SHIP BUOYANCY AND STABILITY

Lecture 02 – Ship equilibrium and introduction to ship hydrostatics
Literature

J. Matusiak: Laivan kelluvuus ja vakavuus

Biran A. B., Ship Hydrostatics and Stability, 2003

J. Matusiak: Short Introduction to Ship Theory (Part 1)

Rawson, K. J J. Basic Ship Theory Volume 1. 2001

Buoyancy and Stability of a Ship, Finnish text [book]


Shorten version of the Finnish textbook, in English

Check the library
BEFORE THIS LECTURE

- General notes on ship stability
- Notes on hydrostatic pressure
- Floating bodies: Archimede’s law

- Notes on Ship definitions
- Definition of Buoyancy and center of buoyancy
- Equilibrium of a ship

Now, you should be able to:

- Estimate the hydrostatic pressure in any points of a fluid
- Describe the influence of fluid density on the immersed volume
- Describe the parameters of an equilibrium floating condition for a ship
- Describe how the draft change by changing the ship weight
Following lecturers

- Introduction
- Ship equilibrium and introduction to ship hydrostatics
  - Center of Buoyancy
  - Center of Flotation
- Ship initial Stability
- The stability curve (GZ curve)
- Preparation for the laboratory test
- Dynamic stability
- Second generation of intact stability criteria
- Ship Damage Stability (Part I & II)
- Stability special topics
\( A_w \): water plan area; \( F \): center of the water plan area;
\( S_w \): wetted surface; \( V \): immersed volume;
\( B \): center of buoyancy

Floating plane, coincident with the undisturbed sea surface
Reference System 1

Waterplane

K: origin of the reference system XYZ; B: origin of the reference system xyz.

Center of buoyancy

Waterplane

K: origin of the reference system XYZ; B: origin of the reference system xyz.

Midship
Reference System 2

φ: heel angle
θ: trim angle
ψ: yaw angle

Note that ship immersed hull i.e. displacement does not vary with the yaw angle.
TRIM of the ship:

- $T_F - T_A$
- $T_F > T_A$: Bow down, stern up
- $T_F < T_A$: Stern down, bow up
How is the trim in this case?

ZERO TRIM, ZERO HEEL i.e. EVEN-KEEL CONDITION
There is no difference in terms of equations if we draw the ship inclined or the floating plane inclined. What matters is that the reference system, fixed with the ship, remains body-fixed and the gravity remains normal to the sea surface.
Equilibrium of the ship

\[
\begin{align*}
\sum_{i=1}^{N} \vec{F}_i &= 0 \\
\sum_{i=1}^{N} \vec{r}_i \times \vec{F}_i &= 0
\end{align*}
\]

\[
\begin{align*}
\vec{W} + \vec{\Delta} &= \vec{0} \\
(G - B) \land \vec{W} &= \vec{0}
\end{align*}
\]

\[
\begin{align*}
\vec{W} &= -\vec{\Delta} \\
(G - B) \parallel \vec{W}
\end{align*}
\]

- The weight \( W \) is applied in the center of gravity \( G \). That means that for equilibrium weight and buoyancy \( \Delta \) vectors have to stay on the same line, that is normal to the floating plane.
How can I define the floating condition of the ship at the equilibrium?

\[
\sum_{i=1}^{N} \vec{F}_i = 0
\]

\[
\sum_{i=1}^{N} \vec{r}_i \times \vec{F}_i = 0
\]
Floating plane

• The weight and the buoyancy are normal to the equilibrium floating plane i.e. the sea surface;
• Using the general equilibrium equation we need to determine the floating plane, i.e. the draft and the trim and heel angle;
• Anyway, it is possible to individuate the floating plane even with equivalent parameters;
• Let’s introduce some geometric features regarding an inclined floating plane with respect to the reference system OXYZ parallel to the main reference system KXYZ
The
\[ \mathbf{W} + \mathbf{\Delta} = \mathbf{0} \]
\[ (G - B) \wedge \mathbf{W} = \mathbf{0} \]
\[ G \equiv (x_g, y_g, z_g) \]
\[ B \equiv (x, y, z) \]

\[ (G - B) = \begin{bmatrix} x_g - x \\ y_g - y \\ z_g - z \end{bmatrix} \]

\[ \mathbf{W} = -\mathbf{\Delta} \rightarrow W = \Delta \]

\[ \mathbf{W} = \begin{bmatrix} \Delta \cos \alpha \tan \theta \\ \Delta \cos \alpha \tan \phi \\ -\Delta \cos \alpha \end{bmatrix} \]
Geometric Features

- O is the intersection point between WL and Z
- s is the intersection line between WL and YZ
- t is the intersection line between WL and XY
- r is the intersection line between WL and XZ
- $\phi$ is the angle between the line s and the Y axis
- $\theta$ is the angle between the line r and X axis
- $\alpha$ is the angle between the line n (the normal to the plane WL) and the Z axis
- $\beta$ is the angle between the line t and the X axis
Gleijeses Equations – equilibrium equation of a ship

\[ \begin{align*}
\vec{W} + \Delta &= \vec{0} \\
\Rightarrow \quad \vec{W} &= -\Delta \\
W &= \Delta = \rho g \nabla \\

(G - B) \wedge \vec{W} &= \vec{0}
\end{align*} \]

\[ \begin{align*}
M_x &= (y - y_g) \Delta \cos \alpha - (z_g - z) \Delta \cos \alpha \tan \phi = 0 \\
M_y &= (x_g - x) \Delta \cos \alpha + (z_g - z) \Delta \cos \alpha \tan \theta = 0 \\
M_z &= (x_g - x) \Delta \cos \alpha \tan \phi - (y_g - y) \Delta \cos \alpha \tan \theta = 0
\end{align*} \]

\[ \begin{align*}
\tan \phi &= \frac{(y - y_g)}{(z_g - z)} \\
\tan \theta &= \frac{(x - x_g)}{(z_g - z)} \\
\frac{\tan \phi}{\tan \theta} &= \frac{(y - y_g)}{(x_g - x)}
\end{align*} \]
Equilibrium equation-floating condition

\[ \Delta = \rho g \nabla \]

\[ \tan \phi = \frac{(y - y_g)}{(z_g - z)} \]

\[ \tan \theta = \frac{(x_g - x)}{(z_g - z)} \]

\[ \nabla (T_0, \phi, \theta) \]

\[ x(T_0, \phi, \theta); \ y(T_0, \phi, \theta); \ z(T_0, \phi, \theta) \]

- The tern \( T_0, \theta, \phi \) determine the equilibrium floating position of the ship (midship draft and floating plan).
- The mathematical problem is characterized by three equations for three variables.
- Ship immersed volume and its center of buoyancy vary with the floating condition.
Archimedes Principle

- **Center of Gravity (G):**
  - Changes position only by change/shift in mass of ship
  - Does not change position with movement of ship

- **Center of Buoyancy (B):**
  - Changes position with movement of ship
  - Underwater geometric center moves
  - Also affected by displacement G
Ship equilibrium with no trim and heel

- In order to have $\phi$ and $\theta$ equal to zero, from the equilibrium equations comes out that:

$$y_G = y_B = 0, \quad x_G = x_B.$$  

**Even-keel condition**

- The ship being symmetrical respect to the diametral plan will have the transversal coordinates of the center of buoyancy as zero for a null $\phi$.

- This condition ensures that Weight and Buoyancy forces act on the same line.

- The ship assumed to have no trim and heel need to comply only the first equation to find equilibrium. $\Delta = \rho g V$

- Ship will submerge and emerge till it doesn't find the draft at which the displaced volume balances the ship weight.
How to find numerically the equilibrium equation?

• Analytically
  volume and centre of buoyancy approximated expressions
  - Cylindric bodies method, small angles
  - Metacentric method i.e. wedge method

• Grafically
  volume and centre of buoyancy curves evaluated for different draft, trim and heel angles
  - Interpolation of hydrostatics calculation

• Numerically
  volume and centre of buoyancy numerically evaluated step by step
  - Iterative computer program

\[
\begin{align*}
\Delta &= \rho g \nabla \\
\tan \phi &= \frac{(y - y_g)}{(z_g - z)} \\
\tan \theta &= \frac{(x_g - x)}{(z_g - z)}
\end{align*}
\]
Floating bodies

- Due to the external actions, the body is forced to move: waterplane of the floating body is free to change as its displaced volume.
- The center of buoyancy for a generic body, is linked to the geometry of the immersed volume, according to the body position.
- The ship geometry is not a box, not a cylinder, not described by an analytical equation.
- The centers of buoyancy do not belong to a known surface
- Some assumption could be done in analyzing the center of buoyancy variation:
  - Qualitatively analysis
  - Approximation of the center of buoyancy curve with a circumference for small angle
ISOHULL CONCEPT

• A ship, in normal operational conditions has a constant displacement.
• This means that under external moments, the buoyancy vector has always the same magnitude, but applied in different points.
• The immersed volume of the ship, i.e. the amount of the displaced water is constant.
• A family of hull with different shapes, characterized by the same volume, is called ISOHULL
HOW IS THE CURVE OF THE CENTER OF BUOYANCY FOR A SHIP?

HOW DOES THE CENTER OF BUOYANCY VARY?

HOW WE CAN EXIMATE IT?
Example of a center of buoyancy curve

Let’s assume fixed volume and fixed trim:
\[ \nabla = \nabla_0 \] (for small angles the draft is the same)
\[ \theta = 0 \]
Let’s incline the ship from 0° to 20°

\[ B = B(\nabla, \phi, \theta) = B(\nabla_0, \phi, 0) \quad \phi \in [0°, 20°] \]

For a hull with a constant volume, inclining the ship only transversally, the center of the immersed volume (center of buoyancy) belong to a curve in the space.
Projection of the curve

\[ B = \mathbf{B}(\nabla_0, \phi, 0) \]
\[ \phi \in [0^\circ, 20^\circ] \]

Projection on the horizontal plane XY

The curve of the center of buoyancy for finite angles comes out from the YZ plane

Measured from the aft perpendicular
Projection of the curve

Projection on the vertical plane XZ

$$B = B(\nabla_0, \phi, 0)$$

$$\phi \in [0^\circ, 20^\circ]$$

Graphs showing measured data for different angles:

- XZ plane
- 3D representation

 measured from the aft perpendicular
Projection of the curve

Projection on the lateral plane YZ

\[ B = B(\nabla_0, \phi, 0) \]
\[ \phi \in [0^\circ, 20^\circ] \]

\( BM \) is called “Metacentric radius”.

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Maria Acanfora
Ship Buoyancy and Stability
HOW CAN WE EXIMATE HOW THE CENTRE OF BUOYANCY CHANGES WITH THE FLOATING PLANE?

approximated methods:
1. vertical variation
2. variation with heel and trim: wedge method
Vertical center of buoyancy variation.

How does the center of buoyancy change only with ship draft?

\[
\delta \overline{KB} = \frac{1}{\overline{V} + \delta \overline{V}} \left( \int_{\overline{V}} z \, d\overline{V} + \int_{\delta \overline{V}} z \, d\overline{V} - \frac{1}{\overline{V}} \int_{\overline{V}} z \, d\overline{V} \right)
\]

\[
\delta W = \delta \Delta = \rho \ g \ \delta V = \rho \ g \ A_w \ \delta T
\]

\[
\int_{\delta V} z \, d\overline{V} = \delta V \left( \overline{T} + \frac{\delta \overline{T}}{2} \right)
\]

\[
\delta \overline{KB} = \frac{\delta \overline{V}}{\overline{V} + \delta \overline{V}} \left( \overline{T} + \frac{\delta \overline{T}}{2} - \overline{KB} \right)
\]
**Variation with heel: Wedge method**

VERTICAL SIDE ASSUMPTION

\[ \nabla' + v_2 = \nabla' + v_1 = \nabla \]

\( \nabla \) is the ship volume

Equivalent hull or Isohull assumption \( v_1 = v_2 = v \)  
Emerged wedge=immersed wedge

\( \nabla' \) is the common volume (white part in the picture)

\[
(B_0 - B_\phi) \equiv (\delta y'_B, \delta z'_B)
\]

\[
(g_1 - g_2) \equiv (y'_g, z'_g)
\]

Vector going from \( B_0 \) to \( B_\phi \)

Vector connecting the wedge centers
Wedge method - transversal

Pure transversal inclination. Small angle assumptions, metacentric assumption.

The volume between the new waterplane and the initial waterplane are two wedges.

The volume \( v_2 \) is now emerged while the volume \( v_1 \) is now immersed with respect to the initial position.

For the equivalent hull hypothesis \( v_1 = v_2 \).

The variation of the center of buoyancy (observed in the transversal plan) is proportional to the variation of the centers of the wedges by means of \( v/\nabla \)

\[
\nabla \delta y'_B = \nabla 0 + v_2 \left( \frac{1}{2} y'_g - \left( -v_1 \right) \right) \frac{1}{2} y'_g, \text{ josta } \nabla \delta y'_B = v \ y'_g \\
\n\nabla \left( z'_B + \delta z'_B \right) = \nabla z'_B + v_2 \left( T + \frac{z'_g}{2} \right) - v_1 \left( T - \frac{z'_g}{2} \right), \text{ josta } \nabla \delta z'_B = v \ z'_g \\
\]

\[ B_0B_1 = \frac{v}{\nabla g_1g_2} \]
Demonstration of the metacenter definition

The metacentric radius for small heeling angle could be written as:

\[
B_0M_0 = \frac{B_0B_1}{\delta \phi} = \frac{v}{\nabla} \frac{g_1g_2}{\delta \phi}
\]

- Assuming \( y \) as half breadth \( s = 0.5 \ y \ (y \ \delta \phi) \)
- The distance between the two centers of the wedges is \( r = 2 \ (2/3) \ y \).
- The moment of inertia of the waterplan, \( I_T \) is given by:

\[
I_T = \frac{2L}{3} \int_0^L y^3 \ dx
\]

\[
v \ g_1g_2 = \int_0^L \left[ \frac{1}{2} y^2 \ \delta \phi \right] 2 \frac{2}{3} y \ dx = \delta \phi \ \frac{2}{3} \int_0^L y^3 \ dx
\]
What is the center of Floatation of a ship?
How it is defined?

Let’s introduce the topic starting from the SMALL ANGLE assumption ....
**Isohull properties: Euler Theorem**

The intersection of two very close floating planes is also very close to their centers. That means that the axis of inclination is central for both the waterplanes.

**Euler Theorem**

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Euler Theorem

The intersection of two very close floating planes is also very close to their centers. That means that the axis of inclination is central for both the waterplanes.

• **F** is the geometrical center of the initial waterplane
• **F** belongs to the axis of inclination **x**: this axis is a central axis of inertia or central.
• After a small inclination, the center the new waterplane is very close to **F**
• They are considered to be the same
• The x axis is baricentral also for the new waterplane
Euler Theorem applied to ship

- A ship that inclines of small angles, rotates around a constant central axis in $F$ and the immersed volume remains constant!!!
The center of floatation $F$ is the centroid of the waterplane area.

It is a POINT, which means it has 3 coordinates $(x_F, y_F, z_F)$.

For small angles $F$ is assumed to be constant and equal to $F$ calculated in the even-keel condition!

- In general the most important coordinate is the longitudinal one: $x_F$ (or LCF);
- For symmetrical hull $y_F = 0$, $F$ belongs to the longitudinal axis!
- $F$ belongs to the waterplane area i.e. to the floating plane, then $z_F$=draft!
Center of floatation

- In order to have an isohull longitudinal inclination, the ship will rotate around a transversal axis y', passing thorough the point F.
- The longitudinal axes x'≡ x
- The point F is called center of floatation.
- Assuming this point as origin of the x' axis then:

\[
\int_{-L}^{L} y(x') x' \, dx' = 0
\]

\[
x_F \text{ is the longitudinal coordinate of the center of flotation and depends on the reference frame. If the reference frame has its origin in F then } x'_F = 0
\]

\[
A_w = \int_{-L/2}^{L/2} y(x) \, dx = \int_{-L_A}^{L_F} y(x') \, dx' \quad x_F = LCF = \frac{1}{A_w} \int_{-L/2}^{L/2} y(x) x \, dx
\]

The waterplane area does not change with the reference frame
Central Moments of Inertia of the waterplane: $I_T$ and $I_L$

The moments of inertia of the waterplane represent important characteristics in assessing the ship initial stability.

In the main reference frame $Kxyz$, let’s define $I_{xx}$ and $I_{yy}$

$$I_{xx} = \int dI_{xx} = \frac{2}{3} \int_{L} y^3(x) \, dx \quad \text{ja} \quad I_{yy} = \int dI_{yy} = 2 \int_{L} y(x) \, x^2 \, dx$$

$x = x'$ is of symmetry for the waterplane; $I_{xx}$ is a central moment of inertia.

$$I_T = I_{xx} \quad \text{ja} \quad I_L = \frac{1}{12} \int 2 \, y(x)^3 \, dx$$

$$dI_{xx} = \int_{X} 2 \, y(x)^3 \, dx \quad \text{ja} \quad dI_{yy} = \int_{X} x^2 \, dA_W = \int_{X} 2 \, y(x) \, dx$$

$$I_T = I_{xx} \quad \text{ja} \quad I_L = \int_{X} \left( x - x_F \right)^2 \, dA_W = \int_{X} x^2 \, dA_W - 2 \, x_F \, x \, dA_W + x_F^2 \, dA_W$$

$$I_L = \int_{X} x^2 \, dA_W - 2 \, x_F \, x \, dA_W + x_F^2 \, dA_W = I_{yy} - 2 \, x_F^2 \, A_W + x_F^2 \, A_W = I_{yy} - x_F^2 \, A_W.$$
To Keep in Mind:

For a given draft (even-keel)

\[ A_w = \text{area of the waterplane (m}^2) \]

\[ F = \text{center of flotation, i.e. center of the waterplane (m)} \]

\[ LCF = \text{longitudinal coordinate of the center of flotation (m)} \]

\[ I_T = \text{moment of inertia of the waterplane about the longitudinal centroidal axis(m}^2) \]

\[ I_L = \text{moment of inertia of the waterplane about the transversal centroidal axis(m}^2) \]

\[ BM_T = \text{BM metacentric radius (sometimes transversal metacentric radius) (m)} \]

\[ BM_L = \text{longitudinal metacentric radius (m)} \]

These values, after small inclinations, remain constant!!!

These values, for large inclinations, CHANGE!!!
Central axes of inertia

- Let’s assume $I_T$ and $I_L$ respectively as the transversal and the longitudinal central moments of inertia of the waterplane.

**EVEN-KEEL** floating condition with $\theta=0$ and $\phi=0$

\[
I_T = I_{xx}
\]
\[
I_L = I_{yy} - x_F^2 A_w
\]

Still valid for small angle inclinations from the E-K. Basically the waterplane for small inclination angles remains the same.

**Generic floating condition with $\theta \neq \phi \neq 0$**

\[
I_{x\alpha} = \int \frac{y^2}{A_w} \cdot dA_w = I_{\bar{x}} \cdot \cos^2 \alpha + I_{\bar{y}} \cdot \sin^2 \alpha - 2 \cdot I_{\bar{xy}} \cdot \sin \alpha \cdot \cos \alpha
\]
\[
I_{y\alpha} = \int \frac{x^2}{A_w} \cdot dA_w = I_{\bar{x}} \cdot \sin^2 \alpha + I_{\bar{y}} \cdot \cos^2 \alpha + 2 \cdot I_{\bar{xy}} \cdot \sin \alpha \cdot \cos \alpha
\]

Product of inertia

\[
I_{xy} = \int \frac{x \cdot y}{A_w} \cdot dA_w
\]

\[
\alpha = \frac{1}{2} \arctan \left( \frac{2 I_{\bar{xy}}}{I_{\bar{yy}} - I_{\bar{xx}}} \right)
\]

The waterplane for finite inclination angles **does not** remain the same.

Still valid for small angle inclinations from the E-K. Basically the waterplane for small inclination angles remains the same.
SUMMARY

• Center of Buoyancy curve
• Center of Flotation
• Wedge method
• Small angle assumption
• Metacenter and metacentric radius

You should be able to:

• Describe qualitatively how the center of buoyancy varies, according to draft change or heeling angle
• Describe the ship inclination axes, defined by the center of flotation
• Use the wedge method results
• Describe the main outcomes of the small angle assumption
• Define the metacenter and its relation with the curve of the center of buoyancy