

5.1 (L6.3) (Two correlated assets) The correlation ρ between assets A and B is 0.1, and other data are given in Table 1 (Note $\rho = \sigma_{AB}/(\sigma_A\sigma_B)$).

Table 1: Two Correlated Cases

Asset	\bar{r}	σ
A	10.0%	15%
B	18.0%	30%

- Find the proportions α of A and $(1-\alpha)$ of B that define the portfolio of A and B which has the minimum standard deviation.
- What is the value of this minimum standard deviation?
- What is the expected return of this portfolio?

5.2 (L6.7) (Markowitz fun) There are just three assets with rates of return r_1, r_2 and r_3 , respectively. The covariance matrix and the expected rates of return are

$$\mathbf{V} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad \bar{\mathbf{r}} = \begin{bmatrix} 0.4 \\ 0.8 \\ 0.8 \end{bmatrix}$$

- Find the minimum-variance portfolio.
- If the risk-free rate is $r_f = 0.2$, find the efficient portfolio of risky assets.

5.3 (L6.1) (Shorting with margin) Suppose that to short a stock you are required to deposit an amount equal to $1.5X_0$, where X_0 is the initial price of the stock. At the end of first year the stock price is X_1 and you liquidate your position. If R is the total return of the stock, what is the total return on your short?

5.4 (L6.5) (Rain insurance) Kalle Virtanen's friend is planning to invest 1 M€ in a rock concert to be held 1 year from now. The friend figures that he will obtain 3 M€ revenue from his 1 M€ investment - unless, my goodness, it rains. If it rains, he will lose his entire investment. There is a 50% chance that it will rain the day of the concert. Kalle suggests that he buys rain insurance. He can buy one unit of insurance for 0.50€, and this unit pays 1€ if it rains and nothing if it does not. He may purchase as many units as he wishes, up to 3 M€.

- What is the expected rate of return on his investment if he buys u units of insurance? (The cost of insurance is in addition to his 1 M€ investment.)
- What number of units will minimize the variance of his return? What is this minimum value? And what is the corresponding expected rate of return? (*Hint* Before calculating a general expression for variance, think about a simple answer.)

- 5.5 (L6.6) Suppose there are n assets which are uncorrelated. You may invest in any one, or in any combination of them. The mean rate of return \bar{r} is the same for each asset, but the variances are different. The return of an asset i has a variance of σ_i^2 ($i = 1, 2, \dots, n$).
- Show the situation on an $\bar{r} - \sigma$ diagram. Describe the efficient set.
 - Find the minimum-variance point. Express your result in terms of

$$\bar{\sigma}^2 = \left(\sum_{i=1}^n \frac{1}{\sigma_i^2} \right)^{-1}.$$

- 5.6 (L6.8) (Tracking) Suppose that it is impractical to use all the assets that are incorporated into a specified portfolio (such as a given efficient portfolio). One alternative is to find the portfolio, made up of a given set of n stocks, that tracks the specified portfolio most closely - in the sense of minimizing the variance of the difference in returns.
- Specifically, suppose that the target portfolio has (random) rate of return r_M . Suppose that there are n assets with (random) rates of return r_1, r_2, \dots, r_n . We wish to find the portfolio rate of return

$$r = \alpha_1 r_1 + \alpha_2 r_2 + \dots + \alpha_n r_n$$

(with $\sum_{i=1}^n \alpha_i = 1$) minimizing $\text{Var}[r - r_M]$.

- Find a set of equations for the α_i 's.
- Although this portfolio tracks the desired portfolio most closely in terms of variance, it may not have the desired the mean. Hence a logical approach is to minimize the variance of the tracking error subject to achieving a given mean return. As the mean is varied, this results in a family of portfolios that are efficient in a new sense - say, tracking efficient. Find the equation for the α_i 's that are tracking efficient.