Chapter 7

DC-to-AC Converters

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Chapter 7 DC-AC Converters
Voltage-source inverter supplied from a diode rectifier
Single-phase voltage-source inverter

Fig. 7.2
Each inverter leg can assume two states only: either the upper (common-anode) switch is on and the lower (common-cathode) switch is off, or the other way around. Thus, two switching variables $a$ and $b$ can be assigned to the inverter legs, and defined as

$$a = \begin{cases} 0 & \text{if SA is OFF and SA' is ON} \\ 1 & \text{if SA is ON and SA' is OFF} \end{cases}$$

$$b = \begin{cases} 0 & \text{if SB is OFF and SB' is ON} \\ 1 & \text{if SB is ON and SB' is OFF} \end{cases}$$

An inverter state is designated as $ab_2$. Four states are possible. The output voltage, $v_o$, of the inverter can be expressed as

$$v_o = V_i(a - b)$$

and the voltage can assume three values only: $V_i$, 0, and $-V_i$, corresponding to state 2, states 0 or 3, and state 1, respectively.

The basic version of the so-called square-wave operation mode of the inverter is described by the following control law:

$$a = \begin{cases} 1 & \text{for } 0 < \omega t \leq \pi \\ 0 & \text{otherwise} \end{cases}$$

$$b = \begin{cases} 1 & \text{for } \pi < \omega t \leq 2\pi \\ 0 & \text{otherwise} \end{cases}$$

where $\omega$ is the fundamental output radian frequency of the inverter. Only states 1 and 2 are used.
States, switching variables, and waveforms of output voltage and current in a single-phase VSI in the basic square-wave mode.

Fig. 7.4
Phase-shift control

• The total harmonic distortion of the output voltage can be minimized by interspersing states 1 and 2 with states 0 and 3 lasting in the $\omega t$ domain 0.81 rad ($46.5^\circ$) each, as shown in Figure 7.5.

• Then, in comparison with the basic square-wave operation, the fundamental output voltage decreases by 8%, to 0.828 V, but the total harmonic distortion is reduced by as much as 40%, to 0.29. The control law yielding the optimal square-wave mode is

  $a = \begin{cases} 
1 & \text{for } \alpha_d < \omega t \leq \pi + \alpha_d \\
0 & \text{otherwise}
\end{cases}$

  $b = \begin{cases} 
1 & \text{for } \pi + \alpha_d < \omega t \leq 2\pi - \alpha_d \\
0 & \text{otherwise}
\end{cases}$

  where $\alpha_d = 0.405$ rad ($23.2^\circ$)

• Fundamental component can be adjusted by using different phase-shifts but of course THD is not then minimized
States, switching variables, and waveforms of output voltage and current in a single-phase VSI in the optimal square-wave mode
Pulse width modulation

• The quality of operation of the inverter can be improved further by pulse width modulation.

• The single-phase inverter has only one output voltage, so the space vector PWM technique is not applicable.

• Here, for illustration purposes, a simple PWM strategy based on a sinusoidal modulating function, \( F(m, \omega t) = m \sin(\omega t) \), is assumed.

• As in all PWM converters, the operating time is a sequence of short switching cycles. Denoting the duty ratios of switching variables \( a \) and \( b \) in the \( n \)th switching cycle by \( d_{an} \) and \( d_{bn} \), the control law is

\[
\begin{align*}
    d_{an} &= \frac{1}{2} [1 + F(m, \alpha_n)] \\
    d_{bn} &= \frac{1}{2} [1 - F(m, \alpha_n)]
\end{align*}
\]

• where \( m \) denotes the modulation index and \( \alpha_n \) is the phase angle of the output voltage in the center of the switching cycle.
Waveforms of output voltage and current in a single-phase VSI in the PWM mode, $N = 10$: (a) $m = 1$, (b) $m = 0.5$

Fig. 7.6
Waveforms of output voltage and current in a single-phase VSI in the PWM mode, $N = 20$: (a) $m = 1$, (b) $m = 0.5$

Fig. 7.7
Harmonic spectra of output voltage in a single-phase VSI: (a) basic square-wave mode, (b) optimal square-wave mode

Harmonic spectra of output voltage in a single-phase VSI in the PWM mode \((m = 1)\): (a) \(N = 10\), (b) \(N = 20\)

Fig. 7.8

Fig. 7.9
Harmonic spectra of output current in a single-phase VSI: (a) basic square-wave mode, (b) optimal square-wave mode

Harmonic spectra of output current in a single-phase VSI in the PWM mode \((m = 1)\):
(a) \(N = 10\), (b) \(N = 20\)

Fig. 7.10

Fig. 7.11
Synthesis of a Sinusoidal Output by PWM

- High frequency triangle (switching frequency \(f_s\)) and sinusoidal modulating function (output frequency \(f_o\)) are compared.
- Bipolar switching.
- Harmonics are in multiples of switching frequency plus/minus multiples of output frequency.
- \(m_a\) is here amplitude of the sinusoidal function and \(m_f = f_s / f_o\).
Details of a Switching Time Period

- Control voltage can be assumed constant during a switching time-period.

**Figure 8-6** Sinusoidal PWM.

\[ V_{Ao} = \frac{v_{control}}{v_{tri}} \cdot \frac{V_d}{2} \]
Input current in a single-phase VSI: (a) optimal square-wave mode, (b) PWM mode \((m = 1, N = 20)\)

- Input current of the inverter or the current from the dc-bus is given by
  \[ i_i = (a - b) \, i_o \]

- As \(a\) and \(b\) has values 0 and 1, current is equal to \(-i_o, 0\) or \(i_o\)

Harmonic spectra: (a) optimal square-wave mode, (b) PWM mode \((m = 1, N = 20)\)

Fig. 7.12

Fig. 7.13
Analysis assuming Fictitious Filters

- Small fictitious filters eliminate the switching-frequency related ripple

**Figure 8-13** Inverter with “fictitious” filters.
DC-Side Current

Figure 8-14  The dc-side current in a single-phase inverter with PWM bipolar voltage switching.

• Bi-Polar Voltage switching
Output Waveforms: Unipolar Voltage Switching

- Instead of one modulating function we are using two, which are complements
- Unipolar output voltage
- Harmonic components around the switching frequency are absent
DC-Side Current in a Single-Phase Inverter

Figure 8-16 The dc-side current in a single-phase inverter with PWM unipolar voltage switching.

• Unipolar voltage switching
Three-phase voltage-source inverter

Fig. 7.14
It is easy to show that in the three-phase inverter the instantaneous line to line output voltages, $v_{AB}$, $v_{BC}$, and $v_{CA}$, are given by

$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = V_i \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

In a balanced three-phase system, the instantaneous line-to-neutral output voltages, $v_{AN}$, $v_{BN}$, and $v_{CN}$, can be expressed as

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix}$$

which yields

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{V_i}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$  

The line-to-line voltages can only assume three values, 0 and $\pm V_i$, while the line-to-neutral voltages can assume five values, 0, $\pm V_i/3$, and $\pm 2V_i/3$.

If the 5 – 4 – 6 – 2 – 3 – 1 – ... sequence of states is imposed, each state lasting one-sixth of the desired period of the output voltage, the individual line-to-line and line-to-neutral voltages acquire waveforms shown in Figure 7.15. This is the square-wave mode of operation, in which each switch of the inverter is turned on and off once within the cycle of output voltage. The peak value, $V_{LL,1,p}$, of the fundamental line-to-line output voltage equals approximately 1.1 $V_i$ and that, $V_{LN,1,p}$, of the line-to-neutral voltage, 0.64 $V_i$. Both voltages have the same total harmonic distortion, $THD$, of 0.31. As in the square-wave single-phase inverter, the magnitude control of the output voltage must be realized on the dc supply side.
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<th>State</th>
<th>$abc$</th>
<th>$v_{AB}/V_i$</th>
<th>$v_{BC}/V_i$</th>
<th>$v_{CA}/V_i$</th>
<th>$v_{AN}/V_i$</th>
<th>$v_{BN}/V_i$</th>
<th>$v_{CN}/V_i$</th>
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<td>0</td>
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<td>0</td>
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</table>
Switching variables and waveforms of output voltages in a three-phase VSI in the square-wave mode

Fig. 7.15

Chapter 7 DC-AC Converters
Waveforms of output voltage (line-to-neutral) and current in a three-phase VSI in the square-wave mode (RL load)

Fig. 7.16
Waveform of input current in a three-phase VSI in the square-wave mode

• Input current or current taken from the dc bus depends on the position of the switches and currents of all three phases

\[ i_i = ai_A + bi_B + ci_C \]

Fig. 7.17
Output current waveform

- **Voltage-sourced inverter, VSI**
  - Inverter is a voltage source => output voltage waveform
  - Output current depends on the impedance of the load
    - Fundamental component and harmonics can be analysed separately in linear circuits, superposition principle

- **Current-sourced inverter, CSI**
  - Output current is defined by the inverter
  - Output voltage depends on load
Equivalent circuit of an induction machine

- $f = \text{supply frequency}$
- $\omega = 2\pi f \text{ angular frequency}$
- $\omega_m = \text{rotor mechanical speed}$
- $\omega_M = p \omega_m \text{ rotor electrical speed}$
- $\omega_R = \omega - \omega_M = s \omega \text{ angular frequency of induced currents in the rotor}$
- $p = \text{number of poles}$
- $s = \text{slip}$
High frequency impedance

- Harmonic n of voltage, angular frequency $\omega_n$ and $\omega_{Rn}$
  \[ \omega_n = n\omega \]
  \[ \omega_{Rn} = n\omega \pm (\omega - \omega_R) = (n \pm 1)\omega \approx (n \pm 1)\omega \]
  - Plus sign e.g. with 5th harmonic
  - Minus sign e.g. with 7th harmonic

- Magnetising reactance
  - High when compared to rotor resistance and reactance
  - Can be neglected

- High frequency impedance is
  \[ Z_n = R_s + R_R \frac{n}{n \pm 1} + jn\omega (L_{\sigma S} + L_{\sigma R}) \approx jn\omega (L_{\sigma S} + L_{\sigma R}) \]
5th harmonic

- Fundamental, phases A, B, C
- At 5th harmonic phases, A, C, B, order reversed
- => 5th harmonic system rotates in reverse direction
7th harmonic

- Fundamental, phases A, B, C
- At 7th harmonic phases, A, B, C, same as fundamental
- => 7th harmonic system rotates in the same direction
Harmonic current

- When induction machine is at stand still connected to nominal supply $U_N$, starting current $I_s$ is

$$Z_k = \frac{U_N}{I_s} \approx j\omega_N (L_{\sigma S} + L_{\sigma R})$$

- High frequency impedance can be estimated with starting impedance

$$Z_n \approx n Z_k \frac{\omega}{\omega_N}$$

- Harmonic current

$$I_n = \frac{U_n}{Z_n} = \frac{U_n}{n} \frac{I_s}{U_N} \frac{\omega_N}{\omega} = \frac{U_n}{\omega} \frac{I_s}{n}$$

- $(\omega/\omega_N)$ $U_N$ is the wanted output voltage $U_1$ at frequency $\omega$

- Harmonic current when compared to the starting current

$$\frac{I_n}{I_s} = \frac{U_n}{n U_1}$$
Example

- 5th voltage harmonic is 10%
  - $I_5 = 0,1/5 \times I_s = 0,02 \times I_s$
  - Starting current of the motor is $I_s = 5\ldots7 \times I_N$
  - $I_5 = 0,1\ldots0,14 \times I_N$ fifth harmonic is therefore larger than the corresponding voltage harmonic

- Harmonic currents are the higher
  - the higher the starting current of the motor is
  - i.e. the smaller leakage inductances are

- Motors with higher power
  - Leakage inductances are getting smaller
  - Harmonic currents are higher than in smaller machines
Current harmonics in CSI

- Output current is 120° wide positive and negative pulses separated by 60° zero
  - Harmonics $I_n = \frac{I_1}{n}, \quad n = 6k \pm 1, \quad k = 1, 2,\ldots$
  - Harmonics depend on the fundamental component, not on the structure (impedances) of the machine

- VSI
  - Current harmonics depend on motor leakage inductance and not on the motor load
Current in induction machine

- a) output voltage and fundamental component
- b) harmonics of the voltage
- c) harmonics of the current
  - integral of voltage harmonics
- d) current at no load
- e) current at load

\[ i_0 = \frac{\int (u - u_1)dt}{L_{\sigma S} + L_{\sigma R}} = \int \sum_{n=2}^{\infty} u_n(t)dt \]

\[ i_0 = \frac{1}{L_{\sigma S} + L_{\sigma R}} \int (u - u_1)dt \]
Side effects of harmonics

- In the inverter
  - Peak values of current are higher, higher current rating for the power semiconductor devices needed

- Motor
  - RMS value of current includes harmonics
  - Fundamental current component is reduced and torque production is lower
  - More losses, magnetising and winding losses
  - Motor rating must be higher than with sinusoidal current
Torque harmonics

- Torque harmonics caused by current harmonics are often small but not in six step (square wave) operation.
- Fundamental component of airgap flux and current harmonics.
  - Torque harmonics
  - 5th and 7th current harmonic are producing 6th torque harmonic.
- Small inertia
  - Angular speed starts to change, oscillate.
- Also mechanical resonances possible.
Switching variables and waveforms of output voltages in a three-phase VSI in the PWM mode

Fig. 7.18
Waveforms of output voltage and current in an RL load of a three-phase VSI in the PWM mode:
(a) load angle of $30^\circ$, (b) load angle $60^\circ$

Fig. 7.19

Input current: (a) load angle of $30^\circ$, (b) load angle $60^\circ$

Fig. 7.20
The principle of the so-called carrier-comparison method is illustrated in Figure 7.21. Reference waveforms, $r_A$, $r_B$, and $r_C$, given by

$$r_A(\omega t) = F(m, \omega t)$$
$$r_B(\omega t) = F\left(m, \omega t - \frac{2}{3}\pi\right)$$
$$r_C(\omega t) = F\left(m, \omega t - \frac{4}{3}\pi\right)$$

where $F(m, \omega t)$ denotes the modulating function employed, are compared with a unity-amplitude triangular waveform $y$. Values of the switching variables, $a$, $b$, and $c$, change from 0 to 1 and from 1 to 0 at every sequential intersection of the carrier and respective reference waveforms.

The sinusoidal modulating function, $F(m, \omega t) = m \sin(\omega t)$, is simple, but the voltage gain of the inverter can significantly be increased using a non-sinusoidal modulating function, many of which were developed over the years. All those functions consist of a fundamental and triple harmonics, which are not reflected in the three-phase output voltages and currents of the inverter. The popular *third-harmonic modulating function*, shown in Figure 7.22 with $m = 1$, is given by

$$F(m, \omega t) = \frac{2}{\sqrt{3}} m \left[ \sin(\omega t) + \frac{1}{6} \sin(3\omega t) \right]$$

having thus only the fundamental and third harmonic. At $m = 1$, the fundamental equals $2/\sqrt{3} \approx 1.15$, which represents a 15% increase in the voltage gain at no changes to the inverter.
Carrier-comparison PWM technique \((N = 12, \ m = 0.75)\)

Fig. 7.21
Overmodulation

- When $m_a < 1$, linea area
  - Harmonics around multiples of switching frequency + output frequency
  - Output voltage is not reaching its maximum

- Overmodulation, $m_a > 1$
  - Also lower frequency harmonics
  - Output voltage depends also on frequency ratio of output/switching, $m_f$
Effect of switching frequency

- Same amplitude of reference but with lower fs
- Only one pulse/half cycle
  - Fundamental of output is higher
  - Harmonics at lower frequencies and amplitudes higher

\[ m_a = 1.5 \quad m_f = \frac{f_o}{fs} = 5 \]

\[ m_a = 1.5 \quad m_f = 15 \]
Adding third harmonic

- No effect in load
  - Eliminated in line-to-line voltages
  - In wye or delta connection loads have no path for zero-sequence current
- Amplitude of third harmonic can be 16.7 %
  - Flattens the sinusoid at its peak
  - Modulator is not saturating
- If added amplitude were higher than 16.7 % the result would be higher than 1 at \( \pi/3 \)
Modulating function when 16.7% third harmonic added
$m_a = 1,167 \quad m_f = 15$

- Adding third harmonic prevents pulses from merging together
- Modulator is linear in 0 - 1,167

No third harmonic added

Added third harmonics is 16,7 %
Third-harmonic modulating function and its components at $m = 1$
Digital modulation

- Previous methods were based on analog technologies
  - Ideal sinusoid easy to realise
  - Often called **natural sampling**
- Digital electronics
  - Everything is replaced by sampled signals
  - Symmetrical sampling (also called uniform, regular)
  - Asymmetrical sampling
Natural sampling

- Comparison is done on both edges of the pulse, leading and trailing
- Result contains much information
Symmetrical sampling

Modulation function
Sampled modulating

Carrier

Fundamental of output
Symmetrical sampling

- Pulse width depends on equal distance samples
  - => symmetric
- Distance between pulse centers is constant
- Pulse width is calculated from

\[ t_p = \frac{T_c}{2} \{1 + m_a \sin \omega_m t_1\} \]
Comparison

- Symmetric
  - Pulse edges modulated similarly

- Natural
  - Pulses are not symmetric around the center points
Asymmetrical sampling

- Sampling at $2^* f_k$
- Every pulse edge is modulated separately
- Contains more information than symmetric sampling
  - Harmonics are reduced
- Pulse width is calculated from

$$t_p = \frac{T}{2} \left\{ 1 + \frac{m_a}{2} \left( \sin \omega_m t_1 + \sin \omega_m t_3 \right) \right\}$$
Sampling and harmonics

- Modulating function is replaced by a sampled waveform
  - Harmonics of output voltage are changed when compared to natural sampling
  - Fundamental component is not any more equal to the amplitude of modulating function
Space-vector modulation
The voltage space vector plane with the reference vector $\vec{v}^*$ of line-to-neutral voltages is shown in Figure 7.23(a) and repeated in Figure 7.23(b) in the per-unit format, with the maximum available magnitude of that vector as the base. The modulation index, $m$, constitutes the magnitude of per-unit $\vec{v}^*$. Neglecting the voltage drops in the inverter, the highest available peak value of the output line-to-line voltage equals the dc input voltage, $V_i$. Thus

$$m = \frac{V_{LL,p}^*}{V_i}$$

where $V_{LL,p}^*$ denotes the reference peak line-to-line voltage. With $m = 1$ the maximum available peak value of the fundamental line-to-line output voltage equals the dc input voltage.

In the steady-state, when the fundamental output voltage and current maintain fixed magnitude and frequency, $m$ is constant and $\vec{v}^*$ rotates with a constant speed. However, the space vector PWM technique allows synthesis of an instantaneous voltage vector, which may change in magnitude and speed from one switching cycle to another.

The angular position, $\beta$, of the reference vector allows determination of the sector of the complex plane in which the vector is located within the given sampling cycle of the digital modulator. Specifically,

$$S = \text{int} \left( \frac{3}{\pi} \beta \right) + 1$$

where $\beta$ is expressed in radians and $S$ is the sector number (I to VI). The in-sector position, $\alpha$, of $\vec{v}^*$ is then given by

$$\alpha = \beta - \frac{\pi}{3} (S - 1)$$
Voltage space vector plane of a three-phase VSI: (a) in volts, (b) per unit

Fig. 7.23

Chapter 7 DC-AC Converters
Control of the voltage

- Induction machine is a typical load for VSIs
  - Inductances are smoothening current
- Voltage is caused by a changing flux
  \[ u_s = \frac{d\psi_s}{dt} \]
- Flux is an integral of voltage
  \[ \psi_s = \int_0^t u_s \, dt + \psi_{s0} \]
- Resistances assumed to be small
- Describes the air gap flux of an induction machine
- More detailed models needed in accurate control, discussed more in courses on electric drives
Requirements

- Ideal voltage vector causes flux in VSI
- Only six non-zero voltage vectors
- Nevertheless, output voltage integral should be similar to ideal
- This is true
  - When flux has constant amplitude
  - Rotates smoothly with the wanted angular frequency

\[ \psi_s = \frac{\hat{u}_s}{\omega_1} e^{j(\omega t - \pi/2)} \]
Flux vector

- Circle is created by sinusoidal voltage
- VSI
  - Integral stops when zero vector is used
  - Non-zero vectors move flux with constant speed in the direction of the voltage vector

Method where two space-vectors separated by zero vector is used

Flux generated by sinusoidal voltage

Method where one space-vector separated by zero vector is used
SVM, Space vector modulator

- Every 60° wide sector is divided either to
  - Constant angle slices $\Delta \alpha$
  - Or constant duration time segments $\Delta T$
- Flux change is same with ideal voltage or with VSI

$$\Delta \psi_s = \int_0^{\Delta T} u_s(\alpha) \, dt = \int_0^{\Delta T} \hat{u}_se^{j\omega t} \, dt$$
The revolving reference voltage vector is synthesized from stationary active (non-zero) vectors, $\vec{V}_x$ and $\vec{V}_y$, framing the sector in question, and a zero vector, $\vec{V}_0$ or $\vec{V}_7$. Durations, $T_X$, $T_Y$, and $T_Z$, of states generating those vectors are given by:

\[
T_X = mT_{sw} \sin(60^\circ - \alpha) \\
T_Y = mT_{sw} \sin(\alpha) \\
T_Z = T_{sw} - T_X - T_Y.
\]

Times $T_X$, $T_Y$, and $T_Z$ indicate only how long a given state should last in the given switching cycle, but how the cycle is divided between the employed states must also be specified. The two most commonly used state sequences can be called a high-quality sequence and a high-efficiency sequence. The high-quality sequence is

$X - Y - Z_1 - Y - X - Z_2 \ldots$

where each state in the sequence lasts half of the allotted time. States $Z_1$ and $Z_2$, complementarily 0 and 7, are placed in such an order that a transition from one state to another involves switching in one inverter leg only. The number of commutations can further be reduced, at the expense of slightly increased distortion of output currents, when the high-efficiency state sequence

$X - Y - Z - Y - X \ldots$

is employed. Now, states $X$ and $Y$ last $T_X/2$ and $T_Y/2$ seconds respectively, and state $Z$ lasts $T_Z$ seconds. Moreover, $Z = 0$ in the even sectors (II, IV, and VI) and $Z = 7$ in the odd sectors (I, III, and V).

With this state sequence, the average number of pulses of a switching variable per cycle of output voltage is $2N/3 + 1 \approx 2N/3$. As a result, the switching losses decrease by about 30% in comparison with the high-quality state sequence,
Example high-quality space sequence

Fig. 7.24
Example high-efficiency space sequence

Fig. 7.25
Switching pattern with half- and quarter-wave symmetries

Fig. 7.26
Optimal primary switching angles as functions of the magnitude control ratio \((K = 5)\)

Harmonic spectrum of line-to-neutral voltage with the harmonic-elimination technique \((K = 5, M = 1)\)

Fig. 7.27

Fig. 7.28
Switching patterns and voltage and current waveforms:
(1) carrier-comparison PWM with sinusoidal reference,
(2) space vector PWM with high-efficiency state sequence,
(3) programmed PWM with harmonic elimination

Fig. 7.29
Waveforms of output current in a three-phase VSI: (a) regular PWM, (b) random PWM

Frequency spectra of the line-to-neutral output voltage in a three-phase VSI: (a) regular PWM, (b) random PWM

Fig. 7.30

Fig. 7.31
Comparison of random PWM techniques with the regular PWM

**Fig. 7.32**
Hysteresis current control scheme

Fig. 7.33

Chapter 7 DC-AC Converters
Characteristic of the hysteresis current controller

![Diagram of hysteresis current controller]

Fig. 7.34
Waveforms of output currents in a VSI with hysteresis current control:
(a) 20% tolerance, (b) 10% tolerance

Fig. 7.35
Waveform of output currents in a VSI with hysteresis current control at a rapid change in the magnitude, frequency, and phase of the reference current

Fig. 7.36
Space vector version of the hysteresis current control scheme

Best control effects are obtained when state 0 or 7 is imposed for \((z_d, z_q) = (0, 0)\), state 1 for \((0, 1)\) and \((1, 1)\), state 2 for \((1, -1)\), state 3 for \((1,0)\), state 4 for \((-1, 0)\), state 5 for \((-1, 1)\), and state 6 for \((-1, -1)\) and \((0, -1)\).
Characteristic of a current controller for the space vector version of the hysteresis current control scheme

Fig. 7.38
Ramp-comparison scheme for current-controlled VSI

Fig. 7.39
Waveforms of the output current in a VSI with the ramp comparison current control:
(a) $f_r/f_1 = 10$, (b) $f_r/f_1 = 20$

Fig. 7.40
Current-regulated delta modulation scheme for a current-controlled VSI

Fig. 7.41
Linear current control scheme for a VSI

Fig. 7.42
Control methods of VSI supplied induction machines

- Modulator
  - PWM based eg. on comparisons or space vectors
- Reference comes from outer motor control
  - Scalar control
  - Vector control
- Direct Torque Control
  - Combines both motor control and modulator
Scalar control

- Motor is controlled by changing supply frequency
- Voltage is increased simultaneously to keep flux constant
  - Modulator gets both voltage amplitude and frequency reference (ohje in the figure below)
- Motor current depends on the load (slip of the machine)
Vector control

- More accurate motor model
- Controls separately flux and torque of the machine
- Modulator receives reference (ohje) to amplitude and frequency of voltage

![Diagram of Vector control system]
Direct Torque Control, DTC

• Flux vector and torque are controlled simultaneously with hysteresis control
• Suitable voltage vector changing flux and torque in correct way is selected => separate modulator is not needed
Flux control

- Complex plane is divided to six sectors
  - 0-5, dashed lines in the right hand figure
- Two voltage vectors are used and they depend on the direction of rotation
  - One vector to increase
  - One vector to decrease flux
Torque control

- Induction machine torque can be expressed as

\[ T = -\frac{3}{2} p \psi_S \times i_S = \frac{3}{2} p \frac{1 - \sigma}{\sigma L_m} \psi_R \times \psi_S = \frac{3}{2} p \frac{1 - \sigma}{\sigma L_m} \psi_R \psi_S \sin \gamma \]

- If angle between rotor flux
  - Increases torque increases
  - Decreases torque decreases
Optimal selection table

- Space vector is selected based on the sector $\theta$ of flux
- Torque
  - $\tau = -1$, too large
  - $\tau = 1$, too small
  - $\tau = 0$, ok, use zero vector
- Flux
  - $\phi = 0$, decrease flux
  - $\phi = 1$, increase flux

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<th>2</th>
<th>3</th>
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<tr>
<td>$\tau = 1, \phi = 1$</td>
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Block diagram of DTC
CSI, Current Source Inverter
Current-source inverter supplied from a controlled rectifier

Fig. 7.43
Three-phase current-source inverter

Note:
Symmetrical power semiconductors like punch-through IGBTs don’t require series connected diodes

Fig. 7.44
Switching variables in a three-phase CSI in the square-wave mode

Fig. 7.45
Idealized waveforms of output currents in a three-phase CSI in the square-wave mode

Wye-connected load

\[
\begin{bmatrix}
i_A \\
i_B \\
i_C
\end{bmatrix} = \begin{bmatrix}
a & -d' \\
b & -b' \\
c & -c'
\end{bmatrix} I_1
\]

Delta-connected load

\[
\begin{bmatrix}
i_{AB} \\
i_{BC} \\
i_{CA}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix} \begin{bmatrix}
i_A \\
i_B \\
i_C
\end{bmatrix}
\]
Waveforms of output voltage and current in a three-phase CSI in the square-wave mode:

- Load current is given by the CSI
- In an inductive load current cannot change instantaneously => induced voltage spike in voltage waveforms

Fig. 7.47
Three-phase PWM current-source inverter

Fig. 7.48
Carrier-comparison method for the PWM CSI

Fig. 7.49
Optimal switching pattern for the PWM CSI with two primary switching angles

Fig. 7.50
Waveforms of the output current, capacitor current, and output voltage in a three-phase PWM CSI (wye-connected RL load, $P = 9$)

Fig. 7.51

Chapter 7 DC-AC Converters
Generic five-level inverter

Fig. 7.52
Half-bridge voltage-source inverter

Fig. 7.53
Three-level neutral-clamped inverter

Fig. 7.54

Chapter 7 DC-AC Converters
In practice only three states of a leg are used, which makes for the total of twenty-seven states of the three-level inverter. The limiting condition is that two and only two adjacent switches must be ON at any time. A ternary switching variable can thus be assigned to each inverter phase and, for phase A, defined as

\[
a = \begin{cases} 
0 & \text{if } S3 \& S4 \text{ are ON} \\
1 & \text{if } S2 \& S3 \text{ are ON} \\
2 & \text{if } S1 \& S2 \text{ are ON}
\end{cases}
\]

Switching variables \( b \) and \( c \) for the other two phases are defined analogously. It is easy to see that the potential a given output terminal of the inverter with respect to the "ground" (inverter's neutral), \( G \), can be expressed in terms of the associated switching variable and input voltage. For instance, the voltage, \( v_A \), at terminal A is

\[
v_A = \frac{a - 1}{2} V_i.
\]

Consequently, the output line-to-line voltages are given by

\[
\begin{bmatrix}
v_{AB} \\
v_{BC} \\
v_{CA}
\end{bmatrix} = \frac{V_i}{2} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}
\]

and the line-to-neutral voltages by

\[
\begin{bmatrix}
v_{AN} \\
v_{BN} \\
v_{CN}
\end{bmatrix} = \frac{V_i}{6} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}
\]

Listing all possible values of the line-to-line and line-to-neutral voltages it can be seen that they can assume five and nine values, respectively. Generally, these numbers in an \( l \)-level inverter are \( 2l - 1 \) and \( 4l - 3 \).
Voltage space vectors of a three-level neutral-clamped inverter

Fig. 7.55
States, switching variables, and waveforms of output voltages in a three-level neutral-clamped inverter in the square-wave mode

Fig. 7.56
Waveforms of output voltage and current in a three-level neutral-clamped inverter in the square-wave mode.
Three-level flying-capacitor inverter

\[
a = \begin{cases} 
0 & \text{if S3&S4 are ON} \\
1 & \text{if S1&S3 or S2&S4 are ON} \\
2 & \text{if S1&S2 are ON}
\end{cases}
\]

Fig. 7.58

Chapter 7 DC-AC Converters
Cascaded H-bridge inverter:
(a) block diagram, (b) constituent bridge

Fig. 7.59
An H-bridge can generate three voltage levels between its output terminals, namely $-V_{dc}$, 0, and $V_{dc}$. The number $N$ of H-bridges in an $l$-level inverter is $(l - 1)/2$, thus $N$ bridges form an inverter with $2N + 1$ levels. In the constituent H-bridge two and only switches can be ON at any time. For the $k$-th bridge in a leg of a three-phase inverter, the ternary switching variable, $a_k$, is defined as follows:

$$
a_k = \begin{cases} 
0 & \text{if } S2\&S3 \text{ are ON} \\
1 & \text{if } S1\&S3 \text{ or } S2\&S4 \text{ are ON} \\
2 & \text{if } S1\&S4 \text{ are ON}.
\end{cases}
$$

The output voltage, $v_{o,k}$, of the bridge is then given by

$$
v_{o,k} = (a_k - 1)V_{dc}.
$$

The voltage of terminal A, $v_A$, with respect to the inverter’s neutral, G, is a sum of output voltages of all the bridges. Consequently, a switching variable of phase A of the inverter can be defined as

$$
a = \sum_{k=1}^{N} a_k
$$

where, depending on control of the individual bridges, $a$ can assume any integer value from the 0 to $2N$ range. Then,

$$
v_A = (a - N)V_{dc}.
$$
Switching variables $b$ and $c$ are defined analogously, yielding the following equations for the line-to-line and line-to-neutral output voltages of the inverter:

\[
\begin{align*}
\begin{bmatrix}
v_{AB} \\
v_{BC} \\
v_{CA}
\end{bmatrix} &= V_{dc} \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\end{align*}
\]

and

\[
\begin{align*}
\begin{bmatrix}
v_{AN} \\
v_{BN} \\
v_{CN}
\end{bmatrix} &= \frac{V_{dc}}{3} \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\end{align*}
\]
Approximation of a sinewave by a stepped waveform in the H-bridge cascaded inverter

Fig. 7.60
Endpoints of line-to-neutral vectors of two-bridge cascaded inverter

Fig. 7.61
Cells with diode rectifiers of two-bridge cascaded inverter:
(a) single-phase, (b) three-phase

Fig. 7.62

Chapter 7 DC-AC Converters
Cells with PWM rectifiers for ac-supplied cascaded inverter: (a) single-phase, (b) three-phase

Fig. 7.63
7.4 Soft-switching inverters

Note: additional reading, not required in the exam

Switched network for illustration of the operating principle of a resonant dc link

Fig. 7.64
Waveforms of voltage and current in the resonant dc link

Fig. 7.65
Three-phase resonant dc link inverter with an active clamp

Fig. 7.66
Waveforms of line-to-line output voltages in a resonant dc-link inverter

Fig. 7.67
Auxiliary resonant commutated pole inverter: (a) one phase with the auxiliary circuit, (b) the entire inverter

Fig. 7.68

Chapter 7 DC-AC Converters
Idealized line-to-neutral voltage and line current waveforms in a VSI in the square-wave mode

Fig. 7.69
Block diagram of a photovoltaic utility interface

Fig. 7.70
Block diagram of an active power filter

Fig. 7.71

Chapter 7 DC-AC Converters
Waveforms of voltage and current in an active power filter

\[ u_{AN} \]

\[ \omega t \]

\[ i'_A \]

\[ \omega t \]

\[ i_A \]

\[ \omega t \]

\[ i''_A \]

\[ \omega t \]

Fig. 7.72
UPS System

Chapter 7 DC-AC Converters

Fig. 7.73
Block diagram of an ac drive system with scalar speed control

Fig. 7.74
Use of the modular frequency changer of Figure 2.24 in an ac drive: (a) system with a braking resistor, (b) system with a step-up chopper

Fig. 7.75

Chapter 7 DC-AC Converters
PWM rectifier-inverter cascades for bidirectional power flow in ac motor drives: (a) current-type rectifier, inductive dc link, and current-source inverter, (b) voltage-type-rectifier, capacitive dc link, and voltage-source inverter

Fig. 7.76