# THE MARCH 11TH SESSION

- Energy in water utilities (20 min)
- Introduction to pumping and pump design (45 min)
- Pump modeling exercise (30 min)
- Break (10 min)
- Introduction to optimization (25 min)
- Optimization in water sector (25 min)
- Modeling exercise – energy, leakage, pumping (30 min)

# INTRODUCTION TO OPTIMIZATION

Aalto University – 2019-03-11

# QUESTION TIME (5 MIN)

*Agitate your gray cells together with your neighbors and think what optimization is?*

*What can be optimized in water utility?*

*When optimization should be done and by whom?*

# LEARNING OUTCOMES

- Describe the meaning and importance, and give examples of
  - Objective function
  - Design variables
  - Constraints
- Describe the difference between classical and meta-heuristic optimization

# OPTIMIZATION

- Optimization means finding the best possible solution, that minimizes or maximizes the objective function value and fulfills the constraints for the problem

$$\min_{\bar{x} \in X} \quad f(\bar{x})$$
$$\text{subject to} \quad g_i(\bar{x}) \leq 0, \quad i = 1, \ldots, m$$

- Where objective function *f*, solution **x**, search space *X*, and constraint function $g_i$
- Elements in vector **x** are called design variables

- Usually optimization is understood as computer or algorithm based method, but optimization can be done, and it often is done – implicitly – as past of engineering process
  - Trial and error and exhaustive search are also forms of optimization

# OPTIMIZATION PROCESS

**Objective**
- Defining objective function

**Structuring**
- Choosing design variables
- Choosing search space

**Constraining**
- Defining constraints

**Optimization**
- The algorithm
- Evaluating candidate solution performance
- Choosing among the results

**Analysis**
- Analyze the results
- Choose the overall best solution
- Validating the solution

**Execution**
- Implement the best solution

# OBJECTIVE FUNCTION

- The most important thing in optimization is deciding, **what will be optimized**?
  - Life cycle costs (LCC)
  - Operational costs
  - Energy use
  - Green house gases
  - Quality
  - Smoothness of pumping
  - Reliability
  - Customer satisfaction
  - Leakage
  - Error between measured and modeled values

- The function that will be optimized is called the **objective function**
  - Usually minimization of (whole) costs
  - Could as well be, for example, maximizing quality or minimizing squared errors (=calibration)

# OBJECTIVE FUNCTION

- Many objectives are contractionary
  - Construction costs VS operational costs
  - Reliability VS operational costs
  - Reliability VS water quality
  - Energy use VS reliability
- On the other hand, many objectives can be affected positively at once
  - Smooth pumping – customer satisfaction – energy use – leakage – (quality)
- Result of optimization is always a compromise between different factors
- Problem formulation has major impact on the results of optimization
- Multi-objective optimization is possible, and it makes the policy decision and implications more transparent

# CONSTRAINTS

- Defining constraints the second most important task
  - What solutions can be considered as feasible?
  - Typically minimum and maximum pressures, velocities, levels etc.

- Different factors can be guided to desired region by defining them as constraints. For example pressure range or water age can be limited.

- Optimization methods always give a solution
- Make sure the solution really is valid and practical – usually testing with model
- It’s important to understand the implications of the problem formulation on the results

# DESIGN VARIABLES

- Design variables are parameters that can be changed and for what the optimization searches the optimal value
  - Hourly flows at different water sources
  - Water tower volume, shape and location
  - Pump model
  - Pipe diameter and location
  - Pressure reducing valve locations and settings
  - Location and size of leaks

- Choosing the design variables, their number, formulation and allowed values has huge effect on the size of search-space and thus on the complexity and computational time of the optimization

- Using integer or binary values as design variables limit the possible algorithm choices w.r.t. real valued variables

# THE CURSE OF DIMENSIONALITY

- The search space size grows exponentially as more design variables are added
  - Finding optimal hourly frequency setting (25-50 Hz) for a pump has search space of $25^{24}=3.6 \cdot 10^{33}$
  - The same for 5 pumps is $5.7 \cdot 10^{167}$ and for 10 pumps $3.2 \cdot 10^{335}$ – $10^{302}$ times larger search space than for a single pump! At 1000 solutions/second it would take $1.0 \cdot 10^{325}$ years to try all solutions.

- Intuition and general engineering skills don’t always take far when optimizing manually – computerized optimization can give surprising results

- A lot is required from the optimization problem formulation and solution implementation
  - Cunning choice of design variables (deltas and fractions instead of direct values etc.) and using pre-optimization can reduce the number of dimensions and the design variable value range considerably
  - In my dissertation the number of pumps to be optimized is 78. In my formulation there’s only 100 design variables instead of 78 x 24 = 1872 in the most obvious formulation. Search space is reduced from $1.48 \cdot 10^{2765}$ to $2.55 \cdot 10^{283}$ – $5.8 \cdot 10^{2481}$ times smaller

# EVALUATION

- The optimization part of the process, the optimization algorithm, always needs to find out the value of a) the objective function and b) values for the constraint functions
- This is called evaluation
- Many optimization methods also require partial derivatives of either a) or both a) and b) with respect to all design variables
- Water distribution network and sewerage equations cannot be solved analytically or differentiated without heavy simplification and mathematical violence
- Thus a numerical method is required for solving the network state, and values for a) and b) – in practice EPANET or EPASWMM model

# OPTIMIZATION ALGORITHMS

- The choice of an actual optimization method or algorithm depends on the properties of the problem (formulation) and computational budget available
- Especially whether the objective function and constraint functions are linear or non-linear, whether they can be solved analytically and are they differentiable
- Often the available optimization algorithms or algorithm classes are limited and known beforehand, and problem is formulated so that it fits the method’s requirements

# OPTIMIZATION ALGORITHMS

- Practically all applicable optimization algorithms are iterative

- Classical algorithms are called “programming”
  - Linear programming (LP), non-linear programming (NLP) and dynamic programming (DP)
  - The algorithms are computationally efficient, but usually unusable in network optimization
  - Require a lot of case-specific math juggling

- Meta-heuristic algorithms are nowadays commonly used
  - Treat the system to be optimized as a black box
  - Only require a way to calculate the objective function value and feasibility given design variable values
  - Can solve practically any problem
  - But they come with a price – they are computationally heavy

# CHOICE OF OPTIMIZATION ALGORITHM

- Optimization algorithm choice effects the quality of the results and computational time required
  - Typically compromise between computational time and the goodness of the solution

- Different optimization algorithms behave differently
  - Good in finding the approximate area in search space where the optimal solution is – global method – gives somewhat good solutions quickly
  - Good in finding the exact optimal solution in a relative small neighborhood – a local method – can converge on locally best solution
  - Find global optimum directly – often unavailable for water sector use cases

- Usually multiple algorithms from the same class can be readily applied for the problem, if the different algorithms accept the same formulation
  - A lot of different programming libraries exist, that support switching algorithms
  - Makes it easier to evaluate the relative performance of different algorithms
  - Algorithms can be also nested or otherwise hybridized

# ANALYSIS AND EXECUTION

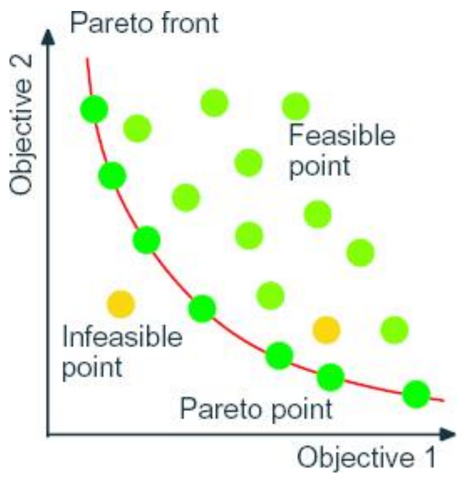
- When the results are at hand, it's time to analyze the proposed solutions carefully
  - Do the solutions actually work in different scenarios?
  - Are they really feasible as a part of the grand scheme of things?
  - Could some of the constraints be more lenient or should they be stricter?
  - How much savings they give?
  - Can they be practically implemented?
- Typically further optimization round can be necessary
- Solutions are further refined during the analysis and design phases
- Make sure, the modified solutions still work and give the anticipated added value, before executing the plan

# SOME FURTHER REMARKS

- Most often life cycle costs, for example, or some other single objective is optimized and all the other important factors are included as constraints – this is called single-objective optimization
- Furthermore the constraints can be included in the objective function value, as the case typically is with meta-heuristic optimization, by normalizing the constraint violations and including them as penalty cost in the objective value
    - Often simplifies the optimization problem
    - Choosing the right penalty coefficients for constraint violations can be tricky and has major impact on the results
    - The penalty coefficients can be them self optimized using an algorith – this is called meta-optimization

- Multi-objective optimization (MO) is also possible, tough it requires more computational budget
- MO produces more information for supporting the decision-making
- Makes the effects of different choices visible and thus policy choices transparent
- Optimization is never value neutral as the choice of constraints and objective implies policy choices – it's good if this can be made clearer to the decision makers

# MULTI-OBJECTIVE OPTIMIZATION & PARETO FRONT




Reference: http://iopscience.iop.org/article/10.1088/1367-2630/11/1/013019

- The best feasible solutions w.r.t. to all objectives form a so-called Pareto front
- When a solution is Pareto optimal impossible to make any objective better without making at least one objective worse
- Every solution at front is optimal in a way
- A choice must be made between the possible solutions, for example based on the available budget or desired minimum level of reliablity

# OPTIMIZATION TOOLS

- Optimization problems typically require at least some level of programming skills as a problem specific configuration and implementation as a form of a program or a script is required
- Spreadsheet programs, Matlab, Scilab and such provide tools for optimization too
- WaterGEMS and some other modeling solutions include some sector specific optimization tools built in, that don’t require programming *per se*
- If there are only a few possible solutions
  - For example <20–50 all possible solutions can be evaluated by hand
  - For example <10 000 – 1 000 000 an exhaustive search or brute-force search can be performed

# CONCLUDING REMARKS ON GENERAL OPTIMIZATION

- There are innumerable number of possibilities to implement an optimization
  - Objective function(s)
  - Constraints
  - Design variables
  - Algorithms

- The only limit to what and how can be optimized is imagination

- Optimization must be done against normal or typical case: average, median or mode demand, for example
- The solution must function correctly and stay feasible in minimum and maximum situations too, but then it doesn’t need to be very efficient or optimal – most of the time the system is functioning under ”normal conditions”