Traffic Assignment Modeling

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Course Outline

• Forecasting overview and data management
• Trip generation modeling
• Trip distribution modeling
• Mode split modeling
• Traffic assignment modeling
• Network theory
• Activity-based modeling
• Modeling practices
Outline

• Time-of-day and directional volume factoring
• Traffic assignment overview
• Modeling strategy and components
• Traffic assignment models
From activities to traffic

Daily activities → Trips → Traffic
Hourly Travel Time Variation

Percent of Daily Trips

Start Time of Trips

Overnight
Morning
Midday
Afternoon
Evening
Factoring

• Time-of-day networks require that the link capacity be adjusted to reflect the capacity appropriate to the time period, for example, the peak 60 minutes, the peak 90 minutes, or the peak 120 minutes, so that the volume delay functions used in the equilibrium assignment algorithm will function correctly.

• Highway and transit network link data (capacity and speed) must be coded appropriately for the time-specific assignments.
Factoring

• Before Traffic Assignment
  • Peak-period factors are applied to the mode-specific trip tables
  • Production and attraction formatted trip tables are factored
  • Directional split factors convert P and A trip tables to O and D trip tables

• After Traffic Assignment (post-assignment)
  • Daily link volumes are multiplied by the peak period factor
  • Directional split percentages derived from observed traffic are applied to the link-level peak volumes
  • Factors can vary by area type (CBD, urban, suburban, rural) and functional classification (freeway, arterial, collector)
Time of Day Factoring before Assignment

1. Trip Generation
2. Trip Distribution
3. Mode Choice
4. Daily Trip Tables (by trip purpose & by mode)
5. Time-of-Day Factors (by area type and by facility type)
6. Time-of-Day Modeling
7. Directional Split Factors (by area type and by facility type)
   - PM Peak Period (or PM Peak Hour Trip Tables)
   - AM Peak Period (or AM Peak hour Trip Tables)
   - Off-Peak Period (or Off-Peak Hour Trip Tables)
8. Trip Assignments (AM, PM, Off-Peak)

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Time of Day Factoring after Assignment

1. Trip Generation
2. Trip Distribution
3. Mode Choice
4. Daily Trip Assignment
5. Time-of-Day Factors (by area type and by facility type)
6. Time-of-Day Modeling
7. Directional Split Factors (by area type and by facility type)
8. Link-level, Peak-hour, and Peak-period Traffic Demands
Inputs and Outputs

Before traffic assignment
• I: Time-of-day factors by trip purpose and by mode
• I: Directional split factors by trip purpose and by mode
• O: Time period trip tables by purpose and by mode in O & D format
• Data: from travel surveys (trip type and direction)

After traffic assignment
• I: Time-of-day factors by area type and by facility type
• I: Directional split factors by area type and by facility type
• O: Link-level, time period traffic volumes by direction
• Data: from traffic counts (per 15 minutes)
# Example - Time of Day and Directional Factors

<table>
<thead>
<tr>
<th>Type</th>
<th>Percent of Trips</th>
<th></th>
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<th></th>
<th></th>
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<tr>
<td></td>
<td>6:30 to 8:30</td>
<td>8:30 to 3:30</td>
<td>3:30 to 6:30</td>
<td>6:30 to 8:30</td>
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<td>15.26</td>
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<td>HB School</td>
<td>45.20</td>
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<td>18.56</td>
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<tr>
<td>HB Shop</td>
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<td>3:30 to 6:30</td>
<td>6:30 to 8:30</td>
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<td>0.450</td>
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</table>
Four-Step Travel Modeling
Overall (Matrix Manipulation) Process

TRIP GENERATION *(TRIP FREQUENCY CHOICE)*
- Total Person Trip Ends
- Total Truck Trip Ends

↓

TRIP DISTRIBUTION *(DESTINATION CHOICE)*
- Total Person Trip Tables
- Total Truck Trip Tables

↓

MODE CHOICE
- Mode Shares
- Mode Trip Tables

↓

TRAVEL ASSIGNMENT *(PATH CHOICE)*
- Trips on Link and Lines
- Times and Speeds
Overall (Matrix Manipulation) Process

Trip Generation

Trip Productions ($P_i$)

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<thead>
<tr>
<th>Zone</th>
<th>1</th>
<th>2</th>
<th>3</th>
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</thead>
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<td>3</td>
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Trip Attractions ($A_j$)

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</thead>
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<td></td>
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Trip Distribution

$T_{ij}$

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<th>To Zone 1</th>
<th>To Zone 2</th>
<th>To Zone 3</th>
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<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>18</td>
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<td>32</td>
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<td>3</td>
<td>5</td>
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Mode Choice

$T_{ijm}$

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<tr>
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<tbody>
<tr>
<td>Mode I</td>
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<td></td>
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<tr>
<td>Mode II</td>
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</table>

Trip Assignment

$T_{ijmr}$

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<tr>
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<tbody>
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<td>Route a</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Route b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Route c</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Traffic Assignment Purpose

- Testing of alternatives
- Establishment of short range priority programs for traffic flow improvements
- Analysis of the location of transport facilities within a corridor
- Providing input to other planning tools (e.g., air quality modeling)
- Detailed study of the effects of a traffic generator (such as a new stadium) on traffic flows

- Network vs. System Equilibrium
Network vs. System Equilibrium

- Network equilibrium – when travelers can no longer find better routes to their destinations
- System equilibrium – when there are no more (significant) changes in time, destination, and mode choice of trips (impedances)

- Computation time
Traffic Assignment Objectives

- Obtain good aggregate network measures (e.g., total road flows, total revenue by the bus service)
- Estimate zone-to-zone travel costs (times) for a given level of demand
- Obtain reasonable link flows and identify heavily congested links
- Estimate the routes used between each O-D pair
- Analyze which O-D pair use a particular link or route
- Obtain turning movements at intersections
Traffic Assignment Input/Output

Input

- Network geometry
- Network parameters for each link
- O & D matrix or trips to be “loaded”
- Assignment rule or hypothesis (such as “all trips take the min. time path”)

Output

- “Traffic Flows” or travel volumes on each segment of the transportation network

<table>
<thead>
<tr>
<th>From Node</th>
<th>To Node</th>
<th>Dist. (mi)</th>
<th>1st Time (min)</th>
<th>1st Speed (mph)</th>
<th>Capacity (vph)</th>
<th>Vol.</th>
<th>New Time</th>
<th>New mph</th>
<th>v/c</th>
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<tr>
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</table>
Traffic Assignment Method Steps

1. Identify a set of routes considered attractive to travelers (skim tree structure)
2. Assign suitable proportions of the trip matrix to these routes (flows on the links)
3. Search for convergence
   • Proximity to the solution
   • Computation time / # of iterations

• Skim Tree – a list of minimum paths between an origin node and all the possible destination nodes
• Minimum Path Algorithm – a logical set of rules used to find the shortest part through a complex network
Traffic Assignment Network

Generally, it is the center of trip activity rather than the geographic center.
Sample Road Network

Nodes 1-99 are Zones
Nodes 100-199 are external stations
Nodes 200-1,000 are node numbers

Rail Line
Centroid Connectors
Roadway Link

Link Length, No. of Lanes, Volumes, Capacities, Turning Penalties, Parking, Transit Stations, Routes, Schedules, Capacitates, etc.
Example of Coded Network

<table>
<thead>
<tr>
<th>From Node</th>
<th>To Node</th>
<th>Length</th>
<th>Link Type</th>
<th>No. of Lanes</th>
<th>Volume/Delay Function</th>
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</table>
Macroscopic Traffic Flow

![Graph showing the relationship between speed and density in traffic flow.](image)

- **Speed $S$**
  - $s_f$ (free flow speed)
  - $v_f$ (maximum speed)
  - $s_{max}$ (maximum stable speed)
  - $v_{max}$ (maximum unstable speed)
- **Density $k$**
  - $k_j$ (critical density)
- **Flow $V = k_s$**

- **Stable regime equation**
- **Unstable regime equation**

**Graphical Representation:**
- The graph illustrates the relationship between speed and density, showing the transition from stable to unstable flow regimes.
- Key points include:
  - Free flow speed ($s_f$)
  - Maximum speed ($v_f$)
  - Maximum stable speed ($s_{max}$)
  - Maximum unstable speed ($v_{max}$)
- The graph highlights the critical density ($k_j$) and the transition point from stable to unstable flow.
Network Flows

Macro vs. Micro
Link Parameters

- Travel Time/Free Flow Ratio
- Volume/Capacity Ratio
- Capacity
- Route travel time
- Traffic flow

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Link Parameters

- Bureau of Public Roads Method
  \[ T_N = T_o \left[ 1 + 0.15 \left( \frac{V}{C} \right)^4 \right] \]

- Davidson’s Method
  \[ T_N = T_o \left[ 1 + J \frac{V}{C - V} \right] \]

\( T_N \) – new travel time
\( T_o \) – free flow travel time
\( V \) – link volume
\( C \) – link capacity
\( J \) – calibration parameter
CATS LINK PERFORMANCE CURVES

\[ S = t_o \left( \frac{2^{\frac{V}{C}} + 1}{2} \right) \]
MODIFIED BPR LINK PERFORMANCE FUNCTION

\[ S = t_o \left[ 1 + 0.15 \left( \frac{V}{C} \right)^4 \right] \quad \text{if } (V / C) \leq 1.0 \]

\[ S = t_o \left[ 1 + 1.0 \left( \frac{V}{C} \right)^4 \right] \quad \text{if } (V / C) \geq 1.0 \]
Shortest Routes in the Network

 Routes and Times
A → B; T = 5
A → C; T = 3
...

function Dijkstra(Graph, source):
  for each vertex v in Graph:
    dist[v] := infinity
    previous[v] := undefined
  dist[source] := 0
  Q := the set of all nodes in Graph
  while Q is not empty:
    u := node in Q with smallest dist[ ]
    remove u from Q
    for each neighbor v of u:
      alt := dist[u] + dist_between(u, v)
      if alt < dist[v] // Relax (u,v)
        dist[v] := alt
        previous[v] := u
  return previous[ ]
Route Choice

• Rational traveler – min perceived (anticipated) costs
• Factors
  • Travel time
  • Distance
  • Monetary cost
  • Congestion and queues
  • Type of maneuvers required
  • Road type
  • Scenery
  • Road works
  • Reliability of travel time
  • Habit
In Reality

- Differences in individual perceptions
- The level of knowledge of alternative routes
- Congestion effects affecting shorter routes first

- Weighting?
- Multiple user classes
- Introducing “stochasticity”
- Congested assignment and equilibrium
Braess’ Paradox

- Demonstrates that, under certain conditions, adding capacity to a road network when driver seek to minimize their own costs can actually make everyone worse off.
- Travel time-flow formulation in minutes.
- There are 1000 cars that want to travel from A to B, and none from F.
- Logical choice is 500 cars take ADB, and 500 cars take ACB.

ACB: $2 + 10 + 25 + 1 = 38$ min
ADB: $25 + 1 + 10 + 2 = 38$ min
Braess’ Paradox

• Green link is a new (planned) link
• At A, it is rational to choose AC
  ACD: 2 + 10 + 1 + 1 = 24 min
  AD: 25 + X min

• At C, it is rational to choose CD
  CDB: 1 + 1 + 2 + 10 = 24 min
  CB: 25 + X min

Everyone ACDB: 2 + 20 + 1 + 1 + 2 + 20 = 46 min > 38 min

• What if there are 1000 vehicles traveling from F to D?
All-or-nothing Assignment

1. Compute Skim Trees via Moore’s Maximum Path Algorithm

2. Develop a trip interchange matrix from the T/D process or from collected O & D data

3. Develop a “link-interchange” table

4. Sum link volumes for each interchange utilizing that link to estimate total traffic volume
Moore’s Minimum Route Algorithm

• Comparing to Dijkstra’s algorithm, Moore’s algorithm selects the top (the oldest) entry in the loose-end table (containing nodes already reached by the algorithm, but not fully explored as predecessors for further nodes)

• On the contrary, Dijkstra’s algorithm selects node nearest to the origin

• To find the minimum time path between any two nodes, the links are followed backward from the destination node to the origin node.
Moore’s Minimum Route Algorithm

• This algorithm does not require all possible routes between an origin and destination to be individually investigated to find the shortest route.

• Rather, a minimum “tree” is developed by fanning out from the origin to all other accessible nodes in increasing order of their impedance summation from the origin.
Moore’s Algorithm – Step 1

- Determine the time to the nodes connected to node 1.
- The time to node 2 is 1 and to node 5 the time is 2.
Moore’s Algorithm – Step 2

- From the node closest to the home Node 1, which is node 2, the connections are to nodes 3 and 6 (backtracking is not permitted).
- The corresponding times are the time to node 2 plus the outbound link times from node 2 which are 4 and 5 minutes, respectively.
Moore’s Algorithm – Step 3

• The node closest to the home node is node 5.
• Proceeding from node 5 to node 6 the time is the time to node 5 plus the link time on 5-6, or a total of 4 minutes.
• Likewise the time to node 9 is 4 minutes (time to node 5 plus link time is 5-9).
Moore’s Algorithm – Step 4

- The time to the node 6 via link 5-6 is less than the previous route via link 2-6.
- Therefore, the link 2-6 is deleted from the tree.
Moore’s Algorithm – Step 5

- The node now closest to the home node is node 3 which is 4 minutes away.
- Adding the corresponding link time to nodes 4 and 7 the corresponding times are 10 and 6 minutes, respectively.
- By convention, the node with the lowest number of those equidistant from the home node is taken first.
Moore’s Algorithm – Step 6

- The node closest to the home node is now node 6. The times to nodes 7 and 10 are 6 and 7 minutes respectively.
- Since node 7 was reached previously in 6 minutes, there is no time savings via the route entering on link 6-7.
- For this reason the connection is shown in dashed lines and will not be part of the tree as can be seen in step 7.
Moore’s Algorithm – Step 7

• Building proceeds from node 9 to nodes 10 and 13, with respective times from the home node of 7 and 6 minutes.
• The dashed line indicates that link 9-10 will not be in the final tree for the same reason mentioned in step 6 above.
Moore's Algorithm – Step 8 and 9

- Building proceeds from node 7 to nodes 8 and 11
- Building proceeds from node 13 to Node 14.
Moore’s Algorithm – Step 10

- Building proceeds from node 10.
- The time node 11 is longer than a previous entry so that link 10-11 will not be in the tree.
- The time to node 14 is the same as a previous entry so that link 10-14 will not be in the tree.
Moore’s Algorithm – Step 11 and 12

- Tree building proceeds from node 11 to node 12 and node 15
- Tree building proceeds from node 8. Note that the time via node 8 to nodes 4 and 12 are less than previous.
Moore’s Algorithm – Step 13 and 14

- Link 3-4 and 11-12 are removed from the tree due to finding in step 12
- Building proceeds from node 15 to node 14 and to node 16.
Moore’s Algorithm – Step 15 and 16

- Building proceeds from node 12 to node 11 and to node 16
- The final tree as built by the process

To go from node 1 to node 4, the route would be found by first looking at node 4 and proceeding to back nodes 8, 7, 3, 2, and 1
All-or-nothing Assignment

T_{ij} matrix

\begin{array}{cc}
C & D \\
A & 400 & 200 \\
B & 300 & 100 \\
\end{array}
All-or-nothing Assignment

$T_{ij}$ matrix

\[
\begin{array}{cc}
  & C & D \\
 A & 400 & 200 \\
 B & 300 & 100 \\
\end{array}
\]
All-or-nothing Assignment

- Resulting traffic assignment
All-or-nothing Assignment

- The method assumes there are no congestion effects (link costs are fixed regardless of traffic volume)
- The method assumes that all drivers consider the same attributes for route choice and that they perceive them in the same way
- These assumptions might work in sparse or uncongested networks
- Useful to represent a sort of “desire line” effect, i.e., what would drivers do in the absence of congestion
- Building block for other assignment techniques, such as equilibrium or stochastic methods
Advanced Traffic Assignment Techniques

- Iterative Assignment
- Incremental Assignment
- User Equilibrium Assignment
- System Optimum Assignment
- Stochastic User Equilibrium Assignment
Example Network

Total volume from 1 to 2 = 8000

Link performance functions

\[ S_A = 15[1 + 0.15(V_A/1000)^4] \]
\[ S_B = 20[1 + 0.15(V_B/1000)^4] \]
\[ S_C = 21[1 + 0.15(V_C/1000)^4] \]
Incremental Assignment

1. Each origin-destination flow is divided into \( n \) equal parts, typically four.
2. Assign each part using all-or-nothing assignment, updating the link-loading and travel times following the assignment of each increment.
3. After the assignment of the \( n^{th} \) part, the link loadings are summed to determine the final loading.

- Easy to program
- Can be interpreted as the build-up of congestion during the peak period
# Incremental Assignment

<table>
<thead>
<tr>
<th>Increment</th>
<th>Step</th>
<th>Link A</th>
<th>Link B</th>
<th>Link C</th>
<th>Equilibrium Objective Function</th>
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<td>4000</td>
<td>29.5</td>
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</table>

177 967
Iterative Assignment

1. Compute the travel time on each link $S_a(v_a)$ corresponding to the flow $v_a$ in the current solution;

2. Compute a weighted mean travel time $\left(S_a''\right)$, which consists of the current travel time $[S_a(v_a)]$ and the travel time $S_a'$ from the previous iteration:

   $$S_a'' = 0.75 S_a' + 0.25 S_a(v_a)$$

   $\phi = 0.25$ or generalized $[0,1]$

3. Trace minimum path trees from each origin to all destinations by using the weighted travel times $\left(S_a''\right)$ from step 2;

4. Assign all trips from each origin to each destination to the minimum path (all or nothing assignment); call this link loading; $V_a'$ and

5. Return to step 1 and replace $v_a$ with $V_a'$.

6. After 4 iterations, compute the mean of the four all-or-nothing assignments.
## Iterative Assignment

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Step</th>
<th>Link A</th>
<th>Link B</th>
<th>Link C</th>
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Method of Successive Averages (MSA)

• Difficulty in achieving convergence when the output flow variables are used directly as input
• MSA uses the averages of the link flow variables from all the previous solutions so that the output of the next solution produces convergent variables
• In each iteration, each of the previous solutions is weighted equally
• The first iteration is the standard run of trip distribution, mode choice, and traffic assignment
• The second iteration starts with the travel costs of the first solution, and then is equally averaged with the first solution
• The third solution, is weighted 1/3 and the previous solution is weighted 2/3
Method of Successive Averages (MSA)

- Nth solution is weighted \((n-1)/n\)
- Standard loop, with initial equilibrium assignment
- MSA loop averages variables from the standard loop with fixed weights
- MSA loop is run several times until convergence
### Method of Successive Averages (MSA)

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<th>( \phi )</th>
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<th>Cost town</th>
<th>Flow bypass</th>
<th>Cost bypass</th>
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<td>1400</td>
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</table>

**Cost computations:**

\[ t^1_b = 15 + 0.005(0) = 15 \]

\[ t^1_a = 10 + 0.02(2000) = 50 \]

\[ t^2_b = 15 + 0.005(1000) = 20 \]

\[ t^2_a = 10 + 0.02(1000) = 30 \]

\[ t^3_b = 15 + 0.005(1333) = 21.67 \]

\[ t^3_a = 10 + 0.02(667) = 23.34 \]

etc.
Comparison of Modeled and Observed Speeds for MSA

![Graph showing comparison of modeled and observed speeds for MSA loops.]
System vs. User Optimum

• System optimal → min system total travel time (for all vehicles)
• User equilibrium → min travel time for individual user and equal on all routes

Wardrop’s Principle of User Equilibrium (First principle)
• The journey times in all routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route
• Under equilibrium conditions, traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes
System vs. User Optimum

T(any used routes) \leq T(any unused routes)
System vs. User Optimum

Wardrop’s Second principle (System optimum)

- Under social equilibrium conditions, traffic could be arranged in congested networks in such a way that the average (or total) travel cost is minimized

- Idealistic situation
- Can control entry onto particular routes and control travelers to use routes that do not minimize their own individual travel time
- Difficult to ever realize in practice → there is always the temptation for individual travelers to change routes to improve their own travel time
- Reference point for comparison with user equilibrium
User Equilibrium Assumptions

- Each motorist chooses the minimum travel-time route
- Motorists have full information of the travel time on every possible route
- Motorists consistently make the correct decisions regarding route choice
- All individuals are identical in their behavior
- $X_1$ and $X_2$ represent the traffic flow on two links below

![Diagram of two links with travel time curves and volume equation $q = x_1 + x_2$.]
User Equilibrium Example

$t(q)$ is used to find the equilibrium flows $x_1$ and $x_2$ and the O-D travel time $t$.
User Equilibrium Example

If $q < q'$

- Motorists will choose link 1 only because it provides lower travel time
- No one will use link 2
- Motorists do not have an incentive to switch routes to the longer link (link 2)
- Equilibrium condition exists
User Equilibrium Example

If \( q \geq q' \)

- Both links are used
- Some motorists can change route and lower their own travel time
- The route-switching process will not occur, only if the travel time on both routes is equal
- Then, motorists have no incentive to switch
- Equilibrium condition exists
User vs. System Equilibrium Example

\[ t_1 = 8 + x_1 \]
\[ t_2 = 1 + 2x_2 \]

- \( x \) – vehicle/h in thousands
- \( t \) – time in minutes

- Route travel times?
- Route flows?
- Total travel times?
System Optimum (SO) Approach

\[ S(x) = x_1(8 + x_1) + x_2(1 + 2x_2) \quad x_2 = q - x_1 \]

\[ S(x) = x_1(8 + x_1) + (q - x_1)[1 + 2(q - x_1)] \]

\[ \frac{d}{dx_1} S(x) = 0 \rightarrow 6x_1 - 9 = 0 \quad to\ find\ the\ system\ minimum \]

Route flows:

\[ x_1 = 1.5 \quad x_1 = 1.5 \cdot 1000 = 1500 \text{ veh/h} \]

\[ x_2 = 4000 - 1500 = 2500 \text{ veh/h} \]
SO Approach

Route travel times:

\[ t_1 = 8 + x_1 = 9.5 \text{ min} \]
\[ t_2 = 1 + 2x_2 = 6 \text{ min} \]

Total travel time:

\[ x_1 t_1 + x_2 t_2 = 29,250 \text{ veh} - \text{min} \]
User Equilibrium (UE) Approach

\[ t_1 = 8 + x_1 \quad t_2 = 1 + 2x_2 \quad q = 4 = x_1 + x_2 \]

\[ t_1 = t_2 \rightarrow \text{User equilibrium} \]

\[ 8 + x_1 = 1 + 2(q - x_1) \]

Route flows:

\[ x_1 = 0.333 \rightarrow x_1 = 0.333 \cdot 1000 = 333 \text{ veh/h} \]

\[ x_2 = 4000 - 333 = 3667 \text{ veh/h} \]
Route travel times:

\[ t_1 = 8 + x_1 = 8.33 \text{ min} \]

\[ t_2 = 1 + 2x_2 = 1 + 2 \cdot 3.667 = 8.33 \text{ min} \]

Total travel time:

\[ x_1 t_1 + x_2 t_2 = 33,333 \text{ veh} - \text{ min} \]
User Equilibrium Example 2

Rt. A Capacity = 1,000
Rt. B Capacity = 2,000
Free Flow time on Rt. A = 15 minutes
Free Flow time on Rt. B = 20 minutes

\[ T_A = 15 \left(1 + 0.15 \left( \frac{V_A}{1000} \right)^4 \right) \]
\[ T_B = 20 \left(1 + 0.15 \left( \frac{V_B}{3000} \right)^4 \right) \]
Equilibrium Volumes on Rt. A and B

Flow (000s) vs. (Min.)

Point of Equilibrium

63.3 min

5847

2153

Curve B

Curve A
Equilibrium Volumes on Rt. A and B

![Graph showing equilibrium volumes on Rt. A and B with a peak at 63.3 minutes.]

Aalto University
School of Engineering

10.5.2017
77
## Equilibrium Volumes on Rt. A and B

<table>
<thead>
<tr>
<th>Volume</th>
<th>T&lt;sub&gt;A&lt;/sub&gt;</th>
<th>T&lt;sub&gt;B&lt;/sub&gt;</th>
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User Equilibrium Formulation

\[
\min \sum_a \int_0^{V_a} S_a(x) \, dx
\]

subject to

\[
V_a = \sum_i \sum_j \sum_r \delta_{ij}^{ar} X_{ij}^r
\]

\[
\sum_r X_{ij}^r = T_{ij}
\]

\[
X_{ij}^r \geq 0
\]

Where:

- \( V_a \) = number of vehicles per unit time on link (a) of the network;
- \( S_a(v) \) = generalized travel time on link a, which increases with flow \( v \) (a typical congestion function is

\[
t_a(v) = t_{a0} + c_a v
\]

where \( t_a \) is the travel time with free flow, and \( c_a \) is a measure of the capacity per unit of link (a);

- \( X_{ij}^r \) = number of vehicles from i to j on path r; and
- \( \delta_{ij}^{ar} \) = 1, if link a belongs to path r from i to j,

\[
\delta_{ij}^{ar} = 0, \text{ otherwise}
\]

- \( T_{ij} \) = given trip matrix.
User Equilibrium Formulation

1. Compute the travel time on each link $S_a(v_a)$ that corresponds to the flow $v_a$ in the current solution;
2. Trace minimum path trees from each origin to all destinations by using the travel times from step 1;
3. Assign all trips from each origin to each destination to the minimum path (all-or-nothing assignment); call this link loading ($w_a$);
4. Combine the current solution ($v_a$) and the new assignment ($w_a$) to obtain a new current solution ($v'_a$) by using a value $\lambda$ selected through a one-dimensional search, so as to minimize the following objective function:
   \[ \sum_{a} \int_{0}^{v_a} S_a(x) \, dx \]
   where \[ v'_a = (1 - \lambda) v_a + \lambda w_a \; and \]
5. If the solution has converged sufficiently, stop; otherwise return to step 1.
## User Equilibrium Solution

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Step</th>
<th>Link A</th>
<th></th>
<th>Link B</th>
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*Note: The table shows the flow and time for each iteration and step, along with the equilibrium objective function and λ values.*

---

*Image*
## Comparison of Solutions

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Stochastic User Equilibrium

• Stochastic user equilibrium is reached when no traveler believes that his or her travel time can be improved by unilaterally changing routes
• It includes a random component in traveler’s perception of travel time
• It relaxes the presumptions used in the UE deterministic approach to traffic assignment
Stochastic User Equilibrium Formulation

Discrete choice models (logit-based)

\[ P_{rs}^k = \frac{e^{-\theta c_{rs}^k}}{\sum_{\ell} e^{-\theta c_{\ell}^{rs}}} \quad \forall \ k, r, s \]

\[ c_{rs}^k = \text{The measured travel time between } r \text{ and } s \]

\[ \theta = \text{A positive parameter} \]

\[ \theta c_{rs}^k = \text{The utility of using the } k^{\text{th}} \text{ route between } r \text{ and } s \]
Dial’s Algorithm (STOCH)

• STOCH effectively implements a logit route-choice model at the network level

• Identifies the set of “efficient” paths connecting each O-D pair
• Assigns flows to these paths using logit formula with parameter $\theta$
• “Inefficient” paths would not be considered in the assignment
Dial’s Algorithm (STOCH)

- A path is defined “efficient” if it includes only links that take the motorist further away from the origin and closer to the destination.
- Such links can be identified by associating two labels with each node $i$: $r(i)$ and $s(i)$.
- $r(i)$ denotes the travel time from origin node $r$ to node $i$ along the minimum travel time path.
- $s(i)$ denotes the travel time from node $i$ to destination node $s$ along the minimum path.
- Each “efficient” path includes only links $i \rightarrow j$ such that $r(i) < r(j)$ and $s(i) > s(j)$. 
STOCH Preliminaries

a) Compute the minimum travel time from node $r$ to all other nodes. Determine $r(i)$ for each node $i$

b) Compute the minimum travel time from node $i$ to node $s$. Determine $s(i)$ for each node $i$

c) Define $\theta_i$ as the set of downstream nodes of all links leaving node $i$

d) Define $\tau_i$ as the set of upstream nodes of all links arriving at node $i$

e) For each link $i \rightarrow j$ compute the “link likelihood”, $L(i \rightarrow j)$, where

$$L(i \rightarrow j) = \begin{cases} e^{\theta [r(j) - r(i) - t(i \rightarrow j)]} & \text{if } r(i) < r(j) \text{ and } s(i) > s(j) \\ 0 & \text{Otherwise} \end{cases}$$
STOCH Example

- Assign a flow of 1000 trips between nodes 1 and 9
STOCH Example

- Not all the paths connecting node 1 to 9 are considered efficient

Path $1 \rightarrow 2 \rightarrow 5 \rightarrow 6 \rightarrow 9$ is efficient
Path $1 \rightarrow 4 \rightarrow 7 \rightarrow 8 \rightarrow 9$ is not $[s(4) = s(7)]$
Path $1 \rightarrow 2 \rightarrow 3 \rightarrow 6 \rightarrow 9$ is not $[r(3) = r(6)]$
Link Likelihood Values

- The link likelihoods for all the links (with \( \theta = 1 \)) are shown in the figure below (note that \( L(i \rightarrow j) = 0 \) for all “inefficient” links)
Forward Pass

- Consider nodes in increasing order of $r(i)$, starting with origin node $r$
- For each node $i$, calculated the “link weight” $w(i \rightarrow j)$, for each $j \in \theta_i$ (i.e., for each link emanating from $i$), where

$$w(i \rightarrow j) = \begin{cases} L(i \rightarrow j) & \text{if } i=r \quad \text{(i.e. if node } i \text{ is the origin)} \\ L(i \rightarrow j) \sum_{m \in \tau_i} w(m \rightarrow i) & \text{otherwise} \end{cases}$$

- When the destination node, $s$, is reached this step is complete
Link Weights

\[ W(5 \rightarrow 8) = L(5 \rightarrow 8) \sum_{m \in \tau_5} W(m \rightarrow 5) \]

- Where \( \tau_5 \) includes nodes 2 and 4 (since links 2 \( \rightarrow \) 5 and 4 \( \rightarrow \) 5 enter node 5)

\[ W(5 \rightarrow 8) = 1.0000(0.3679 + 1.0000) = 1.3679 \]
Backward Pass

- Consider nodes in ascending order of $s(i)$, starting with destination $s$.
- When each node $j$ is considered, compute the link flow $x(i \to j)$ for each $i \in \tau_i$ (i.e., for each link entering $j$) as follows:

$$X(i \to j) = \begin{cases} q_{rs} \frac{W(i \to j)}{\sum_{m \in \tau_j} W(m \to j)} & \text{for } j = s \text{ (i.e., if node } j \text{ is the destination)} \\ \sum_{m \in \theta_j} (x_j \to m) \frac{W(i \to j)}{\sum_{m \in \tau_j} W(m \to j)} & \end{cases}$$
Link Flows

\[
x(2 \rightarrow 5) = \frac{[x(5 \rightarrow 6) + (5 \rightarrow 8)]}{W(2 \rightarrow 5) + (4 \rightarrow 5)} \frac{W(2 \rightarrow 5)}{W(2 \rightarrow 5) + (4 \rightarrow 5)}
\]

\[= (731 + 269) \frac{0.3679}{0.3679 + 1.0000} = 269\]
Public Transport Assignment

- Vehicles have varying numbers of people
- People move on multiple vehicles
- Multiple route choices - people realize when there is a set of paths they can use
- Level of service and travel times for both people and vehicles depend on demand
- When there are multiple route paths and frequencies or service, a passenger decision strategy must be modelled instead
- Difference in capacity and flow interaction
- Generalized time and monetary costs

\[ C_{ij} = \alpha_1 t_{ij}^{\text{vehicle}} + \alpha_2 t_{ij}^{\text{walking}} + \alpha_3 t_{ij}^{\text{waiting}} + \alpha_4 t_{ij}^{\text{interchange}} + \alpha_5 \delta_{ij}^{\text{interchange}} + \alpha_5 \text{Fare}_{ij} \]
PT Link Performance Curve

- Time
- Travel time
- Curve likely to be used in model
- Policy headway
- Right of way C
- Right of way A or B
- Passenger volume [pass/hr]
Public Transport Assignment

Origin i

Line 1
20 mins
Line 2
14 mins
Line 3
Line 4
4 mins
Line 5
5 mins
14 mins

Destination j

Node A

F

H

J

Aalto University
School of Engineering
Validation and Calibration

- Model calibration is the process of developing mathematical functions to explain travel behavior and the process of developing the associated coefficients or parameters for these functions.
- The exponent in the BPR formula is an example of a parameter value, predicting changes in travel time as a function of link’s volume to capacity ratio.
- Calibration is performed individually for each of the four steps in the modeling process and for the entire model.
Validation and Calibration

- Model validation is the process of comparing model output with observed data and making judgments about the performance of the model.
- Includes reasonableness and sensitivity check.
- Reasonableness checks include the comparisons of the model output with observed data and the comparison of model parameters with parameters for similar urban areas.
- Sensitivity checks determine how the model output changes in response to changes in input data.
Validation and Calibration

- Model applied to complete model chain
- Base year model compared to observed travel (network traffic volume counts)
- Judgment as to model suitability
- Validation of a previously calibrated model
Trip Assignment Validation

- Comparison of observed and counted traffic
  - Screen line volumes (agreement within about 5%)
  - Corridor volumes
  - Root-mean-square error (RMSE) by volume group
  - RMSE by facility type group (typically <30%)
- Average weekday school year volumes
- Traffic count and transit ridership day-to-day variation

\[
\left( \frac{\sum (\text{Modeled Volume} - \text{Counted Volume})^2}{\text{Number Counts}^{0.5}} \right) \times 100 \left( \frac{\sum \text{Counted Volumes}}{\text{Number of Counts}} \right)
\]
Rules of Thumb – Examples

- Motorists often travel faster than speed limits under free flow conditions
- Household trips average 9 to 10 per day and 3 to 4 per capita
- Intrazonal trips account for less than 5% of total trips
- A 10% transit fare increase will decrease ridership by 3%
- Allowable VMT difference should not exceed 5%
- Difference in observed and modeled travel across screen lines should be 5% to 10%
Limitations of Conventional Models

- Limitations in the node-ling model
  - Incomplete networks
  - Turning movements
  - Intra-zonal trips
- Errors in defining average perceived costs
- Perception of cost by trip makers
- The assumption of perfect information about cost in all parts of network
- Dynamic nature of traffic