Exercise 1. Formulate a research question that involves extreme quantile estimation (the S&P 500 application discussed throughout the course is an example of this; it's okay to use the same example, but be creative if you can). Find a relevant dataset.

Exercise 2. Use the Hill estimator

$$\hat{\gamma}_H = \frac{1}{k_n} \sum_{i=0}^{k_n - 1} \log X_{(n-i,n)} - \log X_{(n-k_n)}$$

to estimate the extreme value index. Calculate the estimator values for $0.01n \leq k_n \leq 0.1n$. Compare the values to the ones produced by the Pickand's estimator

$$\hat{\gamma}_P = \frac{1}{\log 2} \log \left(\frac{\log X_{(n-k_n,n)} - \log X_{(n-2k_n,n)}}{\log X_{(n-2k_n,n)} - \log X_{(n-4k_n,n)}} \right)$$

with $0.01n \leq k_n \leq 0.1n$.

Exercise 3. If $\gamma > 0$, estimate an extreme quantile from the data using the estimator:

$$\hat{x}_{p_n} = X_{(n-k_n,n)} \left(\frac{k_n}{np_n} \right)^{\hat{\gamma}},$$

where the probability p_n and threshold k_n are chosen to depend on n so that $np_n = o(k_n)$ and $\log np_n = o\left(\sqrt{k_n}\right)$ as $n \to \infty$.