



**Aalto University**  
School of Electrical  
Engineering

# Lecture 3: Vector-Controlled Induction Motor Drive

ELEC-E8402 Control of Electric Drives and Power Converters

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# Learning Outcomes

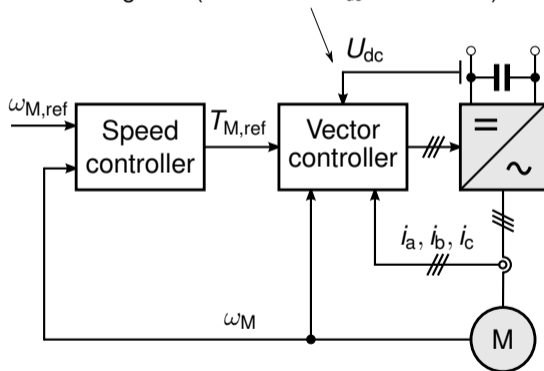
After this lecture and exercises you will be able to:

- ▶ Explain the principle of rotor-flux orientation
- ▶ Derive the rotor-flux orientation equations (torque, flux dynamics, slip relation) using the inverse- $\Gamma$  model
- ▶ Draw block diagrams for the most typical control schemes and explain them
- ▶ Derive the current model and explain its properties

# Vector Control Methods

- ▶ Based on the dynamic motor model
- ▶ Rotor-flux-oriented vector control, direct torque control (DTC)
- ▶ Torque can be controlled
- ▶ High accuracy and fast dynamics
- ▶ Speed measurement can be replaced with speed estimation in most applications

DC-link voltage is typically measured. However, it will be omitted in the following block diagrams (or constant  $U_{dc}$  is assumed).



# Outline

State-Space Representation

Principle of Rotor-Flux Orientation

Flux Estimation With the Current Model

# Review: Model in Synchronous Coordinates

- ▶ Voltage equations

$$\underline{u}_s = R_s \underline{i}_s + \frac{d\underline{\psi}_s}{dt} + j\omega_s \underline{\psi}_s$$

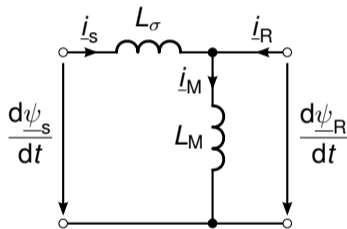
$$\underline{u}_R = R_R \underline{i}_R + \frac{d\underline{\psi}_R}{dt} + j\omega_r \underline{\psi}_R = 0$$

- ▶ Flux linkages

$$\underline{\psi}_s = L_\sigma \underline{i}_s + \underline{\psi}_R$$

$$\underline{\psi}_R = L_M (\underline{i}_s + \underline{i}_R)$$

- ▶ Steady state:  $d/dt = 0$



# State-Space Representation

- ▶ Stator current  $\underline{i}_s$  and rotor flux  $\underline{\psi}_R$  are selected as state variables
- ▶ Derivation: rotor current  $\underline{i}_R$  and stator flux  $\underline{\psi}_s$  are eliminated from the voltage equations by means of the flux equations

$$L_\sigma \frac{di_s}{dt} = \underline{u}_s - (R_s + R_R + j\omega_s L_\sigma) \underline{i}_s + \left( \frac{R_R}{L_M} - j\omega_m \right) \underline{\psi}_R$$
$$\frac{d\underline{\psi}_R}{dt} = R_R \underline{i}_s - \left( \frac{R_R}{L_M} + j\omega_r \right) \underline{\psi}_R$$

- ▶ Dynamics of the stator current are governed by current control
- ▶ Dynamics of the rotor flux are taken into account by rotor-flux orientation

# Outline

State-Space Representation

**Principle of Rotor-Flux Orientation**

Flux Estimation With the Current Model

## Rotor-Flux Dynamics

- ▶ Fast closed-loop stator-current controller is used
- ▶ Stator current is the input from the point of view of the rotor-flux dynamics
- ▶ Rotor equations in synchronous coordinates rotating at  $\omega_s$

$$\frac{d\underline{\psi}_R}{dt} = -R_R \underline{i}_R - j \underbrace{(\omega_s - \omega_m)}_{\omega_r} \underline{\psi}_R$$

$$\underline{\psi}_R = L_M(\underline{i}_s + \underline{i}_R) \quad \Rightarrow \quad \underline{i}_R = \underline{\psi}_R / L_M - \underline{i}_s$$

- ▶ Rotor current can be eliminated

$$\frac{d\underline{\psi}_R}{dt} = - \left( \frac{R_R}{L_M} + j\omega_r \right) \underline{\psi}_R + R_R \underline{i}_s$$



# Rotor-Flux Orientation

- ▶ d-axis of coordinate system is fixed to the rotor flux

$$\underline{\psi}_R = \psi_{Rd} + j\psi_{Rq} = \psi_R + j \cdot 0, \quad \underline{i}_s = i_d + j i_q$$

- ▶ Real and imaginary parts of the rotor-flux dynamics

$$\begin{aligned} \frac{d\psi_R}{dt} &= -\frac{R_R}{L_M}\psi_R + R_R i_d && \text{(in the steady state } \psi_R = L_M i_d) \\ 0 &= -\omega_r \psi_R + R_R i_q \end{aligned}$$

- ▶ Rotor-flux magnitude  $\psi_R$  follows  $i_d$  slowly,

$$\psi_R(s) = \frac{L_M}{1 + s\tau_r} i_d(s) \quad \text{(in the Laplace domain)}$$

due to the rotor time constant  $\tau_r = L_M/R_R$  (typically 0.1... 1.5 s)

# Rotor-Flux Orientation

- ▶ d axis of coordinate system is fixed to the rotor flux:

$$\underline{\psi}_R = \psi_R + j \cdot 0, \quad \underline{i}_s = i_d + j i_q$$

- ▶ Electromagnetic torque

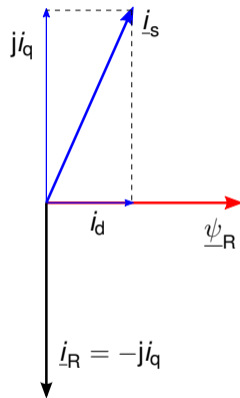
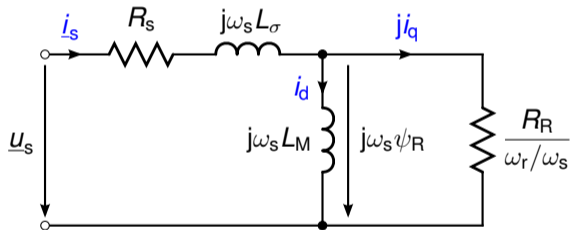
$$T_M = \frac{3p}{2} \operatorname{Im} \left\{ \underline{i}_s \underline{\psi}_R^* \right\} = \frac{3p}{2} \psi_R i_q$$

- ▶ If  $\psi_R$  is constant, **the torque can be controlled using  $i_q$**  (without delays)

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The coordinate system could be fixed to the stator flux  $\underline{\psi}_s$  instead of the rotor flux. This stator-flux orientation would simplify the field weakening, but other parts of the control system would become more complicated.

# Steady-State Equivalent Circuit in Rotor-Flux Coordinates

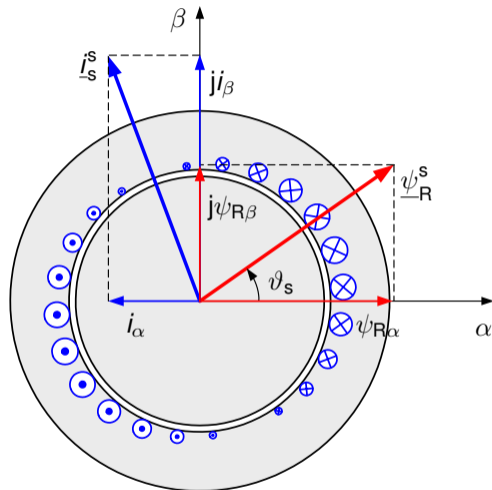


# Stator Coordinates ( $\alpha\beta$ )

- ▶ Vectors are rotating  
(in the steady state  $\vartheta_s = \omega_s t$ )
- ▶ Controlling the torque

$$\begin{aligned} T_M &= \frac{3p}{2} \operatorname{Im} \left\{ i_s^s (\psi_{-R}^s)^* \right\} \\ &= \frac{3p}{2} (i_\beta \psi_{R\alpha} - i_\alpha \psi_{R\beta}) \end{aligned}$$

would be difficult

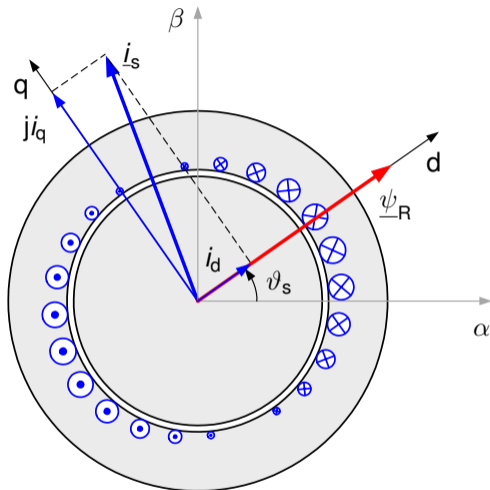


## Rotor-Flux Coordinates (dq)

- ▶ Variables are constant in the steady state
- ▶ Torque

$$T_M = \frac{3p}{2} \operatorname{Im} \left\{ i_s \psi_{-R}^* \right\} = \frac{3p}{2} \psi_R i_q$$

easily controllable via  $i_q$

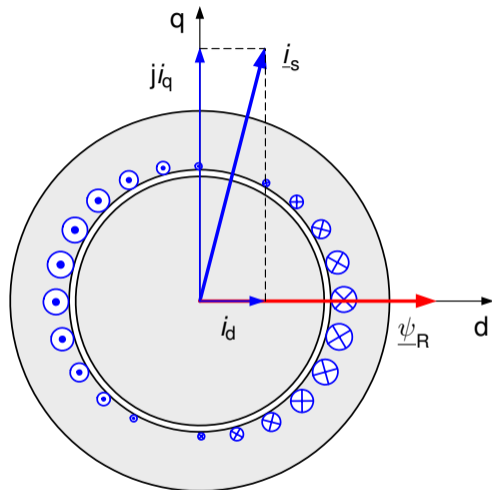


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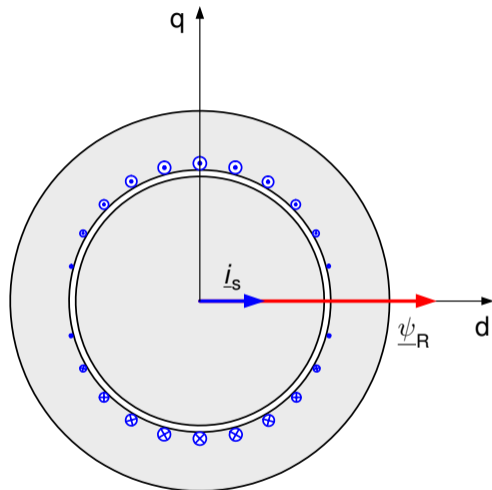


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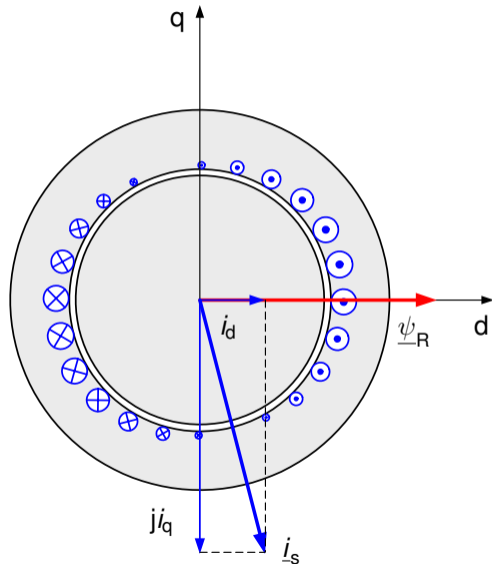


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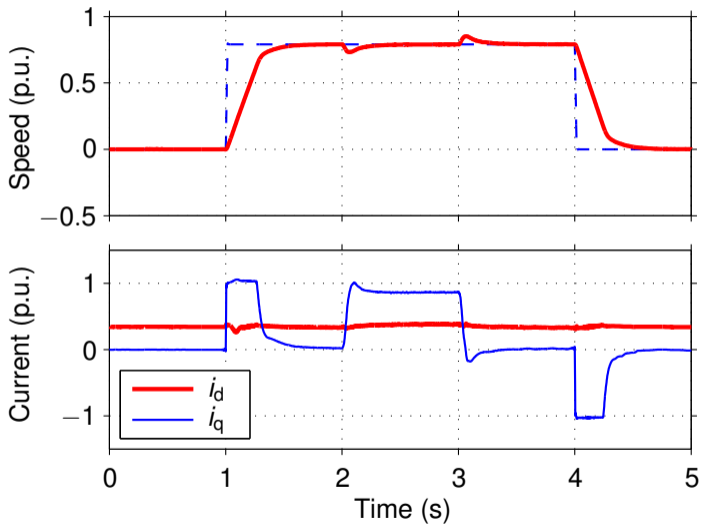
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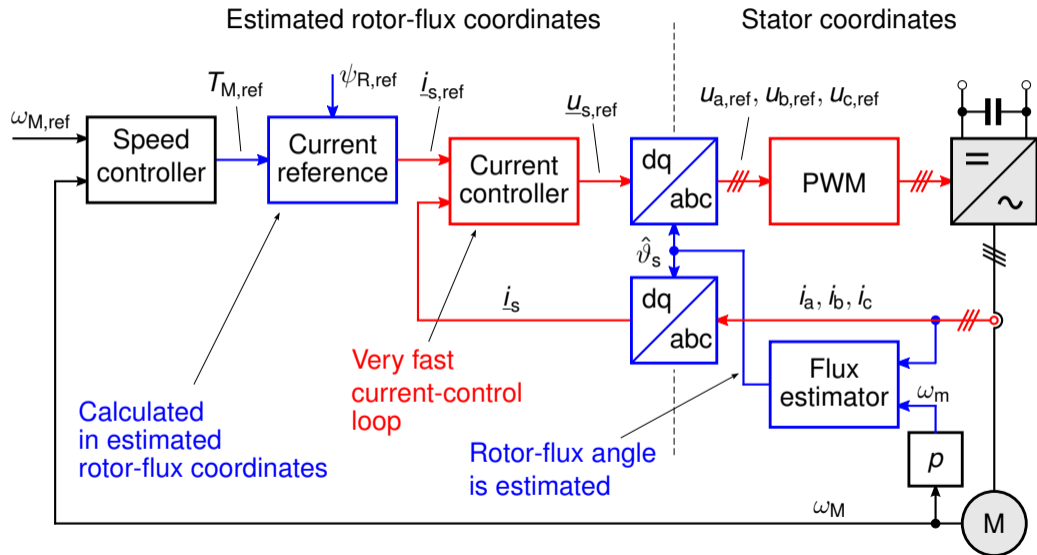




# Example Measured Waveforms: 45-kW Induction Motor Drive



# Rotor-Flux-Oriented Vector Control



# Space-Vector and Coordinate Transformations

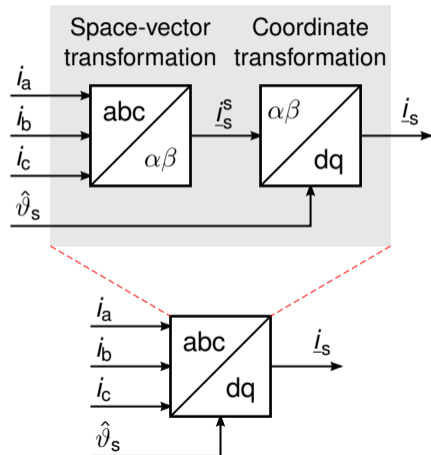
- ▶ Space-vector transformation (abc/ $\alpha\beta$ )

$$\underline{i}_s^s = \frac{2}{3} \left( i_a + i_b e^{j2\pi/3} + i_c e^{j4\pi/3} \right)$$

- ▶ Transformation to rotor coordinates ( $\alpha\beta$ /dq)

$$\underline{i}_s = \underline{i}_s^s e^{-j\hat{\vartheta}_s}$$

- ▶ Combination of these two transformations is often referred to as an abc/dq transformation
- ▶ Similarly, the inverse transformation is referred to as a dq/abc transformation



# Current References

## 1. Flux-producing current reference

$$i_{d,\text{ref}} = \frac{\psi_{R,\text{ref}}}{\hat{L}_M} \quad (\text{where the hat refers to estimates})$$

- ▶ Integral term based on  $u_{\text{max}} - |\underline{u}_{s,\text{ref}}|$  can be used for field weakening
- ▶ Flux controller could be used for improving the flux dynamics in the case of varying  $\psi_{R,\text{ref}}$

## 2. Torque-producing current reference

$$i_{q,\text{ref}} = \frac{2T_{M,\text{ref}}}{3p\psi_{R,\text{ref}}}$$

- ▶ Flux reference  $\psi_{R,\text{ref}}$  is often replaced with the estimate  $\hat{\psi}_R$

# Outline

State-Space Representation

Principle of Rotor-Flux Orientation

**Flux Estimation With the Current Model**

# Current-Model Flux Estimator in Stator Coordinates

- ▶ Current model is based on the rotor voltage equation

$$\frac{d\underline{\hat{\psi}}_R^s}{dt} = - \left( \frac{\hat{R}_R}{\hat{L}_M} - j\omega_m \right) \underline{\hat{\psi}}_R^s + \hat{R}_R \underline{i}_S^s$$

- ▶ Corresponding forward Euler approximation

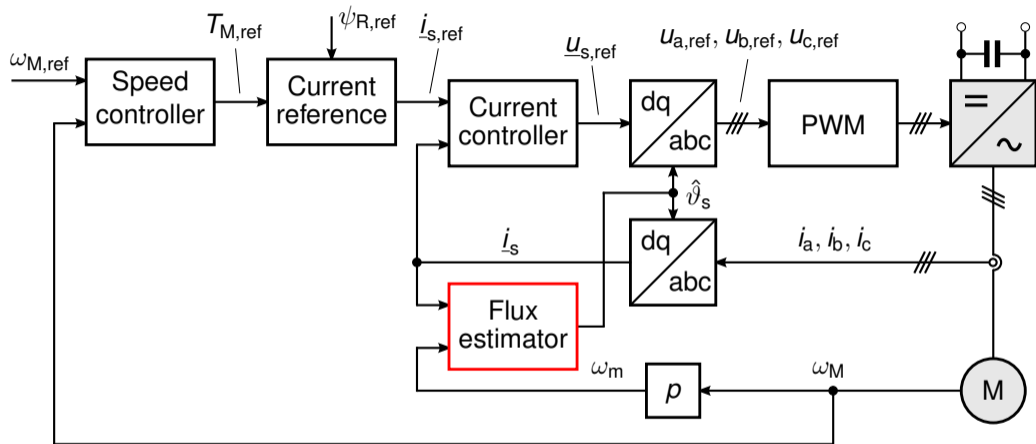
$$\underline{\hat{\psi}}_R^s(k+1) = \underline{\hat{\psi}}_R^s(k) + T_s \left\{ - \left[ \frac{\hat{R}_R}{\hat{L}_M} - j\omega_m(k) \right] \underline{\hat{\psi}}_R^s(k) + \hat{R}_R \underline{i}_S^s(k) \right\}$$

where  $T_s$  is the sampling period and  $k$  is the discrete-time index

- ▶ At each time step, the angle of the flux estimate  $\underline{\hat{\psi}}_R^s = \hat{\psi}_{R\alpha} + j\hat{\psi}_{R\beta}$  is

$$\hat{\vartheta}_s = \text{atan2} \left( \hat{\psi}_{R\beta}, \hat{\psi}_{R\alpha} \right)$$

## Current Model in Estimated Rotor Flux Coordinates



- ▶ Signals fed to the flux estimator are DC in the steady state
- ▶ Discrete-time implementation becomes easier

# Current-Model Flux Estimator in Estimated Rotor-Flux Coordinates

$$\frac{d\underline{\hat{\psi}}_R}{dt} = - \left( \frac{\hat{R}_R}{\hat{L}_M} + j\hat{\omega}_r \right) \underline{\hat{\psi}}_R + \hat{R}_R \underline{i}_s \quad \underline{\hat{\psi}}_R = \hat{\psi}_R + j \cdot 0$$

- ▶ Real and imaginary parts in estimated flux coordinates

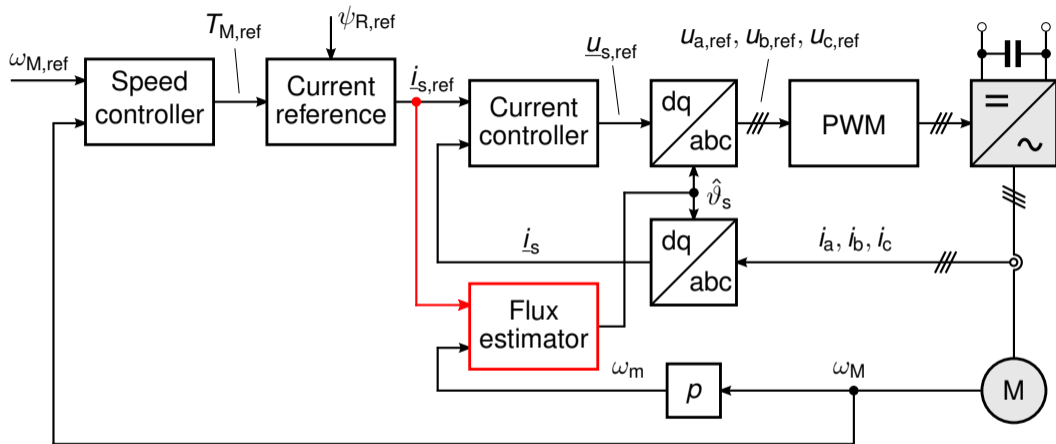
$$\frac{d\hat{\psi}_R}{dt} = - \frac{\hat{R}_R}{\hat{L}_M} \hat{\psi}_R + \hat{R}_R i_d \quad \hat{\omega}_r = \frac{\hat{R}_R i_q}{\hat{\psi}_R}$$

- ▶ Flux-angle estimation

$$\hat{\vartheta}_s = \int \hat{\omega}_s dt = \int (\omega_m + \hat{\omega}_r) dt$$



# Indirect Field Orientation (IFO)



- ▶ Current reference is used as an input of the flux estimator
- ▶ Flux estimator is also simplified (see the following slide)

- ▶ Flux-magnitude dynamics are omitted in the slip relation

$$\hat{\omega}_r = \frac{R_R i_{q,\text{ref}}}{\psi_{R,\text{ref}}}$$

- ▶ Flux-angle estimation

$$\hat{\vartheta}_s = \int (\omega_m + \hat{\omega}_r) dt$$

- ▶ Poor performance if the flux reference  $\psi_{R,\text{ref}}$  is not constant or if the current controller does not work as intended

# Properties of the Current Model and IFO

## Disadvantages:

- ▶ Rotor speed measurement is needed
- ▶ Converges slowly (with the rotor time constant), which can be a problem if the flux reference  $\psi_{R,ref}$  is varied
- ▶ Inaccurate model parameters  $\hat{R}_R$  and  $\hat{L}_M$  cause errors in field orientation  
⇒ degraded control performance

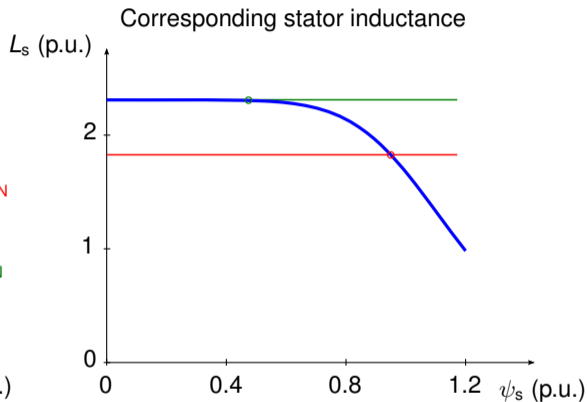
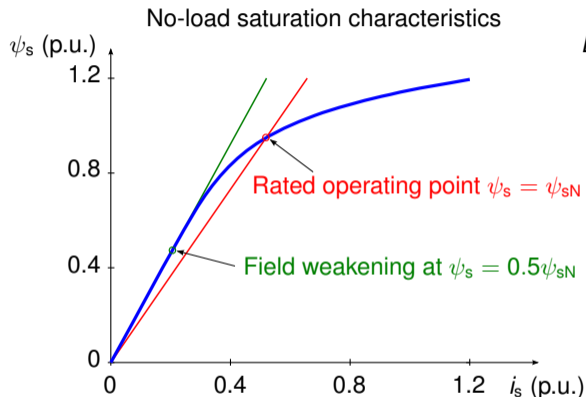
## Advantages:

- ▶ Simplicity
- ▶ Robustness

## Reasons for Parameter Detuning: Actual Motor Parameters Vary

- ▶ Resistances depend on:
  - ▶ Temperature (about 0.4%/K)
  - ▶ Frequency due to the skin effect  
(especially the resistances of the rotor bars)
- ▶ Inductances depend on the magnetic state
  - ▶ Stator (magnetizing) inductance increases as the stator (rotor) flux decreases in the field-weakening region
  - ▶ Torque may also have an effect on the inductances
- ▶ Identification of the motor parameters is never perfect
- ▶ Some phenomena are omitted in the model but exist in the actual machine (e.g. core losses)

# Magnetic Saturation Characteristics: 2.2-kW Motor as an Example



- ▶ Stator inductance  $L_s = L_\sigma + L_M$  depends on the stator-flux magnitude  $\psi_s$
- ▶ Effect should be taken into account in control, if field weakening is used

## Summary: Rotor-Flux Orientation

- ▶ Decoupled control of the flux and the torque, as in the DC machines
- ▶ d-axis of the coordinate system is fixed to the rotor flux vector (or its estimate in practice)
- ▶ Rotor-flux magnitude is controlled using the d-component of the stator current
- ▶ Torque is controlled using the q-component of the current

Similar control structure can also be used in sensorless methods, but a different flux (and speed) estimator is needed.

## Further Reading

### Modeling

- ▶ G. R. Slemon, “Modelling of induction machines for electric drives,” *IEEE Trans. Ind. Applicat.*, vol. 25, no. 6, 1989.

### Flux control, field weakening, loss minimization:

- ▶ Z. Qu, M. Ranta, M. Hinkkanen, J. Luomi, “Loss-minimizing flux level control of induction motor drives,” *IEEE Trans. Ind. Applicat.*, vol. 48, no. 3, 2012.

### Sensorless methods:

- ▶ M. Hinkkanen, L. Harnefors, J. Luomi, “Reduced-order flux observers with stator-resistance adaptation for speed-sensorless induction motor drives,” *IEEE Trans. Pow. Electron.*, vol. 25, no. 5, 2010.
- ▶ Z. Qu, M. Hinkkanen, L. Harnefors, “Gain scheduling of a full-order observer for sensorless induction motor drives,” *IEEE Trans. Ind. Applicat.*, vol. 50, no. 6, 2014.