Lecture#17

Time series III

Dynamic causal effects

- Think of macroeconomic data.
- Difficult to think of an RCT.
- Substitute: different treatments to the same (observation unit) at different points in time.
- Treatment = e.g. level of interest rate t periods ago.

Dynamic causal effects

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + \beta_3 X_{t-2} \ldots + u_t$$

- β_k tell about the effect that a unit change in X_k has on Y_t.
- This is a distributed lags model.

Two exogeneity concepts

- $\mathrm{E}[u_t|(X_t, X_{t-1}, \dots) = 0$.

- Implies that present (period t) shocks to Y are uncorrelated with the whole past of X.

- Implies that if you include k lags, then all effects k + m periods ago, m = 1, ... are zero.

- This is past and present exogeneity (or just exogeneity).

Two exogeneity concepts

- $\mathrm{E}[u_t|(\ldots, X_{t+1}, X_t, X_{t-1}, \ldots) = 0.$

- Implies that present (period t) shocks to Y are uncorrelated with the whole past of X.

- Also implies that the same shock independent of all future values of X.

- This is past, present & future exogeneity (or strict exogeneity).

Distributed lag assumptions

A1: $\mathrm{E}[u_t|(Y_{t-1}, X_{t-1}, Y_{t-2}, X_{t-2}, \dots) = 0.$

A2: i. the random variables Y_t, X_t have a stationary distribution.

ii. Y_t, X_t and Y_{t-j}, X_{t-j} become independent as j grows large.

A3: Y_t, X_t have > 8 nonzero, finite moments (for HAC).

A4: no perfect multicollinearity.

Dynamic effects / multipliers

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + \beta_3 X_{t-2} \ldots + u_t$$

- β_1 = (contemporaneous) impact of X on Y.
- β_2 = the 1-period dynamic multiplier, and so on.
- β_1+β_2 = the 1-period cumulative dynamic multiplier, and so on.
- $\beta_1 + \cdots + \beta_k$ = long-run cumulative dynamic multiplier.

Dynamic effects / multipliers

$$Y_t = \delta_0 + \delta_1 \Delta X_t + \delta_2 \Delta X_{t-1} + \delta_3 \Delta X_{t-2} \ldots \delta_{k+1} X_{t-k} + u_t$$

- β_0 = δ_0(contemporaneous) impact of X on Y.
- $\delta_1 = \beta_1$+β_2 = the 1-period cumulative dynamic multiplier, and so on.
- $\delta_s = \beta_1 + \cdots + \beta_s$ = s-period cumulative dynamic multiplier.
- $\delta_{k+1} = \beta_1 + \cdots + \beta_{k+1}$ = long-run cumulative dynamic multiplier.

What standard errors to use?

- Think of omitted variables, i.e., things that affect Y_t besides X.
- These are likely to be serially correlated (= have autocorrelation).
- ➔ we have an autocorrelated error term.
- ➔ use heteroscedasticity and autocorrelation consistent s.e.'s (HAC / Newey-West).

Linking a DL model with autocorr. & ADL

- Consider the model

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + u_t$$

Such that

$$u_t = \tilde{u}_t + \phi_1 u_{t-1}$$

i.e., with an autocorrelated error.

Linking a DL model with autocorr. & ADL

- We could estimate the model with OLS and use HAC standard errors.

- Alternative is to transform it to a model with an error term that is not autocorrelated.

- Substract $\phi_1 Y_t$ from both sides.

Linking a DL model with autocorr. & ADL

- We get

$$Y_t = (\beta_0 - \phi_1\beta_1) + \beta_1 X_t + (\beta_2 - \phi_1\beta_1)X_{t-1} - \phi_1\beta_2 X_{t-2} + \tilde{u}_t$$

$$Y_t = \delta_0 + \delta_1 X_t + \delta_2 X_{t-1} + X_{t-2} + e_t$$

Vector autoregressions (VAR)

- What if you want to model two or more time series at the same time (simultaneously)?

- What if they interact, i.e., past values of Y affect current value of X and vice versa?

- The answer is to use a VAR.

Vector autoregressions (VAR)

- What is a VAR? It is a model with several equations.

$$Y_t = \beta_{10} + \beta_{11}Y_{t-1} + \beta_{12}Y_{t-2} \ldots + \gamma_{11}X_{t-1} + \gamma_{12}X_{t-2} \ldots + u_{1t}$$

$$X_t = \beta_{20} + \beta_{21}Y_{t-1} + \beta_{22}Y_{t-2} \ldots + \gamma_{21}X_{t-1} + \gamma_{22}X_{t-2} \ldots + u_{2t}$$

- VAR assumptions = time series assumptions (for each eqn.).

Euro-area change in inflation & ue

- This is the so-called Phillips – curve.

```
estimates clear
var ue_euro ddlnpind_euro if year > 2000, lags(1)
estimates store var_1
var ue_euro ddlnpind_euro if year > 2000, lags(1/2)
estimates store var_2
var ue_euro ddlnpind_euro if year > 2000, lags(1/3)
estimates store var_3
var ue_euro ddlnpind_euro if year > 2000, lags(1/4)
estimates store var_4
estimates table var_1 var_2 var_3  var_4, b(%7.4f) star(0.1 0.05 0.01) stat(N r2 r2_a aic bic)
```

```
. var ue_euro ddlnpind_euro if year > 2000, lags(1/2)

Vector autoregression

Sample:  2001m4 - 2013m12                         Number of obs     =        153
Log likelihood =   764.3655                       AIC               =  -9.860986
FPE            =   1.79e-07                       HQIC              =  -9.780528
Det(Sigma_ml)  =   1.57e-07                       SBIC              =  -9.662918

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
ue_euro              5     .085447   0.9960   38247.35   0.0000
ddlnpind_euro        5     .004793   0.2264   44.78908   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ue_euro      |
     ue_euro |
         L1. |   1.421912   .0736158    19.32   0.000     1.277628    1.566197
         L2. |  -.4191532   .0744727    -5.63   0.000     -.565117   -.2731893
             |
ddlnpind_euro|
         L1. |   .6493031   1.349298     0.48   0.630    -1.995272    3.293878
         L2. |    .408793    1.34198     0.30   0.761    -2.221439    3.039025
             |
       _cons |    -.01099   .0486412    -0.23   0.821    -.1063249     .084345
-------------+----------------------------------------------------------------
ddlnpind_euro|
     ue_euro |
         L1. |  -.0010136   .0041293    -0.25   0.806    -.0091069    .0070796
         L2. |   .0010607   .0041774     0.25   0.800    -.0071267    .0092482
             |
ddlnpind_euro|
         L1. |  -.4627587   .0756853    -6.11   0.000    -.6110993   -.3144182
         L2. |  -.3473256   .0752749    -4.61   0.000    -.4948616   -.1997895
             |
       _cons |  -.0004241   .0027284    -0.16   0.876    -.0057716    .0049235
------------------------------------------------------------------------------
```

```
. estimates table var_1 var_2 var_3  var_4, b(%7.4f) star(0.1 0.05 0.01) stat(N r2 r2_a aic bic)
```

Variable	var_1	var_2	var_3	var_4
ue_euro				
ue_euro				
L1.	1.0087***	1.4219***	1.2127***	1.1426***
L2.		-0.4192***	0.2950**	0.3318***
L3.			-0.5110***	-0.3170***
L4.				-0.1619**
ddlnpind_e~o				
L1.	0.3618	0.6493	0.8141	0.4385
L2.		0.4088	0.7331	0.1887
L3.			-0.4233	-1.0723
L4.				-0.9964
_cons	-0.0550	-0.0110	0.0377	0.0480
ddlnpind_e~o				
ue_euro				
L1.	0.0000	-0.0010	-0.0012	-0.0012
L2.		0.0011	0.0023	0.0030
L3.			-0.0011	-0.0024
L4.				0.0007
ddlnpind_e~o				
L1.	-0.3364***	-0.4628***	-0.5283***	-0.5899***
L2.		-0.3473***	-0.4357***	-0.5844***
L3.			-0.1739**	-0.3552***
L4.				-0.3410***
_cons	-0.0002	-0.0004	-0.0005	-0.0007

Granger - causality

- Do lagged values of X predict Y, or the other way round?

- H0: the coefficient on all the lagged values of X in the Y regression are jointly insignificant (and vice versa).

- Granger-causality $\neq$ causality.

- Does it make sense to assume the future does not affect the past?

Granger causality testing

```
. vargranger
```

Granger causality Wald tests

Equation	Excluded	chi2	df	Prob > chi2
ue_euro	ddlnpind_euro	1.6736	4	0.796
ue_euro	ALL	1.6736	4	0.796
ddlnpind_euro	ue_euro	.28146	4	0.991
ddlnpind_euro	ALL	.28146	4	0.991