed under log-space reductions, then $\mathcal{C}_1 \subseteq \mathcal{C}_2$.

3. (a) Define the complexity classes Σ_i^p ($i \geq 0$) and **PH**.

   (b) Prove that **PH** $\subseteq$ **PSPACE**. (*Hint:* Induction.)

4. Show that the following problem is **NP**-complete:

   HITTING SET
   INSTANCE: A family $\mathcal{F} = \{S_1, \ldots, S_n\}$ of subsets of a finite set U and an integer K.
   QUESTION: Is there a subset $H \subseteq U$ of at most K elements in U such that H contains at least one representative from each set in $\mathcal{F}$, i.e. $H \cap S_i \neq \emptyset$ for every $i = 1, \ldots, n$?

   (*Hint:* Think of an element $u \in U$ such that $u \in S_i$ as "covering" the set S_i.)

Grading: Each problem 6p, total 24p.