Aalto University
School of Electrical
Engineering

Lecture 4: Pulse-Width Modulation and Current Control

ELEC-E8402 Control of Electric Drives and Power Converters

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Learning Outcomes

After this lecture and exercises you will be able to:

- Explain the difference between the standard suboscillation PWM method and the symmetrical suboscillation PWM method
- Explain the principle of three-phase synchronous-frame current control
- Understand operation of the current controller in Homework Assignment 1

The current controller is presented here for induction motors, but it can be easily extended for other AC machines and for grid converters (equipped with L filter).

Outline

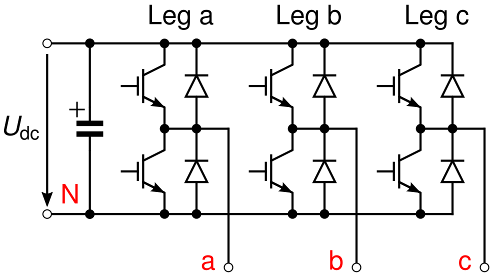
3-Phase Inverter

Pulse-Width Modulation

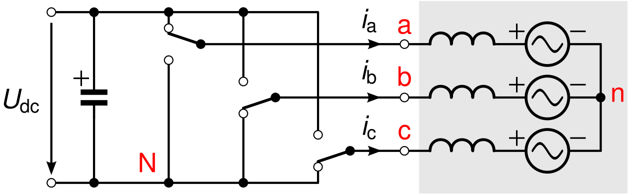
Current Control

Anti-Windup, Sampling, PWM Update

3-Phase Inverter



Space Vector of the Converter Output Voltages



- Zero-sequence voltage does not affect the phase currents
- Reference potential of the phase voltages can be freely chosen

$$\underline{u}_s^s = \frac{2}{3}\left(u_{an} + u_{bn}e^{j2\pi/3} + u_{cn}e^{j4\pi/3}\right) \qquad \text{Neutral n as a reference}$$

$$= \frac{2}{3}\left(u_{aN} + u_{bN}e^{j2\pi/3} + u_{cN}e^{j4\pi/3}\right) \qquad \text{Negative DC bus N as a reference}$$

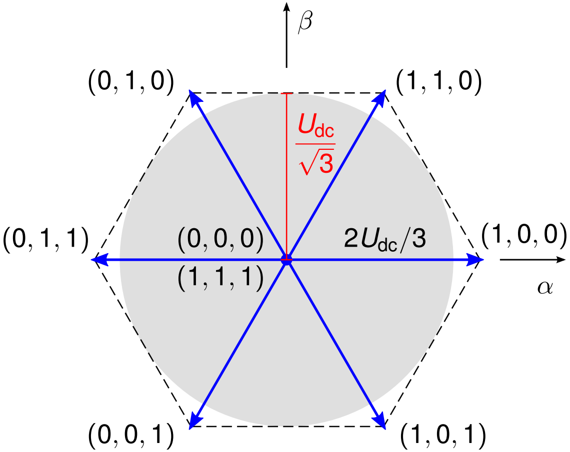
- Converter output voltage vector

$$\underline{u}_s^s = \frac{2}{3}\left(u_{aN} + u_{bN}e^{j2\pi/3} + u_{cN}e^{j4\pi/3}\right)$$
$$= \frac{2}{3}\left(q_a + q_b e^{j2\pi/3} + q_c e^{j4\pi/3}\right) U_{dc}$$

where q_{abc} are the switching states (either 0 or 1)

- Vector $(1, 0, 0)$ as an example

$$\underline{u}_s^s = \frac{2U_{dc}}{3}$$



Switching-Cycle Averaged Voltage

- Using PWM, any voltage vector inside the voltage hexagon can be produced in average over the switching period

$$\overline{\underline{u}}_\mathrm{s}^\mathrm{s} = \frac{2}{3}\left(d_\mathrm{a} + d_\mathrm{b}\mathrm{e}^{\mathrm{j}2\pi/3} + d_\mathrm{c}\mathrm{e}^{\mathrm{j}4\pi/3}\right) U_\mathrm{dc}$$

  where d_abc are the duty ratios (between $0 \ldots 1$)
- Maximum magnitude of the voltage vector is $u_\mathrm{max} = U_\mathrm{dc}/\sqrt{3}$ in linear modulation (the circle inside the hexagon)
- PWM can be implemented, e.g., using the carrier comparison
- Mainly switching-cycle averaged quantities will be needed in this course (overlining will be omitted for simplicity)

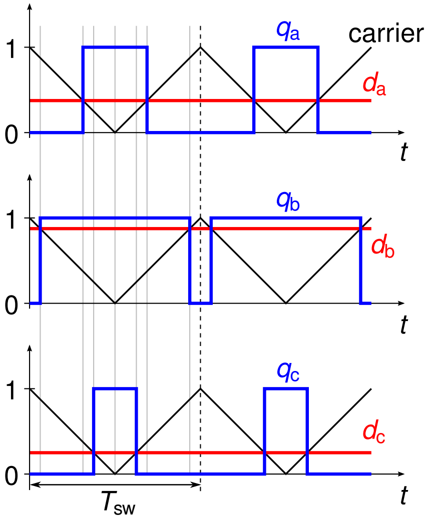
Outline

Suboscillation Method

- Duty ratios for carrier comparison

$$d_\mathrm{a} = \frac{1}{2} + \frac{u_\mathrm{a,ref}}{U_\mathrm{dc}} \quad d_\mathrm{b} = \frac{1}{2} + \frac{u_\mathrm{b,ref}}{U_\mathrm{dc}} \quad d_\mathrm{c} = \frac{1}{2} + \frac{u_\mathrm{c,ref}}{U_\mathrm{dc}}$$

- Voltage vectors during T_sw in the example: $(0,0,0) \rightarrow (0,1,0) \rightarrow (1,1,0) \rightarrow (1,1,1) \rightarrow (1,1,1) \rightarrow (1,1,0) \rightarrow (0,1,0) \rightarrow (0,0,0)$
- Problem: only 87% of the maximum available voltage can be used!
- Proper zero-sequence component should be added to utilize all available voltage



Symmetrical Suboscillation Method

- Zero sequence

$$u_0 = \frac{\min(u_{\mathrm{a,ref}}, u_{\mathrm{b,ref}}, u_{\mathrm{c,ref}}) + \max(u_{\mathrm{a,ref}}, u_{\mathrm{b,ref}}, u_{\mathrm{c,ref}})}{2}$$

- Modified voltage references

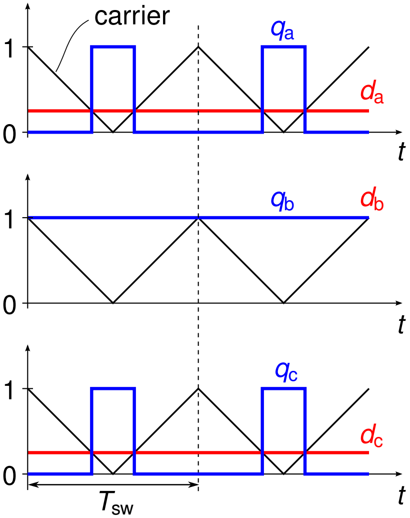
$$u'_{\mathrm{a,ref}} = u_{\mathrm{a,ref}} - u_0 \qquad u'_{\mathrm{b,ref}} = u_{\mathrm{b,ref}} - u_0 \qquad u'_{\mathrm{c,ref}} = u_{\mathrm{c,ref}} - u_0$$

- Duty ratios for carrier comparison

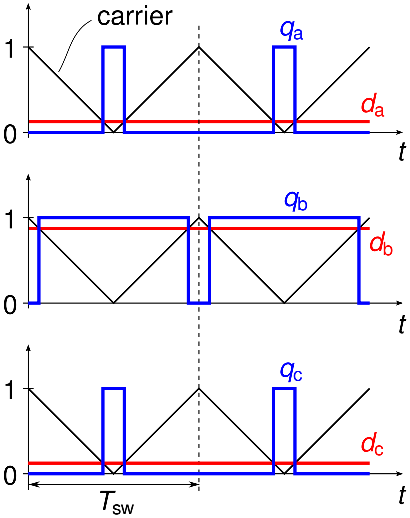
$$d_{\mathrm{a}} = \frac{1}{2} + \frac{u'_{\mathrm{a,ref}}}{U_{\mathrm{dc}}} \qquad d_{\mathrm{b}} = \frac{1}{2} + \frac{u'_{\mathrm{b,ref}}}{U_{\mathrm{dc}}} \qquad d_{\mathrm{c}} = \frac{1}{2} + \frac{u'_{\mathrm{c,ref}}}{U_{\mathrm{dc}}}$$

- Whole voltage hexagon can be now used
- Following example: $u_{\mathrm{a,ref}} = u_{\mathrm{c,ref}} = -U_{\mathrm{dc}}/4$ and $u_{\mathrm{b,ref}} = U_{\mathrm{dc}}/2$

The symmetrical suboscillation method is identical to the continuous space-vector PWM.



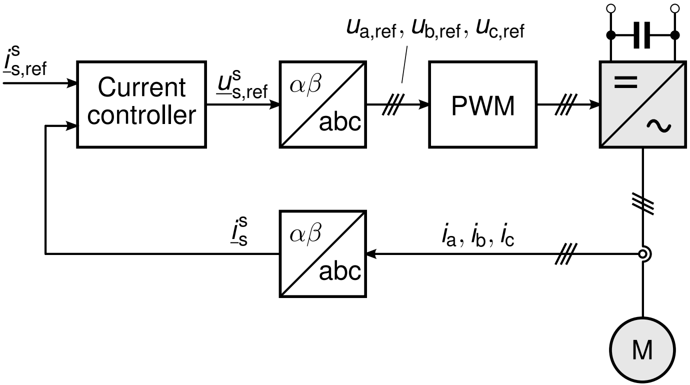
Suboscillation method



Symmetrical suboscillation method

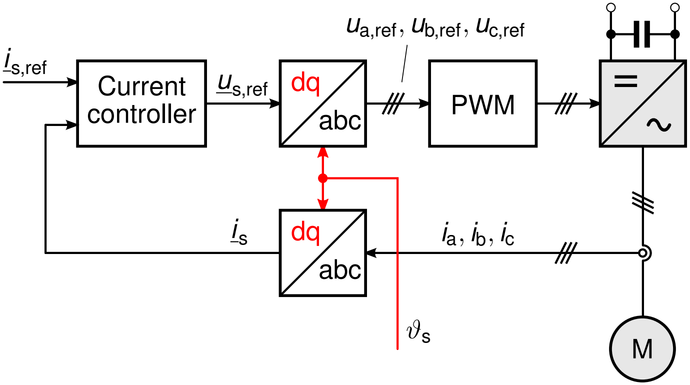
Outline

Current Controller in Stator Coordinates



- PI controller cannot give zero steady-state error for sinusoidal references
- Actual current does not follow its reference in the steady state (phase shift and magnitude error)

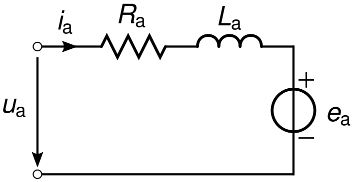
Current Controller in Synchronous Coordinates



- DC signals in steady state, no steady-state error
- Synchronous-frame two-degree-of-freedom PI control will be considered

Review: Electrical Dynamics of the DC Motor and SPM

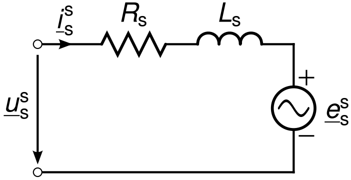
DC motor



$$L_\mathrm{a}\frac{\mathrm{d}i_\mathrm{a}}{\mathrm{d}t} = u_\mathrm{a} - R_\mathrm{a}i_\mathrm{a} - e_\mathrm{a}$$

Back-emf: $e_\mathrm{a} = k_\mathrm{f}\omega_\mathrm{m}$

SPM in stator coordinates



$$L_\mathrm{s}\frac{\mathrm{d}\underline{i}_\mathrm{s}^\mathrm{s}}{\mathrm{d}t} = \underline{u}_\mathrm{s}^\mathrm{s} - R_\mathrm{s}\underline{i}_\mathrm{s}^\mathrm{s} - \underline{e}_\mathrm{s}^\mathrm{s}$$

Back-emf: $\underline{e}_\mathrm{s}^\mathrm{s} = \mathrm{j}\omega_\mathrm{m}\psi_\mathrm{f}\mathrm{e}^{\mathrm{j}\vartheta_\mathrm{m}}$

- Back-emf is a quasi-constant disturbance for the current controller

The DC motor and the surface-mounted permanent-magnet motor (SPM) were considered in the prerequisite course ELEC-E8405 Electric Drives.

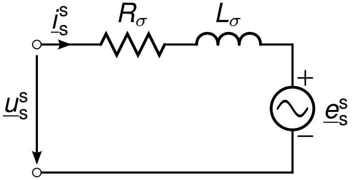
Electrical Dynamics of the Induction Motor

- State-space representation in stator coordinates
- Stator current and rotor flux as state variables

$$L_\sigma \frac{d\underline{i}_s^s}{dt} = \underline{u}_s^s - R_\sigma \underline{i}_s^s - \underbrace{\left(j\omega_m - \frac{R_R}{L_M} \right) \underline{\psi}_R^s}_{\text{back-emf } \underline{e}_s^s}$$

$$\frac{d\underline{\psi}_R^s}{dt} = R_R \underline{i}_s^s + \left(j\omega_m - \frac{R_R}{L_M} \right) \underline{\psi}_R^s$$

where $R_\sigma = R_s + R_R$



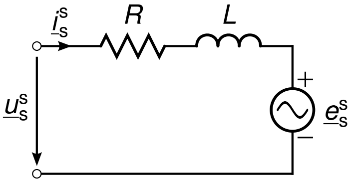
- Rotor-flux magnitude and speed change slowly as compared to the stator current

General 3-Phase Load

- Differential equation in synchronous coordinates

$$L\frac{\mathrm{d}\underline{i}_\mathrm{s}}{\mathrm{d}t} + \mathrm{j}\omega_\mathrm{s} L \underline{i}_\mathrm{s} = \underline{u}_\mathrm{s} - R\underline{i}_\mathrm{s} - \underline{e}_\mathrm{s}$$

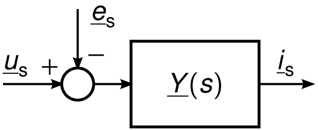
General	SPM	Induction motor
L	L_s	L_σ
R	R_s	R_σ
$\underline{e}_\mathrm{s}$	$\mathrm{j}\omega_\mathrm{m}\psi_\mathrm{f}$	$\left(\mathrm{j}\omega_\mathrm{m} - \frac{R_\mathrm{R}}{L_\mathrm{M}}\right)\underline{\psi}_\mathrm{R}$
ω_s	ω_m	ω_s



Transfer Function in Synchronous Coordinates

- Transfer function of the system to be controlled

$$\frac{\underline{i}_\mathrm{s}(s)}{\underline{u}_\mathrm{s}(s)} = \underline{Y}(s) = \frac{1}{(s + \mathrm{j}\omega_\mathrm{s})L + R}$$



- Back emf $\underline{e}_\mathrm{s}$ can be seen as a load disturbance
- Similar to the transfer function of the DC motor
- Ideal converter and PWM will be assumed ($\underline{u}_\mathrm{s} = \underline{u}_\mathrm{s,ref}$)

Cross-Coupling Compensation and Active Resistance

$$L\frac{\mathrm{d}\underline{i}_\mathrm{s}}{\mathrm{d}t} = \underline{u}_\mathrm{s} - (R + \mathrm{j}\omega_\mathrm{s}L)\underline{i}_\mathrm{s} - \underline{e}_\mathrm{s}$$

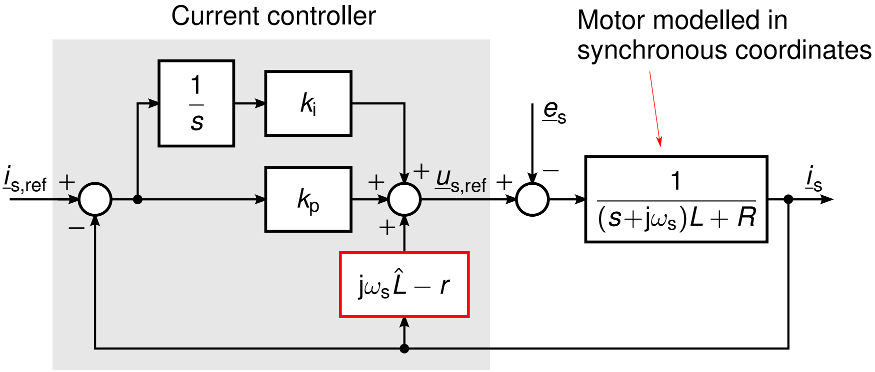
- Term $\mathrm{j}\omega_\mathrm{s}L\underline{i}_\mathrm{s}$ causes the cross-coupling between the axes

$$L\frac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} = u_\mathrm{d} - Ri_\mathrm{d} + \omega_\mathrm{s}Li_\mathrm{q} - e_\mathrm{d}$$
$$L\frac{\mathrm{d}i_\mathrm{q}}{\mathrm{d}t} = u_\mathrm{q} - Ri_\mathrm{q} - \omega_\mathrm{s}Li_\mathrm{d} - e_\mathrm{q}$$

- Cross-coupling is cancelled by adding $\mathrm{j}\omega_\mathrm{s}\hat{L}\underline{i}_\mathrm{s}$ to the controller output
- Active resistance r improves disturbance-rejection capability

$$\underline{u}_\mathrm{s} = \underline{u}'_\mathrm{s} + (\mathrm{j}\omega_\mathrm{s}\hat{L} - r)\underline{i}_\mathrm{s}$$

Control Algorithm



$$\frac{d\underline{I}}{dt} = \underline{i}_{s,ref} - \underline{i}_s$$

$$\underline{u}_{s,ref} = k_p(\underline{i}_{s,ref} - \underline{i}_s) + k_i\underline{I} + (j\omega_s\hat{L} - r)\underline{i}_s$$

Tuning

- PI controller sees the system (assuming $\hat{L} = L$)

$$L\frac{\mathrm{d}\underline{i}_\mathrm{s}}{\mathrm{d}t} = \underline{u}'_\mathrm{s} - (R + r)\underline{i}_\mathrm{s} - \underline{e}_\mathrm{s} \qquad \text{or} \qquad \frac{\underline{i}_\mathrm{s}(s)}{\underline{u}'_\mathrm{s}(s)} = Y'(s) = \frac{1}{sL + R + r}$$

- PI controller and closed-loop transfer functions

$$K(s) = k_\mathrm{p} + \frac{k_\mathrm{i}}{s} \qquad \frac{\underline{i}_\mathrm{s}(s)}{\underline{i}_\mathrm{s,ref}(s)} = \frac{K(s)Y'(s)}{1 + K(s)Y'(s)} = \frac{\alpha_\mathrm{c}}{s + \alpha_\mathrm{c}}$$

  where α_c is the desired closed-loop bandwidth
- Gain selection

$$r = \alpha_\mathrm{c}\hat{L} - R \qquad k_\mathrm{p} = \alpha_\mathrm{c}\hat{L} \qquad k_\mathrm{i} = \alpha_\mathrm{c}^2\hat{L}$$

This tuning principle is covered in more detail in connection with the DC motors in the course ELEC-E8405 Electric Drives.

Outline

Anti-Windup

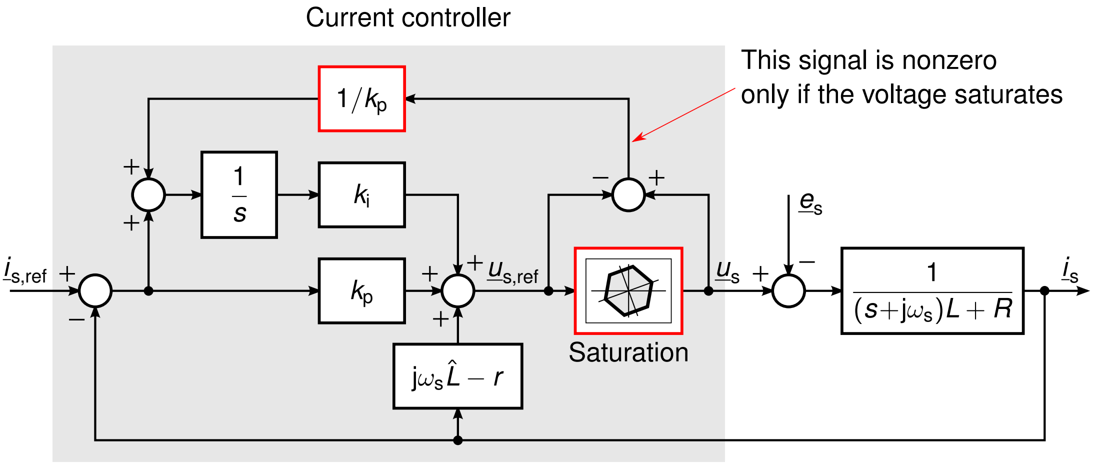
- Maximum converter output voltage is limited: $|\underline{u}_\mathrm{s}| < u_\mathrm{max}$
- Reference $|\underline{u}_\mathrm{s,ref}|$ may exceed u_max for large current steps (especially at high rotor speeds due to the large back-emf $|\underline{e}_\mathrm{s}|$)
- Realisable voltage vector $\underline{u}_\mathrm{s}$ is obtained from the PWM algorithm
- Algorithm is augmented with the anti-windup

$$\frac{\mathrm{d}\underline{I}}{\mathrm{d}t} = \underline{i}_\mathrm{s,ref} - \underline{i}_\mathrm{s} + \frac{1}{k_\mathrm{p}}(\underline{u}_\mathrm{s} - \underline{u}_\mathrm{s,ref})$$

$$\underline{u}_\mathrm{s,ref} = k_\mathrm{p}(\underline{i}_\mathrm{s,ref} - \underline{i}_\mathrm{s}) + k_\mathrm{i}\underline{I} + (\mathrm{j}\omega_\mathrm{s}\hat{L} - r)\underline{i}_\mathrm{s}$$

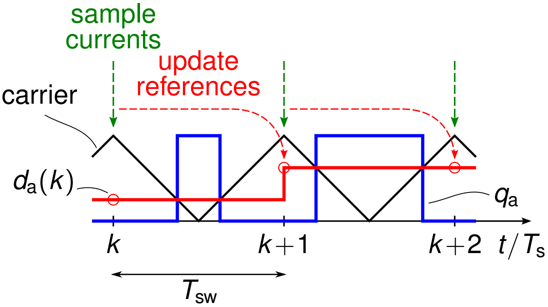
$$\underline{u}_\mathrm{s} = \mathrm{PWM}(\underline{u}_\mathrm{s,ref}, \vartheta_\mathrm{s})$$

Back-Calculation Anti-Windup Method

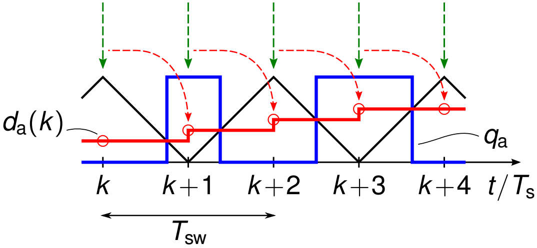


Go through the implementation in Homework Assignment 1. Extra material: M. Hinkkanen, H. A. A. Awan, Z. Qu, T. Tuovinen, and F. Briz, "Current control for synchronous motor drives: Direct discrete-time pole-placement design," *IEEE Trans. Ind. Applicat.*, vol. 52, no. 2, 2016.

Discrete Implementation: Synchronous Sampling and PWM Update



Single-update PWM



Double-update PWM

- No switching ripple in the current samples due to synchronous sampling
- Duty ratios d_b and d_c are updated simultaneously with d_a

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The double-update PWM scheme is used in Homework Assignment 1.