Exercise 1. Show that the Cauchy distribution $F(x) = \frac{1}{2} + \frac{1}{\pi}\arctan x$ is heavy-tailed with $\gamma = 1$.

Exercise 2. Let $E_1, E_2, \ldots$ be i.i.d. and standard exponential. Show that

$$\frac{1}{\log n} E_{(n,n)} \to_P 1.$$

Exercise 3. Let $X_1, X_2, \ldots$ be i.i.d. and assume that the corresponding distribution $F \in \mathcal{D}(G_\gamma)$, with $\gamma > 0$. Show that

$$\frac{\log X_{(n,n)} - \log X_{(n-k_n,n)}}{\log k_n} \to_P \gamma,$$

where k_n is such that $k_n/n \to 0$, $k_n \to \infty$ as $n \to \infty$. *Hint: Refer to the proof of Hill estimator's consistency. Apply similar techniques combined with exercise 2.*