Phase Equalizers - Overview and Experiments in Audio Processing

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Abstract

This paper is an overview on the topic of Phase Equalizers. The concept of phase, especially from the perspective of digital filters is discussed. Types of filters used in phase equalizing and common applications are presented. Additionally the audibility of phase is discussed. Some experimentation on audio effects processing applications of an phase equalizer is implemented and the results visualized.

1 Introduction

An overview of digital phase equalizers is presented in this paper. Phase equalizers are filters that target to change the phase response without altering the magnitude response of a given system. Both allpass IIR filters [1],[2],[3], and FIR filters with flat magnitude response[4], [5],[6], [4], [7]can be used for this purpose. Most common applications are related to group delay equalization of loudspeakers. [3],[4],[5], [6], [7]. These type of filters are also suitable for novel audio processing [8].

First the concept of phase, phase response and phase characteristics of digital filters are introduced in Section 2. Next the two types of phase equalizer filters (allpass and FIR) are discussed is Section 3. In Section 4 selected applications from previous studies are presented, concentrated on group delay equalization and audio effects use of phase equalizers. In Section 5 some experimentation of group delay based allpass type of filters is presented. These experiments intend to take a closer look on the method presented in [8] and to apply the filters to musical audio samples for audible results. Finally, Section 6 concludes the paper and provides further discussion on the topic overall.

2 Phase

Phase is defined as the relative position in time of a periodic function. Usually the phase value is presented as an angle either in degrees or radians. In this paper radians will be used as the unit for phase. Equation 1 shows the equation for a single continuous time sine wave, where θ is the phase.

$$s(t) = sin(2\pi ft + \theta) \tag{1}$$

Figure 1 shows a single cycle of two sine waves. The other one has zero phase while the other one is phase shifted by π radians. The phase shift of π radians results in the phase shifted sine wave to be opposite phase of the first one. As a result the two sine waves would add up to zero.

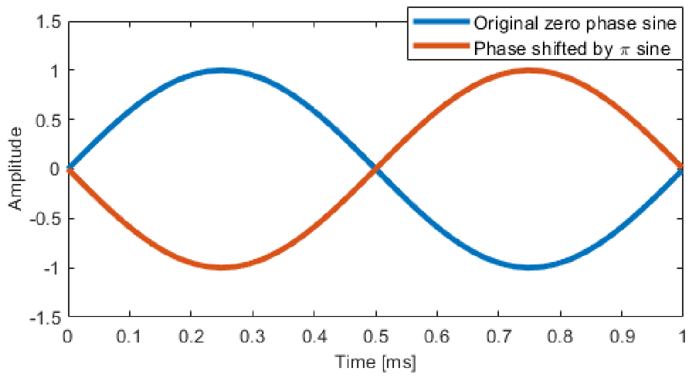

Figure 1: *1 kHz Sine wave and its phase shifted version.*

As real signals are usually composed of multiple frequencies, the overall phase is not sufficient in describing the phase characteristics of real signals and the filters applied to them. That is where phase response is a much more suitable descriptor of phase.

2.1 Phase Response

Phase response of a filter describes the difference in phase in frequency domain. Basically it gives the shift in phase for each sine wave component of the input signal. [9] Figure 4 shows phase response curves of several first order all pass filters. Like the phase, phase response values are given in radians or degrees.

2.2 Phase Delay and Group Delay

Phase Delay and Group Delay are alternative forms to describe the phase response. Phase delay gives the time delay for each input frequency component at the filter output. The formula for phase delay is shown in Equation 2. It is basically the negative phase response divided by frequency.

$$P(\omega) = -\frac{\theta(\omega)}{\omega} \tag{2}$$

Group delay on the other hand is defined as the negative derivative of the phase response as seen in Equation 3. It describes the frequency dependent time delays of the amplitude envelope of the signal. Figure 2 shows a comparison of phase delay and group delay of an amplitude modulated sine wave.

$$\tau(\omega) = -\frac{d\theta(\omega)}{d\omega} \tag{3}$$

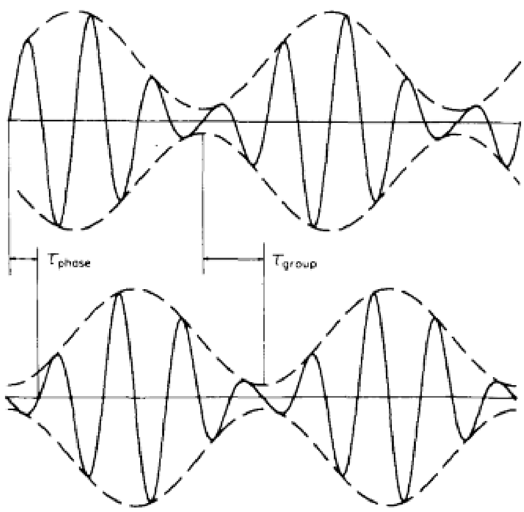

Figure 2: *Phase delay and group delay of an amplitude modulated sine wave. Reprinted from [2].*

As seen from Figure 2, phase delay and group delay are not necessarily equal. Actually, they are only equal for linear phase systems. Linear phase is discussed in Section 2.3.1. It seems that the group delay is much more commonly used quantity at least in the applications researched for this paper.

2.3 Phase Characteristics of Digital Filters

The phase characteristics of digital filters are discussed in this section. Linear phase and minimum phase are the most common types used. Linear phase filters have the benefit of introducing a constant amount of delay for each frequency, whereas minimum phase filters can achieve shorter delays with the expense of non linear phase response [2].

2.3.1 Linear Phase

Linear phase filter's phase response is linear and thus its group delay is constant [10]. Also the phase delay is equal to the group delay in this case as seen in Figure 3. The impulse response of a linear phase filter is always symmetrical w.r.t. its peak value [2]. Figure 3 shows the impulse response, phase response, phase delay and group delay of an order 32 FIR lowpass filter.

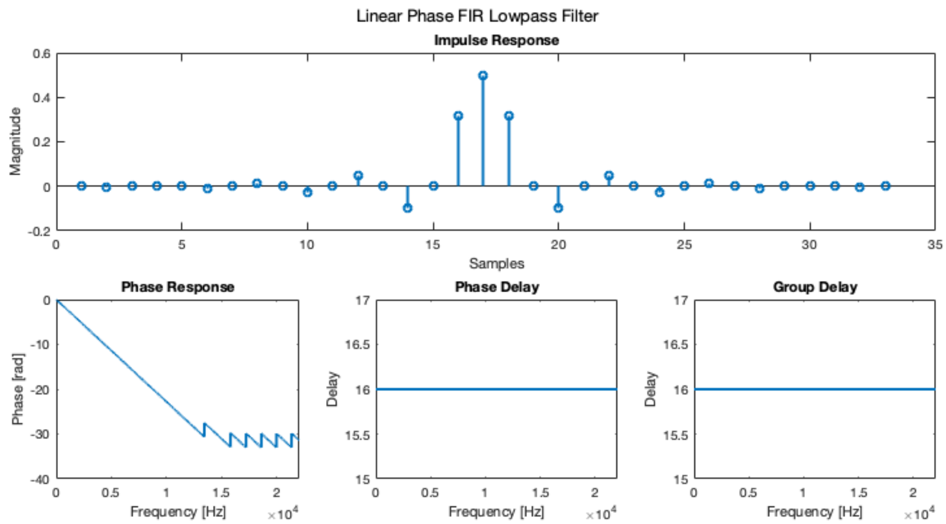

Figure 3: *Impulse response, phase response, phase delay and group delay of an linear phase FIR lowpass filter.*

2.3.2 Zero Phase

Zero phase filter is a special case of a linear phase filter. Its impulse response is symmetrical wrt. time zero. This type of filter is possible only offline [10]. Zero phase filter is not causal since it has values in negative time [2].

2.3.3 Minimum & Maximum Phase

A filter is considered to be minimum phase if all its zeros and poles are inside the unit circle but not on it [9]. It is also called a minimum delay filter having the minimal phase changes for a given magnitude response[2]. Minimal phase filters release their energy as fast as possible within the boundaries of causality [2]. Maximum phase filter is simply the time reversed version of a minimum phase filter.

3 Phase Equalizer Filters

The two common types of phase equalizer filters (IIR allpass and FIR with flat magnitude response) are introduced.

3.1 Allpass Filter

Digital recursive allpass filter is by definition a lossless filter. Also every lossless filter is an allpass filter. [9] The magnitude response of an allpass filter is unity from zero to Nyquist while its phase response is varying w.r.t. frequency [10]. Allpass filters can be designed to compensate non linear phase responses of other filters. When used in that way, they are also called phase equalizers [10].

The transfer function of a Nth order digital allpass filter is shown in Equation 4 below. As seen from the transfer function, the coefficients of the numerator and denominator are the same but in the reversed order.

$$H(z) = \frac{a_N z^{-N} + ... + a_2 z^{-2} + a_1 z^{-1}}{a_1 z^{-1} + a_2 z^z - 2 + ... + a_N z^{-N}} \tag{4}$$

To make the filter stable, the values of all the coefficients a_N should be $-1 < a_N < 1$. Difference equation of a first order allpass filter is shown in Equation 5. As seen from the equation the current value of the output depends on the current value of the input, previous value of the input and previous value of the output.

$$y(n) = a_1 x(n) + x(n-1) - a_1 y(n-1) \tag{5}$$

Figure 4 shows the frequency response (both magnitude and phase) of a first order digital allpass filter. As seen in the Figure 4 the Magnitude response of the filter is at unity gain for all frequencies. The phase response is variable and the shape of it depends on the coefficient a_1. More complex shapes for the phase response curve can be obtained by using higher order allpass filters.

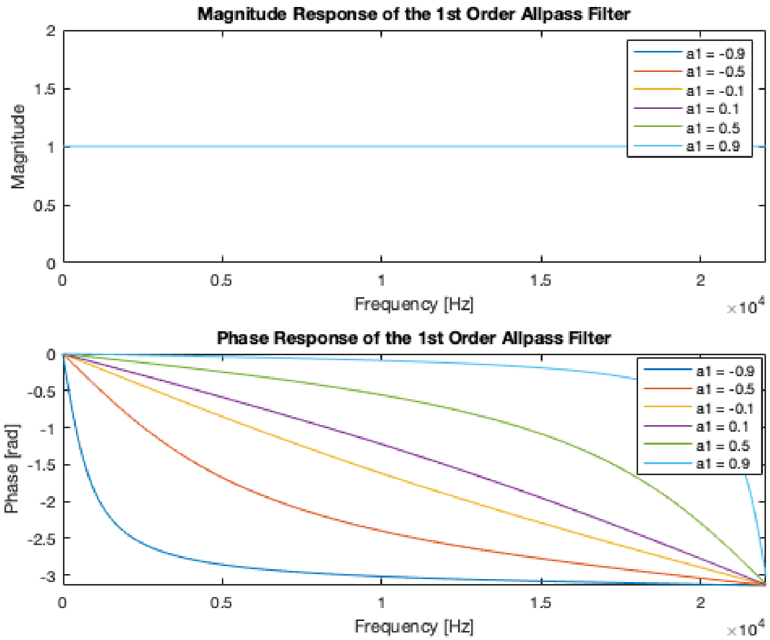

Figure 4: *Frequency response of a first order allpass filter with different coefficient (a1) values.*

3.2 FIR Filters with Flat Magnitude Response

The digital allpass filter seems to be in principle great for phase equalizer purposes. However, in many applications, it may be more flexible to use a FIR filter instead. Carini et al.[7] as well as Canfield-Dafilou et al.[8] use a method to design FIR filters with flat frequency responses based on desired group delay trajectories in the frequency domain. These type of filters resemble allpass filters and can be thought as sampled IIR filters. This type of phase equalizer filter is discussed more in Section 4.3 and used in the experiments in Section 5.

4 Applications

In this section the main applications for phase equalizers filters are discussed. Additionally the audibility of phase is discussed.

4.1 Audibility of Phase

Before putting in the effort to implement any phase equalization to given system it is wise to determine if the phase response will cause any audible effects. According to Eastty [1], phase shift can cause problems in two cases. When the phase response has rapid changes or when multiples of a signal are added together with the other one having different phase.

Karjalainen et al. discuss the perception of phase in loudspeaker equalization applications[6]. The authors state that group delay of approximately 2 ms is just noticeable when using transients and impulses as test signals. However, for actual program material even 3-5 ms group delay errors would be barely noticeable based on informal listening experiments. Some problems due to the phase equalization arise, mainly when equalizing relatively large group delays at cut off frequency. The used FIR techniques introduce additional bulk delay at all frequencies and create audible preecho or prehiss.

More recently, Liski et al. [11] carried out listening test to study the audibility of group-delay characteristics of loudspeakers. They used impulse responses and their time-reversed versions as the test signal pairs Their results indicate group delays of under 1 ms to be inaudible within 300 Hz - 1 kHz. The effect of premasking is also discussed as it is relevant to the audibility of the preecho. No exact data for the level of premasking as function of time is yet to be presented. Approximate window for premasking seems to be around 10 - 20 ms.

4.2 Group Delay Equalization

In this section some selected applications on using phase equalizer filters as group delay equalizers are presented. The most common application in this category is the group delay correction of loudspeakers. Makivirta et al. compare the use of bulk delay (delay line) and a linear phase group delay filter on a two-way loudspeaker [5]. However, the use of linear phase filters is problematic due to their preecho. The problem of the preecho was also encountered by Karjalainen et al. when using FIR based methods for phase equalizing [6].

Herzog and Hilsamer compare two different approaches for low frequency group delay equalization of vented boxes [4]. The authors state that the design of such a filter is a trade of between length needed for a FIR implementation and the accuracy needed for an IIR implementation. In other words, when using IIR filter one can create a sufficient filter with less coefficients but the accuracy of those coefficeints needs to be higher than in the FIR implementation. The paper indicates that the FIR based method can give accurate results but it is computationally heavy. The time reversed IIR method on the other hand is less computationally heavy and allows real time control of the delay. However, the IIR method introduces additional bulk delay to the system.

Also Bharitkar et al. studied group delay equalization of multi-way speakers [3]. The authors use allpass filters designed with the so called Deczky Technique. The method presented manages to flatten the group delays relatively well, however it results in temporal smearing visible in the impulse responses of the group delay equalized loudspeaker. The authors plan on introducing listening test in the second part of this paper to better understand the possible trade of between flat group delay and the time smearing.

Recently Carini et al. suggested a FIR group delay equalizer added to an existing design of a multipoint mimimum phase room response equalizer [7]. The method to design the group delay equalizer is similar to the one used in [8] and the design process is described in Section 4.3 of this paper. The proposed method seems to also introduce preecho which the authors seem to redeem inaudible based on its short length. However, this claim seems not have any concrete evidence backing it.

All and all, it seems that most of the group delay equalizing applications would benefit from further investigation on the audibility of phase characteristics. Many of the group delay equalizer methods trade of either excess bulk delay or preecho to the system versus flat group delay. Further research in especially in temporal masking inherent in the human auditory system would help to optimize these trade offs.

4.3 Audio Effects Processing

Another application for phase equalizer filters is to use them as audio effects. These type of applications for phase equalizer filters are on the contrary purposefully distorting phase response to create audible effects.

Canfield-Dafilou and Abel present novel methods to design allpass filters based on their group delay trajectories. According to the authors, applications of these filters are not limited to only audio effects processing, but they can also be used for abstract sound synthesis, decorrelation, steganography and impulse response measurement.[8]

In the paper, the allpass filter is implemented by constructing its impulse response based on any arbitrary group delay trajectory. By using Equation 6 one can retrieve the phase response from the group delay trajectory. [8]

$$\theta(\omega) = -\int_0^{\omega} \tau(\omega) d\omega \tag{6}$$

The impulse response (IR) can then be computed based on the respective phase response with Equation 7. The authors are computing the IR offline and applying a fast convolution algorithm to the input for real time processing. [8]

$$g(t) = \mathcal{F}^{-1}\left[e^{j\theta(\omega)}\right] \tag{7}$$

At this time, the authors have only demonstrated static implementations of these type of filters. In the future they hope to come up with efficient time-varying designs. [8] This would indeed make these audio effects more useful for practical use.

5 Experiments

In this section audio processing examples were tested in Matlab. The group delay based allpass filter method described in Section 4.3 from [8] was chosen to be implemented. It is relatively simple to implement and it demonstrates definitely audible results by altering just the phase of the signal without touching the magnitude response.

Two different group delay trajectories were tested; a linear trajectory and a modulated trajectory. Both of the tested trajectories were also tested in the reference material. Figure 5 shows the implemented linear group delay trajectory as well as corresponding magnitude and phase responses of the resulting allpass filter. The unwrapped phase response has its minimum at the frequency where the group delay equals pi.

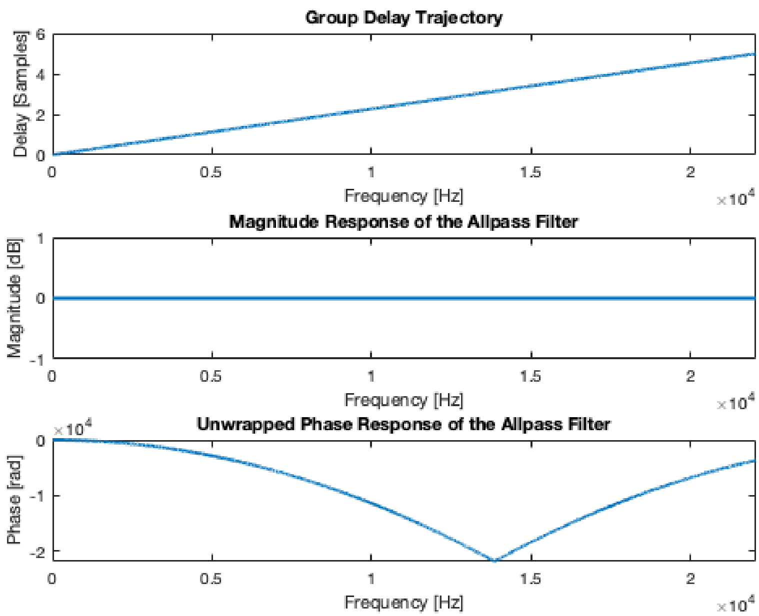

Figure 5: *Linear group delay trajectory and resulting frequency response of the filter.*

Since the group delay trajectory was defined from DC to Nyqvist, in order to implement Equation 7, the negative frequency array was constructed. This was done by appending the mirrored complex conjugate of the group delay array to the original array. Additionally the DC and Nyqvist frequencies need to appear only once in the resulting array and have zero phase. By following these steps the ifft function in Equation 7 results in a real valued signal.

The constructed impulse response of the linear group delay based allpass filter is plotted in Figure 6. As seen from the spectrogram the linear group delay based impulse response becomes a linear chirp.

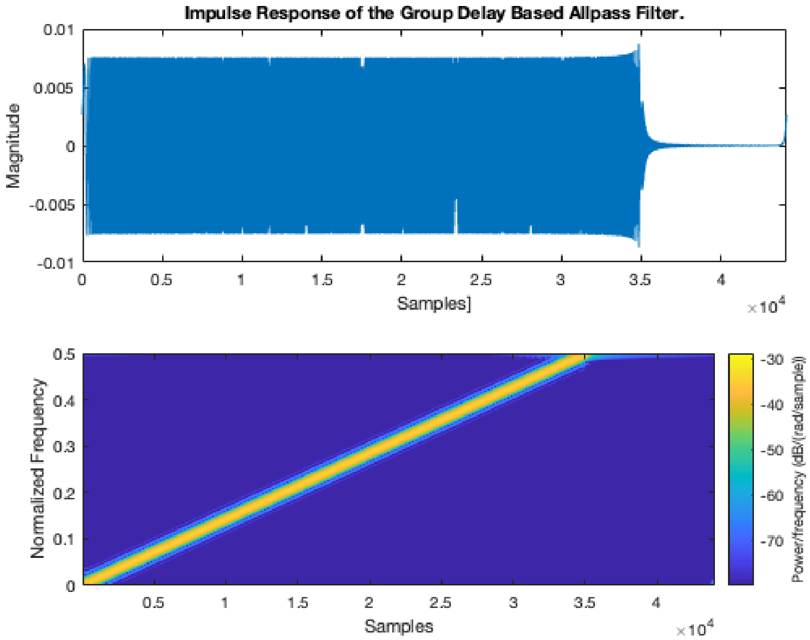

Figure 6: *Constructed impulse response based on linear group delay and its spectrogram. Figure style adapted from [8]*

The impulse response was then convolved with the a short chord progression on electric guitar to have audible results of the filter. The spectrograms of the guitar sample show the first 5 seconds of the signal. The spectrograms of the original sample and the filtered version are shown in Figure 7. As seen from the figure the higher frequencies are delayed compared to the lower frequencies. This can be seen as a tilt in the lower spectrogram. The audible results resemble the sound of a envelope modulated lowpass filter, where the cut off moves rapidly higher in frequency.

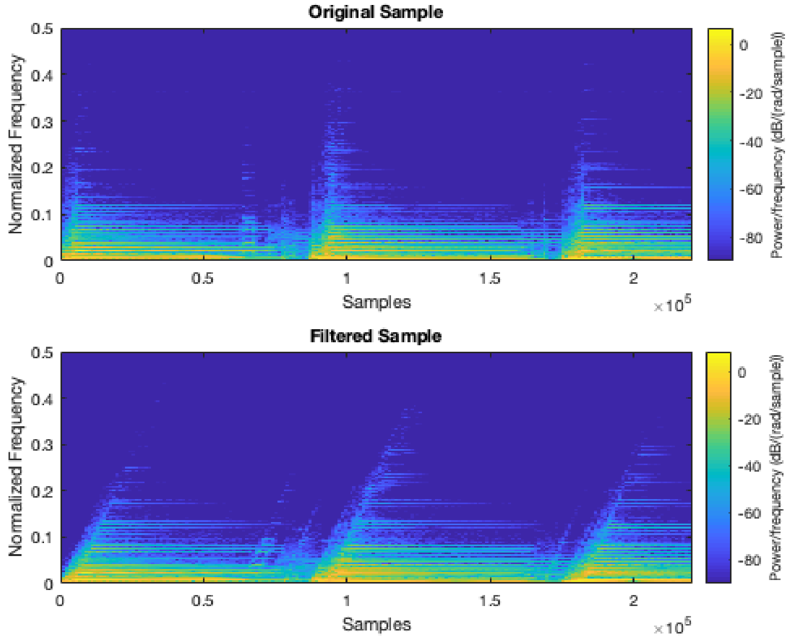

Figure 7: *Spectrograms of the original sample (up) and the linear group delay filtered sample (down). Figure style adapted from [8]*

Figure 8 shows the sine modulated group delay trajectory, and the respective frequency response. The group delay from 0 to Nyquist follows the shape of a single sine wave cycle. The amplitude was set to five. The frequency and the amplitude of the modulator were chosen based on giving pleasant audible results when used on the sample guitar part. As seen from the unwrapped phase response in the bottom most plot in Figure 8 the shape of the phase response resembles a wave shaped sine wave.

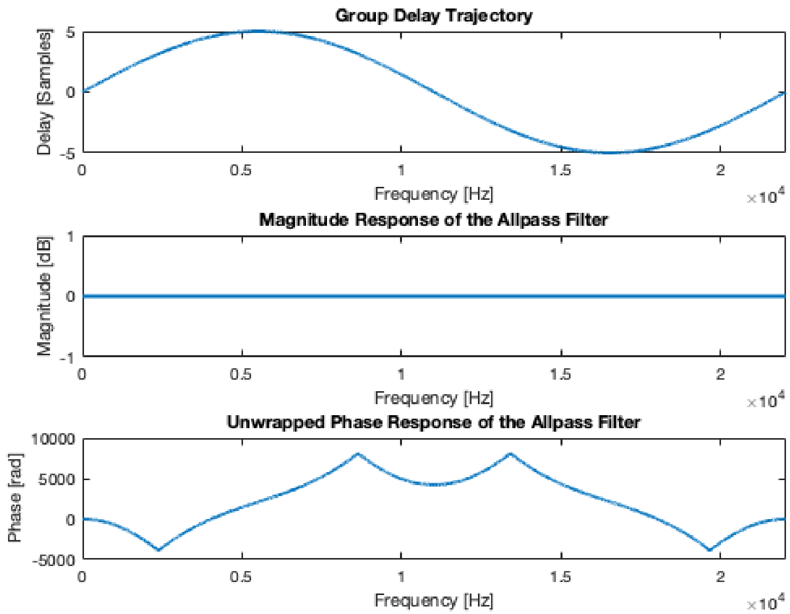

Figure 8: *Sine-modulated group delay trajectory and resulting frequency response of the filter.*

The constructed impulse response and its spectrogram are shown in Figure 10. As seen from the figure the intended group delay seems to have aliasing as the spectrogram shows two mirrored in time "sine shapes" in Figure 9.

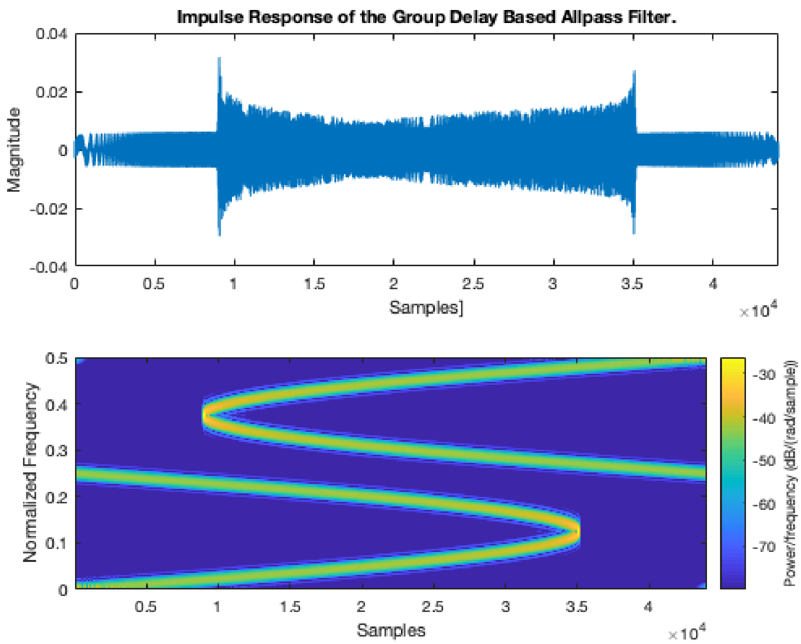

Figure 9: *Constructed impulse response based on the modulated group delay and its spectrogram. Figure style adapted from [8]*

Again, the original sample (up) and the filtered sample (bottom) are plotted in spectrograms in Figure 10. As seen from the figure mostly the bottom ”sine shape” seen in Figure 9 has an effect on the processed signal as the guitar sample is a lowpass signal. Thus the aliasing has very little audible effects here, affecting only above frequencies of 4 kHz. The filtered sample has almost a glassy character to it with a lot of motion in the timbre. Here the audible results are more extreme compared to the linear group delay case.

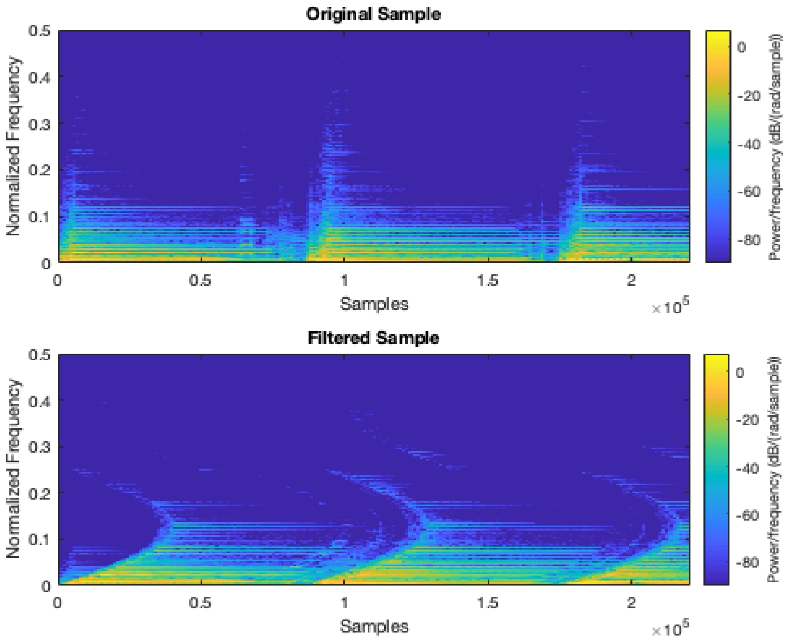

Figure 10: *Spectrograms of the original sample (up) and the modulated group delay filtered sample (down). Figure style adapted from [8]*

The experiments conducted in this section demonstrated the effect of two different group delay based allpass filters on electric guitar processing. Definite audible results were achieved and they were visualized in the spectrograms of Figure 7 and Figure 10. As the authors of this method also stated [8] it would be interesting to implement this type of filters with time-varying properties. However, even with the static implementations some very interesting sounds can be achieved which resemble some phasing or time varying classic filter effects but still have quite an unique sound.

6 Conclusions

In this paper, the concept of phase equalizers was reviewed and some experiments on audio effects processing capabilities of these filters were demonstrated. Also applications in group delay equalizing were introduced including discussion on the audibility of phase.

The biggest problem in designing group delay equalizers seems to be the insufficient data about the concrete amount of temporal masking in the human auditory system. Since all of the methods have some trade offs between flat group delay and temporal smearing, more exact knowledge about the temporal masking would help to decide how much one can and should trade off.

In audio effects processing applications phase equalizer filter are capable of producing novel musical effects.

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