Chapter 14
Design with strut-and-tie models

14.1 Introduction

14.1.1 Plastic analysis

The analysis of shear in reinforced concrete members is based on the so-called truss analogy (see chapter 7 in these course notes). The truss model helps to understand how forces are transferred throughout a structural member. The equivalent truss model is identified on the basis of the crack pattern that appears when the member is loaded; the crack pattern helps to describe load transfer in the member by means of forces that are concentrated in compressive struts and tensile loaded bars.

The truss analogy proposed by RITTER-MÖRSCH (1899) for the calculation of shear reinforcement in reinforced concrete members is the oldest and well-known example of reasoning by means of “strut-and-tie models” (NL: staafwerkmodellen; FR: modèles composés de bielles et tirants). However, the method is also very well suited for the design calculation of compact structural members such as foundation slabs (or blocs) supported by piles, corbels, deep beams, walls and anchorage regions in prestressed members. The strut-and-tie method is essentially founded on the publications (years 1980-90) of SCHLAICH (Univ. Stuttgart).

Designing members by means of the strut-and-tie method belongs to the domain of the application of the plastic design philosophy, and more in particular to the domain of (lower bound) static methods. Indeed, the design is based on a truss model in which acceptable stresses are not exceeded in the members of the truss, and is thus based on a statically acceptable system, which may transfer the loads with sufficient safety. If on top of that, the directions of the compression struts coincide with the orientations of the stresses in uncracked situation, there is no need for large redistributions in order to activate the truss system; consequently, deformations and crack widths remain limited which is a visual proof of the well functioning of the truss system in service conditions.

The philosophy described above is the one of the lower bound methods for plastic design in ULS. According to the lower bound theorem, the structural member does not fail if for the imposed load combination, a stress distribution can be identified for which:

- equilibrium is guaranteed (the system of forces is in equilibrium with a given set of loads), and
- the threshold for plastic behaviour of materials is not exceeded.

The present course notes are essentially based on the following references: LAMBOTTE (1989), WALRAVEN (1995), PROVOST (1998) and WIGHT (2009).

14.1.2 Application domain

Every structural member can be subdivided in so-called B-regions and D-regions.
$B$ stands for BERNOULLI and corresponds to those parts of the member where the assumption of BERNOULLI can be accepted (planar sections remain plane after load has applied). For example: the assumption is accepted for the analysis of long beams loaded in bending where the load transfer is continuous over long distances: cracks and compression struts between the cracks are parallel to each other over the whole region.

$D$ stands for “Discontinuity” and corresponds to the regions where the assumption of BERNOULLI is no longer valid; this is typically the region around:
- a “geometrical discontinuity”: changes in cross-section, openings, nodes in frames, connections between girders and beams, etc.;
- a “static discontinuity”: isolated loads, supports, temperature changes, anchorage of prestressing tendons, etc.

$D$-regions are characterized by the disturbance of the load transfer system, due to sudden changes in dimensions, by changes in orientation of load transfer due to the presence of openings, the connection with other structural members, the introduction of concentrated loads, etc.

Figure 14.1.2-1 shows the truss model in a beam; the figure shows the regular pattern of cracks and struts between the cracks in the $B$-region and the fan-like distribution of the cracks at the position of the concentrated loads (support reaction force included).

Figure 14.1.2-2 shows a frame structure with subdivision into $B$- and $D$-regions. In this figure, the principle of BARRE DE St VENANT is used, saying that the localized effect of a disturbance dies out by about one member-depth from the point of disturbance. On this basis, D-regions are assumed to extend one member-depth each way from the discontinuity.

Although the strut-and-tie method may also be applied in $B$-regions (see chapter 7 for shear design by means of the truss model of MÖRSCH), it is above all THE method to be used for the design of $D$-regions. Strut-and-tie models are helpful tools to design the necessary reinforcement which helps to arrange the transfer of loads through the $D$-regions towards the $B$-regions.

![Figure 14.1.2-1](image)

Truss model for a beam loaded in bending with $B$- and $D$-regions
Figure 14.1.2-2
Frame structure with indication of B- and D-regions. The conceptual principle of DE St VENANT is used as a quantitative guide in selecting the dimensions of D-regions: D-regions are assumed to extend one member-depth each way from the discontinuity

14.2 Principle aspects of the strut-and-tie method, explained by means of an example

14.2.1 Identification of the strut-and-tie model

The principle aspects of the strut-and-tie method are explained in this paragraph by means of the example of a deep beam which has to transfer the concentrated load $P$ of a column towards two foundation piles.

In accordance with the definitions presented in paragraph 2.5.2.1 in these course notes, the beam is characterized by a length $l < 3. \! h$, and is thus identified as a “deep beam” (NL: dragende wand; FR: poutre-cloison). The principal stress trajectories in the deep beam, in uncracked situation, are presented in figure 14.2.1-1(a); this permits to deduce a plausible truss model (figure 14.2.1-1(b)), composed of:

- two concrete compression struts with inclination angle $\theta$ with respect to the horizontal axis;
- a horizontal tie in the lower part of the deep beam.
Each compression strut has to transfer the load:

\[ P_c = \frac{P}{2 \cdot \sin \theta} \quad (14.2.1-1) \]

The tie has to transfer the tensile force:

\[ P_s = \frac{P}{2 \cdot \tan \theta} \quad (14.2.1-2) \]

It is important to limit the concrete compression stress in the nodes and the struts as well as to assure the anchorage of the ties in the nodes. The truss analysis has thus to consider the performance of the struts, the ties and also the nodes.
14.2.2 The ties

The tensile force \( P_x \) may be resisted by a bundle of parallel bars. It is of essential importance that the bars are efficiently anchored in the nodes; in such situation, the design yield strength of the steel \( f_{yd} \) may be used to design (in ULS) the reinforcement area \( A_r \) by the expression:

\[
A_r = \frac{P_x}{f_{yd}}
\]  

(14.2.2-1)

The anchorage of reinforcement bars is discussed in chapter 6 in these course notes, with topics such as anchorage plates (limited usefulness), anchorage length, reduction of anchorage length by means of hooks, loops and welded transverse bars; loops are very efficient because they create a favourable stress distribution in the concrete enclosed within the loop (multi-axial loading of the concrete in the node).

\[ \text{Note:} \]

The truss model in itself does not take care of crack development considerations; two strategies may be applied to limit crack development:

- the stresses in the steel reinforcement should be limited to values lower than \( f_{yd} \);
- the tie-bars should not be concentrated around the axis of the tie in the truss model, but should be distributed in the zone where, according to the elasticity theory, the largest tensile stresses appear in the concrete. It is preferred to arrange for more bars with smaller diameter, but lack of space may hamper this.

14.2.3 The compression struts

The identification of the axis of the concrete compression struts in a truss model is in general not a difficult problem. It is less evident to determine reasonable values for the dimensions of the cross-section of a strut, which in reality has a bottle-like shape (see figure 14.2.1-1(a)). A simplified solution is to consider an idealized prismatic strut. The rectangular cross-section of the strut is determined by the smallest value of \( b_1 \) and \( b_2 \) on the one hand (see figure 14.2.1-1(b)) and by the dimensions of the nodes on the other hand.

With the assumption (for reasons of simplicity) that the width of the struts equals \( b \), the cross-section of the compression strut is (see figure 14.2.3-1):

\[
b_a = \frac{b \cdot a_1 / 2}{\sin \theta}
\]  

(14.2.3-1)
The force $P_c$ in the strut defined by expression (14.2.1-1) is distributed over the cross-section, which leads to the compression stress in the strut:

$$\sigma_{c,\text{schoor}} = \frac{P_c}{b_a} = \frac{P}{2 \cdot \sin \theta} = \frac{P}{b \cdot a_i}$$

(14.2.3-2)

In this example, the stress in the compression strut is equal to the stress in the contact area between the deep beam and the column (the area used for the introduction of the load into the deep beam). Consequently, the node is subjected to a bi-axial stress situation where stresses are the same in all directions; this corresponds to a (planar) state of hydrostatic pressure (represented by 1 point in MOHR’s circle).

Figure 14.2.3-1
Detailed representation of the node where the load of the column is introduced into the deep beam. A hydrostatic stress state is observed

The compression stress in the strut is compared to a threshold value, which is taken as a fraction of $f_{cd}$ (analogous reasoning as in chapter 7 dealing with struts in the analogous truss system for the calculation of shear). EN 1992-1-1:2004; 6.5.2(2) recommends to limit the compression stress in the struts to:

$$\sigma_{c,\text{schoor}} \leq 0.6 \cdot \nu \cdot f_{cd}$$

(14.2.3-3)

where:
\[ f_{cd} = \frac{f_{ck}}{1.5} \]  
(ANB: coefficient 0.85 is not used for strut-and-tie design!)

\[ \nu = 1 - \frac{f_{ck}}{250} \]  
(14.2.3-4)

and \( f_{ck} \) in MPa.

The reasons for the reduction of \( f_{cd} \) are mainly:

- the fact that the strut is subjected to a two-dimensional stress state with compression along the axis of the strut and tension in the transversal direction (which will be taken up by reinforcement). The tensile stress may lead to splicing cracks parallel to the strut’s axis;

- the uncertainty about the exact dimensions of the compression strut. Indeed, the idealized prismatic strut is surrounded by concrete that cooperates in the transmission of a part of the load (see the bottle-like shape of the compression struts in figure 14.2.1-1(a)); this is even becoming clearer when cracks appear perpendicular to the tie: see figure 14.2.3-2.

![Figure 14.2.3-2](image)

Deep beam: principal stress trajectories in cracked situation

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**Note 1:**
The design strength for a concrete strut in a region with transverse compressive stress or no transverse stress may be \( f_{cd} \), without reduction coefficient. It may even be appropriate to assume higher design strength in regions where multi-axial compression exists.

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**Note 2:**
More elaborated models for struts are available in EC2, where the bottle-like shape is translated into a sub-truss system with additional ties: see figure 14.2.3-3.
14.2.4 The nodes

The dimensioning and detailing of concentrated nodes are critical in determining their load-bearing resistance. Concentrated nodes may develop where point loads are applied, at supports, in anchorage zones with concentration of reinforcement or prestressed tendons, at bends in reinforcing bars and at connections and corners of members.

It was already observed in the preceding paragraphs that distinction has essentially to be made between:

- nodes where only compression struts are concurrent; a multi-axial compression stress state is observed in the node, which is a far more comfortable position for the concrete than in the case of the one-dimensionally loaded concrete strut;
- nodes where ties are anchored. The main issue is of course the anchorage of the tensile reinforcement, but attention has also to be paid to the verification of the compression stress in the concrete in the node, which appears to be less resistant due to the presence of the tie.

EN 1992-1-1:2004; 6.5.4 recommends limited design values for the compression stresses within nodes:

- in compression nodes where no ties are anchored at the node (see figure 14.2.4-1): \( \sigma_{Rd,\text{max}} = \nu \cdot f_{cd} \) where \( f_{cd} = f_{ck}/1,5 \)
- in compression-tension nodes with anchored ties provided in one direction (see figure 14.2.4-2): \( \sigma_{Rd,\text{max}} = 0,85 \cdot \nu \cdot f_{cd} \) where \( f_{cd} = f_{ck}/1,5 \)
- in compression-tension nodes with anchored ties provided in more than one direction (see figure 14.2.4-3): \( \sigma_{Rd,\text{max}} = 0.75 \cdot v \cdot f_{cd} \) where \( f_{cd} = f_{ck} / 1.5 \)

Figure 14.2.4-1
Concrete compression stress verification in a compression node without ties (figure 6.26 in EN 1992-1-1:2004; 6.5.4)

![Concrete compression stress verification in a compression node without ties](image)

Figure 14.2.4-2
Concrete compression stress verification in a compression-tension node with anchored ties provided in one direction (figure 6.27 in EN 1992-1-1:2004; 6.5.4)

![Concrete compression stress verification in a compression-tension node with anchored ties provided in one direction](image)
Concrete compression stress verification in a compression-tension node with anchored ties provided in more than one direction (figure 6.28 in EN 1992-1-1:2004; 6.5.4)

**Note 1:**
EN 1992-1-1:2004; 6.5.4 (5) allows to increase the design compressive stress values defined above, by up to 10% where at least one of the following applies:
- triaxial compression is assured;
- all angles between struts and ties are $\geq 55^\circ$;
- the stresses applied at supports or at point loads are uniform, and the node is confined by stirrups;
- the reinforcement is arranged in multiple layers;
- the node is reliably confined by means of bearing arrangement or friction.

**Note 2:**
EN 1992-1-1:2004; 6.5.4 (6) allows to increase the design compressive stress value in a triaxially compressed node, by up to $\sigma_{Rd,max} = 3 \nu f_{cd}$ if for all three directions of the struts the distribution of load is known.

### 14.2.5 Final principle remarks

- In many cases, it appears that the limitation of the concrete stress in the struts is not a critical issue.
- The objective of the design by means of the strut-and-tie method is to get first yielding of the steel reinforcement, before the compression stress limit is reached in the struts.
14.3 Practical development of a strut-and-tie model: a procedure in 6-steps

In the following paragraphs, the elaboration and exploitation of a strut-and-tie model is explained on the basis of a procedure composed of six steps.

14.3.1 Step 1: the identification of the B- and D-regions

The structural member is subdivided into:

- regions with regular transmission of forces according to BERNOULLI’s assumption (B-regions), and
- regions around discontinuities related to changes in dimensions, introduction of concentrated forces, etc (D-regions). The D-region is identified by means of St.-VENANT’s principle: the disturbance of the regular stress distribution extends one member-width from the discontinuity. Yet, figure 14.3.1-1 shows that the D-region not only depends on the depth of the member (a foundation bloc), but also on the width over which the loads have to be smeared out. Figures 14.3.1-2 to 14.3.1-4 present several examples of D-regions caused by geometrical discontinuities (changes of dimensions, presence of corners and openings) or by static discontinuities (application of concentrated loads) (LAMBOTTE, 1989).

![Figure 14.3.1-1](image)

Example of a foundation block with two different loading conditions: the size of the D-region is also determined by the load distribution into the member.
Figure 14.3.1-2
Some examples of $D$-regions caused by geometrical discontinuities

Figure 14.3.1-3
Some examples of $D$-regions caused by static discontinuities (application of concentrated loads)
Some examples of $D$-regions for members with combination of static and geometrical discontinuities

14.3.2 **Step 2: isolation of the $D$-region and identification of the loads at the edges of the $D$-region**

The loads that are applied onto the edges of the $D$-region have to be identified:

- on one edge, the applied forces;
- on the other edges, the forces which are the result of the stress distributions along these edges, determined according to the elasticity theory in the adjacent $B$-region.

Figures 14.3.1-2 to 14.3.1-4 present several examples of forces and resulting stress distributions along the edges of the $D$-regions.

For further calculation purposes, distributed loads along edges are replaced by their resultant forces, in order to obtain later on, the transmission scheme of forces throughout the $D$-region. The $D$-region is calculated for the forces acting on it; it is evident to say that all applied forces (on all edges) have to verify the conditions of static equilibrium.

14.3.3 **Step 3: analysis of the load transfer throughout the $D$-region in view of the identification of a suitable truss model**

A suitable truss model has to be identified for the design calculation. However, in order to find a solution which meets the SLS requirements (avoiding excessive crack widths), the chosen truss system should be as much as possible in agreement with the real stress state situation according to linear elasticity theory (lower bound theorem for plastic
methods). The ideal situation is that the scheme of principal stress trajectories would be known (as a result of a preliminary FEM calculation for example). The stress trajectories show how the imposed loads are transferred throughout the structural member to the supports. Figure 14.3.3-1 illustrates that point for two corbels.

It should be noted that in many cases, it is possible to draw a workable approximate scheme of load transfer without the knowledge of the exact linear elastic stress trajectories. This is illustrated in figure 14.3.3-2 where simple (almost intuitively determined) load paths are shown. It is important to notice that when curvatures in the load path appear (where the action line of the forces changes orientation), reaction forces have to be resisted.

Note:
The passage of the forces between B- and D-regions should be compatible over the whole structural element: detail design should be performed taking account of the adjacent regions.
14.3.4 Step 4: identification of the strut-and-tie model

The analysis performed in step 3 allows to identify the strut-and-tie model which has to assure the internal equilibrium of the member for the imposed loading conditions. In most cases, the aim is to identify a statically determinate system, which allows the determination of the tensile and compression member forces by means of equations expressing the equilibrium of the nodes in the truss. The following points should help with the identification of a suitable strut-and-tie system:

- each time the load path orientation changes, a node should be added to the strut-and-tie system: see figure 14.3.3-2;
- the ties should have orientations that lead to easy arrangement of the reinforcement bars in the execution phase. The reinforcement corresponding to one tie may be realized in practice by means of a bundle of bars so that the resultant force corresponds with the tie force in the idealized truss system. However, it may also be argued that trying to respect scrupulously the tie orientation in the model, is not necessary because the scheme of stress trajectories changes when cracks appear;
- the strut-and-tie model should be as simple as possible, with a minimum value for the total length of the tensile reinforcement. Indeed, the model has to assure load bearing capacity in ULS (thus after crack development) with the least possible forces and deformations. In order to make a choice between two or more truss models which seem to be plausible solutions, the criterion of minimum energy (minimum work done) may be used in its most simple formulation:

\[
\sum F_{si} \Delta l_i = \text{minimum} \quad (14.3.4-1)
\]

where: \( F_{si} \) = tensile force in tie \( N^o \ i \) of the truss model \( (s < \text{steel}) \);
\( \Delta l_i \) = elongation of the tie \( N^o \ i \).
This criterion is used without consideration of the compression struts, because deformations are rather small with respect to the ones of the ties. Figure 14.3.4-1 presents the example of a deep beam for which, on the basis of the stress trajectories, two plausible strut-and-tie models may be identified. It may be shown that the second model (characterized by a longer total tie-length) leads to larger deformations.

![Diagram](image)

Figure 14.3.4-1
Deduction of plausible strut-and-tie models from the scheme of the stress trajectories:
(a) compression arch model; (b) suspension model

Some examples of frequently used strut-and-tie models are presented in figure 14.3.4-2.

Note:
Sometimes, statically indeterminate systems have to be considered. Figure 14.3.4-3 shows a beam which transmits a support reaction force to a wall (beam is perpendicular to the wall). It is recommended to include the part of the wall on top of the beam in the load transfer system; otherwise, cracks may appear in this part.
Figure 14.3.4-2
Examples of frequently used strut-and-tie models
**14.3.4-3**
Example of a statically indeterminate strut-and-tie system, which is needed for the modelling of the transfer of the support reaction force of the beam to the wall

### 14.3.5 Step 5: computation phase

On the basis of the chosen strut-and-tie model, the strength of struts, ties and nodes is verified according to the principles presented in paragraph 14.2. If the method is used for design purposes (determination of dimensions, reinforcement areas), overall dimensions are generally determined on the basis of the conditions for stress limitation in the concrete struts. This leads to the dimensions of the nodes and to limit values of the inclination angles $\theta$ and to the overall dimensions $b$ and $h$ of the structural member. In other cases, overall dimensions are given, except for the reinforcement: reinforcement areas and position of the reinforcement have to be determined. Although the stresses in the compression struts are in general not determining in such cases, sufficient attention should always be paid to the verification of the struts.

### 14.3.6 Step 6: identification of the reinforcement

The tensile forces in the ties have to be taken up by reinforcement bars. The reinforcement may be composed of several bars which are oriented according to the axis of the tie in the strut-and-tie model. In order to prevent surface cracking, additional reinforcement may be necessary, such as links and technological reinforcement.
14.4 Applications and examples

14.4.1 Deep beams

This paragraph focuses back again on the example of deep beam that was already presented in figure 14.2.1-1(b). It is assumed that the span \((2a)\) and the support dimensions \((a_2)\) are known as well as the imposed load \(P\).

Question: to determine \(h\) (and \(d\)) and \(A_s\).

The solution starts with the choice of the strut inclination angle \(\theta\). With a choice \(\theta = 45^\circ\), one finds:

\[
d = a \cdot \tan \theta
\]

The next step is the determination of the cross-section of the prismatic compression struts and the verification of the compression stress in the struts:

\[
\sigma_c = \frac{P}{2 \cdot \sin \theta \cdot A_{\text{strut}}} \leq \sigma_{c, \text{strut, max}}
\]

where:

\[
\sigma_{c, \text{strut, max}} = 0.6 \cdot v \cdot f_{cd} \quad \text{and} \quad v = 1 - \frac{f_{cd} \text{ (MPa)}}{250}
\]

With \(\sigma_c\) much smaller than the maximum value, the depth \(d\) may be reduced (which means that less inclined struts are possible); when \(\sigma_c > 0.6 \cdot v \cdot f_{cd}\), the depth \(d\) has to be increased.

Finally, the reinforcement area \(A_s\) is determined by:

\[
A_s = \frac{P}{2 \cdot \tan \theta \cdot f_{yd}}
\]

EN 1992-1-1:2004; 9.7 specifies that deep beams should be provided with an orthogonal reinforcement mesh near each face, with a minimum \(A_{s,\text{db, min}}\) \((db < \text{deep beam})\):

\[
A_{s,\text{db, min}} \geq 0.1\% \cdot A_c \geq 150 \text{ mm}^2 \text{/m}
\]

in each face and each direction (vertically and horizontally).

where:

\(A_{s,\text{db, min}} = \text{minimum reinforcement in each face and each direction of the deep beam}\)

\(A_c = \text{bruto concrete cross-section for the vertical or horizontal direction}\).
The distance between two adjacent bars (between horizontal bars and between vertical bars):
\[ \leq 2 \times \text{the deep beam thickness}; \]
\[ \leq 300 \text{ mm}. \]

**Note:** Figure 14.4.1-1 shows the transition from deep beams (short span beams) towards long span beams (characterized by the presence of the regularly spaced stirrups).

**Figure 14.4.1-1**
Transition from deep beams (short span beams) towards long span beams (characterized by the presence of the regularly spaced stirrups): (a) strut-and-tie model; (b) main reinforcement

**14.4.2 Walls**
Walls are large \(D\)-regions for which typical \(D\)-models may be applied.
14.4.2.1 Example 1

Figure 14.4.2-1 shows the example of a wall supported by two columns, loaded by a uniformly distributed load on the top side of the wall. A typical strut-and-tie model is presented in figure 14.3.4-2 (case e.). Figure 14.4.2-1 also presents the evolution of the horizontal tensile stress in the mid-section of the wall.

![Figure 14.4.2-1](image)

**Figure 14.4.2-1**

Typical strut-and-tie model for a wall supported by 2 columns, loaded by a uniformly distributed load on the top side of the wall

The main tensile reinforcement is concentrated at the bottom side of the wall. For a wall with a ratio $d/l$ approximately equal to 1 and $a/l = 0.1$, the tensile force reaches the value of about $0.2q.l$. The main horizontal tensile reinforcement should be arranged over the whole length of the wall and should be anchored preferably with hooks and loops, in order to avoid the tearing off of the corners above the supports. For walls with large dimensions, the main tensile reinforcement should also be distributed over a depth $0.15.d$ in order to assure a better crack distribution: see figure 14.4.2-2.

Finally, the region where the load is applied should be reinforced by means of vertical links (see further in these course notes).

![Figure 14.4.2-2](image)

**Figure 14.4.2-2**

Reinforcement of the wall presented in figure 14.4.2-1
Prescriptions EN 1992-1-1:2004; 9.6:

• vertical reinforcement
  - should be provided at both sides of the wall;
  - the area of the vertical reinforcement (for both sides together) should lie between $0.002 A_c$ and $0.04 A_c$;
  - the distance between two adjacent vertical bars ($s_1$ in figure 14.4.2-2) shall not exceed $(3 \times \text{the wall thickness } t)$ or 400 mm, whichever is the lesser (the smallest value should be used).

• horizontal reinforcement
  - horizontal reinforcement running parallel to the faces of the wall (and to the free edges) should be provided at each surface;
  - the area of the horizontal reinforcement (for both sides together) should be:
    $$\geq 25\% \text{ of the total vertical reinforcement for both sides}$$
    $$\geq 0.001 A_c$$
  - the spacing between two adjacent horizontal bars ($s_2$ in figure 14.4.2-2) should not be greater than 400 mm.

• transverse reinforcement
  - where the total area of the vertical reinforcement in the two faces exceeds $0.02 A_c$, transverse reinforcement in the form of links should be provided in accordance with the requirements for columns: the links are necessary to avoid buckling of the vertical bars. The technological provisions for walls are identical to the ones for columns (EN 1992-1-1:2004; 9.5.3); figure 14.4.2-3 presents a summary of the provisions for walls.

![Diagram of reinforcement](image)

**Figure 14.4.2-3**

Detailing of the transverse reinforcement necessary to link the reinforcement meshes at both sides of the wall.
14.4.2.2 Example 2

Figure 14.4.2-4 presents the case of a wall supported by two columns and loaded by a uniformly distributed load applied at the bottom side of the wall.

A typical strut-and-tie model is represented in figure 14.3.4-2 (case f). The model shows the development of a compression arch from which the distributed load and the self-weight is suspended. Consequently, vertical reinforcement is necessary up to the upper side of the wall: figure 14.4.2-5. The stresses in the horizontal direction are practically the same as in example 1 (load applied on top of the wall) which leads to an identical horizontal reinforcement scheme.

Figure 14.4.2-4
Typical strut-and-tie model for a wall supported by two columns and loaded by a uniformly distributed load at the bottom side of the wall

Figure 14.4.2-5
Reinforcement of the wall presented in figure 14.4.2-4
14.4.2.3 Example 3

Figure 14.4.2-6(a) presents a wall with large height. The wall is supported by two columns. A concentrated load (a column) is applied in the middle of the upper side of the wall. The depth of both $D$-regions at the upper and lower side of the wall, is equal to the width $b$ (applying St VENANT’s principle). At the interface between the $B$-region and the two $D$-regions, one observes the compression stress $\frac{P}{b.t}$ where $t =$ wall thickness; see figure 14.4.2-6(b).

- **Analysis of the upper $D$-region**
  The distributed stresses at the $B$-$D$ interface are replaced by two resultant forces $\frac{1}{2}P$. This allows drawing the strut-and-tie model: see figure 14.4.2-6(c). This leads to the identification of the horizontal tensile force $N$, which is translated into a certain number of horizontal ties. The ties embrace the peripheral reinforcement and are distributed over the region where $N$ is active: see figure 14.4.2-6(d).

- **Analysis of the lower $D$-region**
  An analogous reasoning leads to the identification of tensile reinforcement at the bottom of the wall.

![Figure 14.4.2-6](image-url)
Analysis of a high wall by means of strut-and-tie models; (a) subdivision in $B$- and $D$-regions; (b) identification of the load transfer; (c) strut-and-tie models; (d) scheme of the reinforcement

14.4.3 **Reinforced concrete blocs with concentrated loads**

14.4.3.1 **Centrally applied load**

This case has already been discussed in the paragraph above (the upper $D$-region in figure 14.4.2-6). This case is frequently encountered in practice, for example in the context of anchorage regions in prestressed girders, column heads and column footings where concentrated compression forces are applied, etc.

The following notes present the well known results of the analysis by SCHLAICH (1981) of a reinforced concrete bloc with limited thickness, loaded by a centrally applied load.

Figure 14.4.3-1 represents the stress trajectories, the tensile stress in the plane of symmetry and the evolution of the stress in a plane, parallel to the plane of symmetry, at the distance $0.3h$, for different values of $a_1/a$: see figure 14.4.3-1(b). Figure 14.4.3-1(c) represents a plausible strut-and-tie model, figure 14.4.3-1(d) represents the values of $F_i/P$ for $h = a$ and different values of $a_1/a$; figure 14.4.3-1(e) represents (for $a_1/a = 0.1$) the influence of the ratio $a/h$ on the values of $F_i/P$ and $\theta$. SCHLAICH developed the following empirical expression for the maximum stresses $\sigma_i$ shown in figure 14.4.3-1(b):

$$\sigma_{x,max} = 0.5 \frac{P}{b.a} \left(1 - \frac{a_1}{a}\right) \quad \text{(for } h = a\text{)} \quad (14.4.3-1)$$

The curve that represents the evolution of $F_i/P$ in figure 14.4.3-1(d) may be approximated (in a safe way) by the straight line passing through the end points; the equation of this straight line is:

$$F_i = 0.3 \cdot P \left(1 - \frac{a_1}{a}\right) \quad (14.4.3-2)$$

Another possible assumption is to put the action line of the tensile force $F_i$ at a distance $0.4a$ from the bottom edge of the bloc. With this assumption, almost the same approximate relationship is found (with 0.31 instead of 0.3).

The reinforcement $A_y = \frac{F_i}{f_{yd}}$ is arranged by means of closed stirrups, put in horizontal position. The stirrups are distributed in a zone with an approximate height of $a/2$, symmetrically with respect to the action line of $F_i$. 

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14.4.3.2 Eccentrically applied loads

Figure 14.4.3-2 presents the stress trajectories as well as a plausible strut-and-tie model.
This model may be applied to the wall presented in figure 14.4.3-3; the length of the $D$-region is once again $b$. The stress distribution at the $B$-$D$-interface may be determined from theory of elasticity; this leads to the determination of the resultant forces $D_1$ (compression) and $N$ (tensile). There are no discontinuities at the $B$-$D$-interface, which allows the compression force $D_1$ to enter in an undisturbed way into the $D$-region (dashed line passing by point $c$). Regarding the position of the tensile force $N$, as reinforcement is always needed along the edges of walls and blocs, the reinforcement for $N$ is arranged as close as possible to the edges, but taking into account the cover requirements. On the basis of these edge conditions and in combination with the scheme of the stress trajectories, a suitable strut-and-tie model may be identified: figure 14.4.3-3(b).

Special attention has to be paid to point $d$. The compression diagonal $cd$ is situated in the mid-plane of the wall, while the reinforcement is put along the side edges. This means that a transversal tensile component $N_0$ is necessary to assure equilibrium; extra links are thus needed to take up $N_0$. 

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Figure 14.4.3-2

Application of the strut-and-tie model method for the analysis of a thin bloc in reinforced concrete, loaded by an eccentrically applied concentrated load:
(a) stress trajectories; (b) strut-and-tie model
14.4.3.3 Three-dimensional case

With large three-dimensional reinforced concrete blocks, load paths for the concentrated load may be considered in two directions; this is the approach adopted in figure 14.4.3-4, where the strut-and-tie method is applied in two perpendicular planes. The reinforcement is composed of meshes put in planes that are perpendicular to the action line of the concentrated load.
Figure 14.4.3-4
Foundation bloc loaded by a centrically applied concentrated load; the strut-and-tie method is applied in two different planes

Figure 14.4.3-5 presents the case of foundation blocs supported by 3 or 4 piles. The load is transferred in multiple directions. The strut-and-tie model should be based on the principle that loads are always transferred to the piles along the shortest load paths. The ties in the model are thus situated along the shortest distance between the pile heads. The better solution is thus to concentrate the reinforcement above the pile heads instead of having reinforcement distributed over the whole foundation bloc.

For widely spaced piles \((l > 4.\phi_p)\) in figure 14.4.3-5 (d), the regions in between the piles are also reinforced. It is necessary to arrange for “supporting stirrups” along the edges (supporting stirrups are also necessary with indirect supports; see further).
Figure 14.4.3-5
Foundation blocs supported by three or four piles; (a) top view; (b) strut-and-tie models; (c) scheme of the reinforcement; (d) case of blocs with large distances between the pile heads ($l > 4.\phi_p$).
Note: “suspension stirrups” for indirect supports

The load transfer from a transverse beam to a main girder (figure 14.4.3-6 (a)), may be realized by means of compression struts (figure 14.4.3-6 (b)) or by means of an inclined tie (figure 14.4.3-6 (c)). With compression struts, the load “enters” into the main girder at the bottom layer, but has to be suspended to the upper layer by means of stirrups (called suspension stirrups). The design of the main girder is then performed in the same way as for a beam with a concentrated load applied on top of the beam.

Reminder: it was already observed in figure 14.3.4-1 that an inclined suspension tie is less efficient than an inclined strut (which provides a stiffer load transfer mechanism).

Figure 14.4.3-6 (d) shows the process of the load transfer. The load is fixed in the upper layer by the stirrups (bb') in the transverse beam. This load is transferred by the inclined compression strut to the main girder. Suspension stirrups (aa') are used for the transfer of the load to the upper part of the main girder. It is important to note that the suspension stirrups are situated in the same plane as the transverse beam. Figure 14.4.3-6 (e) represents the reinforcement scheme.
Figure 14.4.3-6
Indirect support; (a) two main girders and one transverse beam; (b) load transfer by means of a compression strut; (c) load transfer by means of an inclined tie; (d) detailed view of the load transfer mechanism from the transverse beam to the main girder; (e) reinforcement scheme with side and top view
14.4.4 Corbels

A corbel (or console) is a small cantilever fixed on a column or wall, used to support other horizontal structural members such as transverse beams. In historical masonry or wood buildings, corbels are pieces of stone or timber jutting out of a wall to carry any superincumbent weight. The word "corbel" comes from the Latin *corbellus*, a diminutive of *corvus* (a raven) which refers to the beak-like appearance.

The load applied to the corbel is mainly the vertical load induced by the self-weight. Yet, corbels may also be subjected to important horizontal actions, for example where temperature variations lead to shortening of the transverse beam and to transmission of horizontal actions to the corbel by friction resistance in the contact area between the two members.

Design recommendations are given in Annex J to EN 1992-1-1:2004. The essential recommendations are summarized in the following.

The basic model for corbel design is presented in figure 14.4.4-1. The proposed model is a combination of two strut-and-tie systems:
- a direct strut, characterized by the inclination angle $\theta$;
- a secondary system, including a horizontal tie, which takes account of the bottle shape of a real strut.

EC2 specifies that the inclination of the direct strut should be sufficiently high:

$$1 \leq \tan \theta \leq 2,5$$

or:

$$45^\circ \leq \theta \leq 68,2^\circ$$

If the inclination angle $\theta$ is too small, the model with one direct strut is not appropriate anymore; the corbel should then be treated as a cantilever beam. This condition is reinforced by a second one (see figure 14.4.4-1 for the significance of the symbols):

$$a_c < z_0$$

**Note:** the horizontal force $H_{Ed}$ is indicated in figure 14.4.4-1, but specific models and rules concerning this load are not discussed in EC2 (version 2004).
Two cases are considered in function of the length of the corbel:

- if $a_c \leq 0.5 \cdot h_c$, closed horizontal or inclined links should be provided in addition to the main tension reinforcement (see figure 14.4.4-2(a)), with:
  \[ A_{s,\text{lnk}} \geq 0.25 \cdot A_{s,\text{main}} \]

- if $a_c > 0.5 \cdot h_c$ and $F_{Ed} > V_{Rd,e}$ closed vertical links should be provided in addition to the main tension reinforcement (see figure 14.4.4-2(b)), with:
  \[ A_{s,\text{lnk}} \geq 0.5 \cdot F_{Ed}/f_{yd} \]

**Note:** for corbels with large depth, strut-and-tie models such as illustrated in figure 14.3.4-2 should be used.
14.4.5 Half joints in beams

A corbel is a very visible support system. Sometimes, for aesthetic reasons, it is preferred to work with so called “half joints”, where the corbel is integrated into the beam: see figure 14.4.5-1.

Note:
In case a horizontal force is applied to the beam’s end at the support, it is necessary to increase the vertical stirrup reinforcement: see figure 14.4.5-1 (d).
Figure 14.4.5-1
Beams with half joints (NL: tandoplegging van balken; FRXXX); (a) $D$ and $B$-regions; (b) strut-and-tie model; (c) reinforcement scheme; (d) a horizontal tensile force at the support leads to increased vertical reinforcement at the beam’s end

14.4.6 Corners in frames

14.4.6.1 Corners subjected to “positieve” moments

Figure 14.4.6-1 shows the tie-and-strut model as well as the reinforcement for a corner where column and beam have approximately the same dimensions. More elaborated strut-and-tie models are necessary where the dimensions of the beams and columns are very different: see figure 14.4.6-2; indeed, it is recommended to choose the inclination angle of the compression struts not too different from $45^\circ$. 
14.4.6.2 *Corners subjected to “negative” moments*

Figure 14.4.6-3 presents a strut-and-tie model, with two plausible solutions for the reinforcement scheme. More elaborated strut-and-tie models are available in literature (SCHLAICH): figure 14.4.6-4.
Figure 14.4.6-3
Corner in a frame structure, subjected to a “negative” moment; (a) strut-and-tie model; (b) first solution: a loop is used for the main reinforcement; (c) second solution composed of straight bars and inclined stirrups

Figure 14.4.6-4
Corner in a frame structure, subjected to a “negative” moment; more elaborated strut-and-tie model and reinforcement schemes

14.4.7 Nodes in frame systems
Different load combinations are possible in the nodes of frame structures; the reinforcement scheme should thus be adapted for all these load combinations. This leads to "universal solutions" such as shown in figure 14.4.7-1.
Figure 14.4.7-1
Nodes in framework systems; (a) load combinations; (b) strut-and-tie models; (c) universal reinforcement scheme