Aalto University
School of Electrical
Engineering

Lecture 1: Induction Motor

ELEC-E8402 Control of Electric Drives and Power Converters

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Learning Outcomes

After this lecture and exercises you will be able to:

- Transform the T-model parameters to the inverse-Γ-model parameters (no need to remember the scaling equations, however)
- Express the dynamic inverse-Γ model in stator coordinates

Later in this course, only the inverse-Γ model will be used.

Outline

3-Phase Induction Motor: Structure

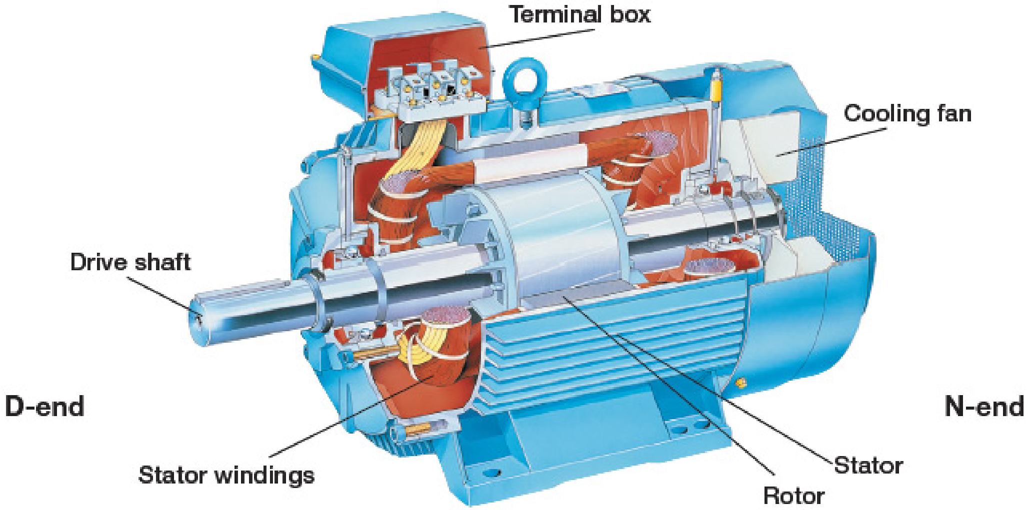
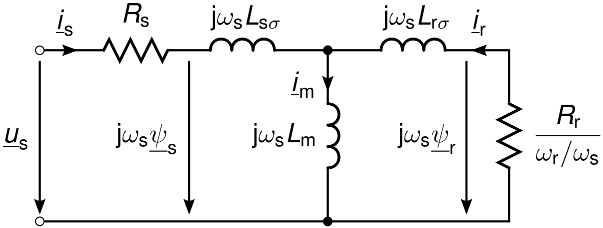


Figure: http://www.ctiautomation.net/About-Motors.htm. See also: http://www.youtube.com/watch?v=N8LUOTQKXlk

Steady-State T-Equivalent Circuit

- Rotor angular speed ω_m (in electrical rad/s)
- Supply angular frequency ω_s
- Slip angular frequency

$$\omega_r = \omega_s - \omega_m$$



3-Phase Induction Motor

- Most common motor in industrial applications
- Advantages
  - Robust, inexpensive, durable
  - Can be started by direct connection to the mains (unlike other standard AC motors)
  - Low-performance control with a frequency converter is very simple (open-loop scalar control aka volts-per-hertz control)
- Disadvantages
  - High-performance control is complicated
  - Less efficient than synchronous machines (but more efficient than DC machines)

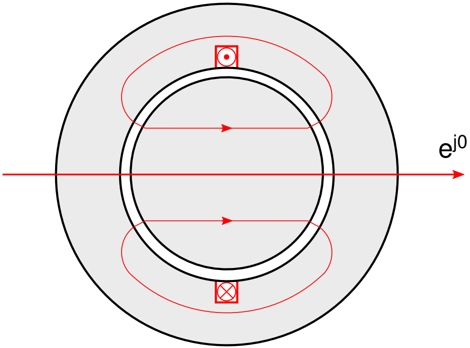
Operating Principle

- Magnetised from the stator
- If the motor rotates synchronously, no currents are induced in the rotor
- Once the motor is loaded mechanically, the rotor starts lagging behind the synchronous rotation of the flux
- Currents are induced in the short-circuited rotor winding
- Torque counteracting the lagging effect is produced

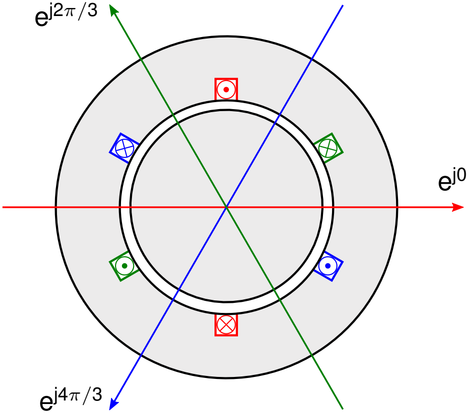
Animation: https://en.wikipedia.org/wiki/Induction_motor#/media/File:Asynchronmotor_animation.gif

Outline

Magnetic Axes in the Complex Plane



Phase a



All 3 phases

Windings are sinusoidally distributed along the air gap

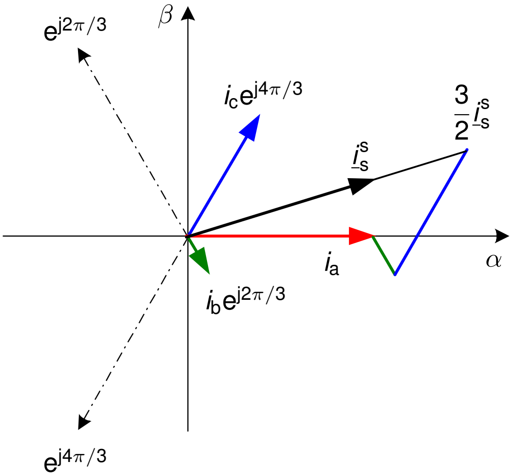
Space-Vector Transformation

- Stator current will be used as an example
- Space vector is a complex variable (signal)

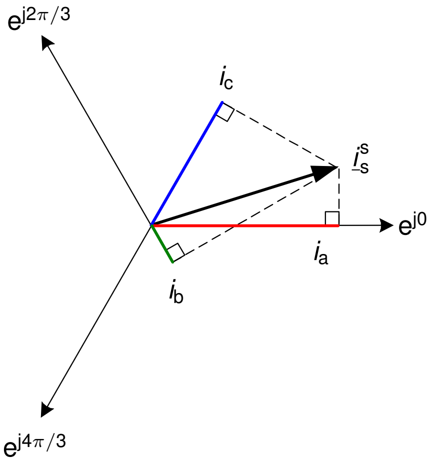
$$\boxed{\underline{i}_\mathrm{s}^\mathrm{s} = \frac{2}{3}\left(i_\mathrm{a} + i_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + i_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right)}$$

  where i_a, i_b, and i_c are arbitrarily varying instantaneous phase variables
- Superscript s marks stator coordinates
- Same transformation applies for voltages and flux linkages
- Space vector does not include the zero-sequence component (not a problem since the stator winding is delta-connected or the star point is not connected)

Peak-value scaling of space vectors will be used in this course. Furthermore, we will use the subscript s to refer to stator quantities, e.g., the stator current vector $\underline{i}_\mathrm{s}$ and the stator voltage vector $\underline{u}_\mathrm{s}$. The subscript r refers to rotor quantities.



$$\underline{i}_s^s = \frac{2}{3}\left(i_a + i_b e^{j2\pi/3} + i_c e^{j4\pi/3}\right)$$



$$i_a = \mathrm{Re}\left\{\underline{i}_s^s\right\}$$

$$i_b = \mathrm{Re}\left\{e^{-j2\pi/3}\underline{i}_s^s\right\}$$

$$i_c = \mathrm{Re}\left\{e^{-j4\pi/3}\underline{i}_s^s\right\}$$

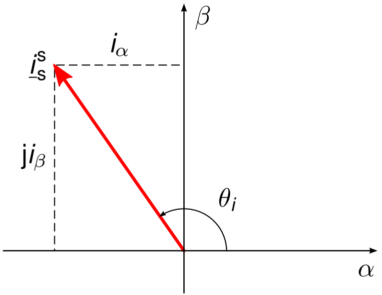
Representation in Component and Polar Forms

- Component form

$$\underline{i}_s^s = i_\alpha + \mathrm{j} i_\beta$$

- Polar form

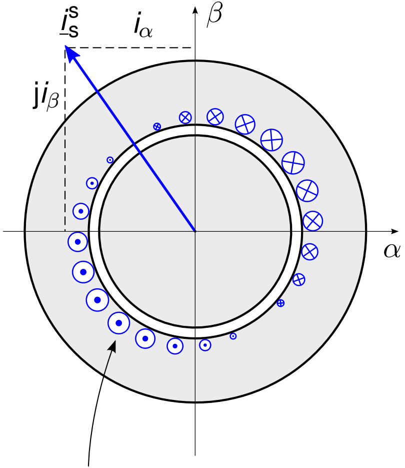
$$\begin{aligned}\underline{i}_s^s &= i_s e^{\mathrm{j}\theta_i} \\ &= \underbrace{i_s \cos(\theta_i)}_{i_\alpha} + \mathrm{j}\, \underbrace{i_s \sin(\theta_i)}_{i_\beta}\end{aligned}$$



- Generally, both the magnitude i_s and the angle θ_i may vary arbitrarily in time
- Positive sequence in steady state: $i_s = \sqrt{2}I$ is constant and $\theta_i = \omega_m t + \phi$

Physical Interpretation: Sinusoidal Distribution in Space

- 3-phase winding creates the current and the mmf, which are sinusoidally distributed along the air gap
- Space vector represents the instantaneous magnitude and angle of the sinusoidal distribution in space
- Magnitude and the angle can vary freely in time



Rotating current distribution produced by the 3-phase stator winding

Outline

Stator Winding

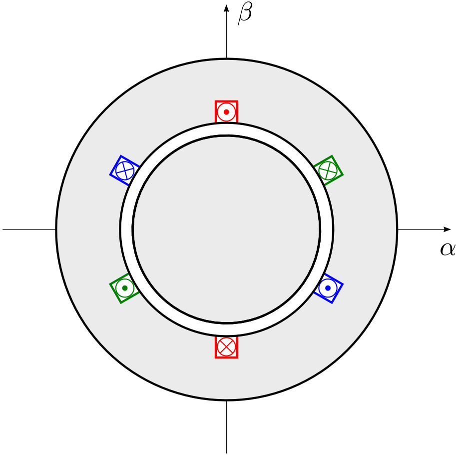
- Stator phase voltages

$$u_a = R_s i_a + \frac{d\psi_a}{dt}$$
$$u_b = R_s i_b + \frac{d\psi_b}{dt}$$
$$u_c = R_s i_c + \frac{d\psi_c}{dt}$$

- Corresponding stator voltage space vector

$$\underline{u}_s^s = R_s \underline{i}_s^s + \frac{d\underline{\psi}_s^s}{dt}$$

- In the following, space vector equations will be directly given



Rotor Winding

- Short-circuited rotor winding is modelled similarly as the stator winding

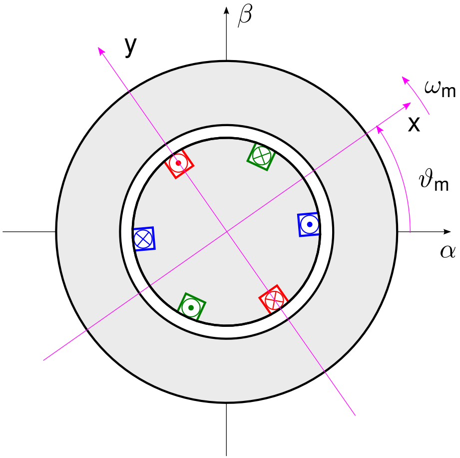
$$\underline{u}_r^r = R_r \underline{i}_r^r + \frac{d\underline{\psi}_r^r}{dt} = 0$$

- Superscript r refers to rotor coordinates
- Coordinate transformations

$$\underline{i}_r^r = e^{-j\vartheta_m} \underline{i}_r^s \qquad \underline{\psi}_r^r = e^{-j\vartheta_m} \underline{\psi}_r^s$$

lead to

$$0 = R_r \underline{i}_r^s + \frac{d\underline{\psi}_r^s}{dt} - j\omega_m \underline{\psi}_r^s$$



$$\omega_m = \frac{d\vartheta_m}{dt}$$

Flux Linkages

- Stator flux linkage

$$\underline{\psi}_{\mathrm{s}}^{\mathrm{s}} = L_{\mathrm{s}\sigma}\underline{i}_{\mathrm{s}}^{\mathrm{s}} + L_{\mathrm{m}}\underline{i}_{\mathrm{m}}^{\mathrm{s}}$$

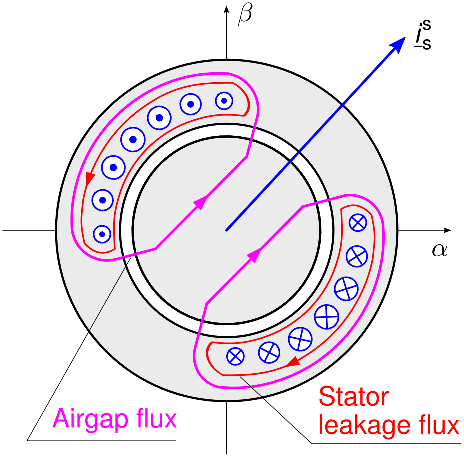
- Rotor flux linkage

$$\underline{\psi}_{\mathrm{r}}^{\mathrm{s}} = L_{\mathrm{r}\sigma}\underline{i}_{\mathrm{r}}^{\mathrm{s}} + L_{\mathrm{m}}\underline{i}_{\mathrm{m}}^{\mathrm{s}}$$

- Magnetising current

$$\underline{i}_{\mathrm{m}}^{\mathrm{s}} = \underline{i}_{\mathrm{s}}^{\mathrm{s}} + \underline{i}_{\mathrm{r}}^{\mathrm{s}}$$

- Figure illustrates the currents and flux paths in a no-load condition (where $\underline{i}_{\mathrm{r}}^{\mathrm{s}} = 0$)



Dynamic T-Model

- Flux linkages

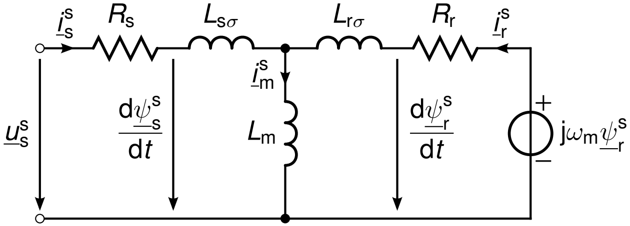
$$\underline{\psi}_s^s = L_s \underline{i}_s^s + L_m \underline{i}_r^s$$
$$\underline{\psi}_r^s = L_m \underline{i}_s^s + L_r \underline{i}_r^s$$

- Stator inductance

$$L_s = L_{s\sigma} + L_m$$

- Rotor inductance

$$L_r = L_{r\sigma} + L_m$$



Outline

Γ Model and Inverse-Γ Model

- T model can be transformed into simpler models with no loss of information
- Rotor variables are scaled

$$\underline{i}_\mathrm{r}^\mathrm{s} = \gamma \underline{i}_\mathrm{R}^\mathrm{s} \qquad \underline{\psi}_\mathrm{r}^\mathrm{s} = \underline{\psi}_\mathrm{R}^\mathrm{s}/\gamma$$

- Scaling factor γ can be chosen arbitrarily
- Scaled rotor voltage equation

$$0 = (\gamma^2 R_\mathrm{r})\underline{i}_\mathrm{R}^\mathrm{s} + \frac{\mathrm{d}\underline{\psi}_\mathrm{R}^\mathrm{s}}{\mathrm{d}t} - \mathrm{j}\omega_\mathrm{m}\underline{\psi}_\mathrm{R}^\mathrm{s}$$

- Stator voltage equation is not affected

Γ Model and Inverse-Γ Model

- Scaled flux equations

$$\underline{\psi}_{\mathrm{s}}^{\mathrm{s}} = L_{\mathrm{s}}\underline{i}_{\mathrm{s}}^{\mathrm{s}} + (\gamma L_{\mathrm{m}})\underline{i}_{\mathrm{R}}^{\mathrm{s}}$$
$$\underline{\psi}_{\mathrm{R}}^{\mathrm{s}} = (\gamma L_{\mathrm{m}})\underline{i}_{\mathrm{s}}^{\mathrm{s}} + (\gamma^2 L_{\mathrm{r}})\underline{i}_{\mathrm{R}}^{\mathrm{s}}$$

- Two feasible selections for γ

$$\gamma = L_{\mathrm{s}}/L_{\mathrm{m}} \quad \Rightarrow \quad \Gamma \text{ model}$$
$$\gamma = L_{\mathrm{m}}/L_{\mathrm{r}} \quad \Rightarrow \quad \text{inverse-}\Gamma \text{ model}$$

- Both these selections decrease the number of model parameters from 5 to 4
- Inverse-Γ model is convenient for control purposes

For details about model transformations, see: G. R. Slemon, “Modelling of induction machines for electric drives,” *IEEE Trans. Ind. Applicat.*, vol. 25, no. 6, 1989.

Dynamic Inverse-Γ Model

- Scaled parameters

$$L_\mathrm{M} = \gamma L_\mathrm{m}$$
$$L_\sigma = L_{\mathrm{s}\sigma} + \gamma L_{\mathrm{r}\sigma}$$
$$R_\mathrm{R} = \gamma^2 R_\mathrm{r}$$

where $\gamma = L_\mathrm{m}/L_\mathrm{r}$

- Flux linkages

$$\underline{\psi}_\mathrm{s}^\mathrm{s} = L_\sigma \underline{i}_\mathrm{s}^\mathrm{s} + \underline{\psi}_\mathrm{R}^\mathrm{s}$$
$$\underline{\psi}_\mathrm{R}^\mathrm{s} = L_\mathrm{M}(\underline{i}_\mathrm{s}^\mathrm{s} + \underline{i}_\mathrm{R}^\mathrm{s})$$