31E11100 - Microeconomics: Pricing

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Lecture 2: Search costs and price dispersion
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Objectives of this lecture

- We saw in the previous lecture that there is substantial price dispersion in many real markets
  - Failure of the Law of One Price
- In this lecture we try to explain this theoretically by looking at a richer model
- The key increadient for the model is that the consumers are imperfectly informed about the prices
- This seems realistic in many situations:
  - Searching for the lowest price is costly (think about opportunity cost of time)
  - Should one bother to search? What is the expected gain in searching? This depends on price dispersion...

- Why would costly search lead to price dispersion?
  - If all the buyers are well informed of all prices, competition should drive all prices low
  - If all the buyers are very poorly informed (say, know only prices in their local store), then the sellers have monopoly power over their local customers and should price high
  - But if some buyers are better informed than others, then perhaps some sellers should price low to attract well informed buyers, while some should price high to sell only to uninformed buyers
  - Is this consistent with buyer's optimal search behavior?
- To analyze this, we need a model

What are economic models?

- Economic models are simplified descriptions of reality
- Their purpose is to isolate key elements of an economic situation to understand causal relationships:
  - How do patent policies affect innovation?
  - How do rental price controls affect housing markets?
  - etc. etc.
- Challenge for modeling: individuals' behavior is not fixed
- Economic models usually build on the following principles:
  - Individual rationality: individuals make choices as if maximizing some objective function.
  - Consistent expectations: what you believe about others is consistent with the others' actual behavior.

Strategic interactions

- Often what is good for you depends on what others do.
  - Do you want to drive on the right-hand side of the road?
  - How much do you want to bid in an auction?
  - For this lecture: Your optimal price depends on prices charged by others.
- Similarly what others want to do depends on what you do.
- Game theory is the tool for analyzing such situations.

Very short introduction to game theory

- To specify a game, one needs to specify *players*, *strategies*, and *payoffs*:
- N economic agents called *players* are engaged in an economic interaction. Set of players $\{1, ..., N\}$.
- Each agent i has a set of feasible choices called *strategies* S_i
- Each player has a preference over vectors of choices $(s_1, ..., s_N)$ represented by a *payoff function* $u_i(s_1, ..., s_N)$.
- We call the collection $\{S_i, u_i\}_{i=1}^N$ a game.

Games in Normal Form

- All players i choose independently and simultaneously a strategy $s_i \in S_{i.}$.
- Payoffs are realized for the vector of choices $(s_1, ..., s_N)$.
- How should each i choose her action?
- The best action of i depends on her beliefs about the choices of s_j for $i \neq j$.
- Consistency of beliefs: Nash equilibrium.

Nash Equilibrium

Definition

A Nash equilibrium is a vector of strategies $s^* = (s_1^*, ..., s_N^*)$ such that for each i, s_i^* solves

$$\max_{s_i \in S_i} u_i \left(s_1^*, ..., s_{i-1}^*, s_i, s_{i+1}^*, ..., s_N^* \right).$$

- This says that at a Nash equilibrium, each player i is maximizing her payoff holding fixed the strategies of the other players.
- Nash equilibrium is a stable situation: starting from a Nash equilibrium, no player has an incentive to change her strategy.
- Conversely, if s is not a Nash equilibrium, then some player has an incentive to change her behavior.
- Notice that we have here N optimization problems that must be solved simultaneously.

- Example: Bertrand Pricing Game:
  - The players are two firms selling identical products. Set of players $\{1,2\}$.
  - Each firm chooses a positive price s_i at which it agrees to sell its product in the market. The size of the market is 1.
  - $S_i = \mathbb{R}_+$ for $i \in \{1,2\}$.
  - All buyers in the market know the prices and buy from a firm charging the lowest price. At equal prices, the market is split equally. Production cost c per unit.
  - Payoffs:

$$u_i = \begin{cases} s_i - c \text{ if } s_i < s_j, \\ \frac{1}{2}(s_i - c) \text{ if } s_i = s_j, \\ 0 \text{ if } s_i > s_j. \end{cases}$$

  - Claim: $s^* = (c,c)$ is the only Nash equilibrium of the game. Hence 2 firms is enough for competitive prices.

- Do all games have a Nash equilibrium? Example: inspection game
  - A firm may commit tax evasion. Gives positive payoff 1 if undetected, but entails loss of 1 due to penalty if detected
  - Tax authority wants to detect fraud, but inspection is costly.
  - Strategies are $S_1 = \{$ Inspect, Do not inspect$\}$ for the tax authority and $S_2 = \{$ Commit evasion, Be honest$\}$ for the firm
  - Assume the following payoffs (The first number in each cell is for the row player = the tax authority, and second for the column player = the firm):

	Commit evasion	Be honest
Inspect	$-1, -1$	$-1, 0$
Do not inspect	$-2, 1$	$0, 0$

- Is there a Nash equilibrium in pure strategies? (i.e. in the sense of our definition above)
- What if the players can randomize over their choices?
- Every finite game has a Nash equilibrium in pure or mixed strategies. See the additional material on game theory for more on this

- Example of a dynamic game: Entry game
  - An entrant considers entry into an industry with a current incumbent firm
  - Entry costs 1 unit
  - Monopoly profit in the industry is 4
  - If entry takes place, the monopolist can either accomodate or fight
  - Accommodation splits monopoly profits, whereas fighting gives zero profit to both firms
  - Will entrant enter, and if so, will incumbent fight or accomodate?
- Dynamic games are normally expressed in *extensive form* (see additional material)
- But can also be be represented in the normal form

- Normal form representation of the entry game:

	Fight if entry	Accommodate if entry
Enter	$-1, 0$	$1, 2$
Stay out	$0, 4$	$0, 4$

- There are now two Nash equilibria: (Enter, Accommodate) and (Stay out, Fight if entry)
- But only (Enter, Accommodate) is a *sub-game perfect* Nash equilibrium
- Sub-game perfect equilibrium can be derived by backward induction: start by analysing the optimal behavior of the incumbent firm once the entry has already taken place

Model with Imperfectly Informed consumers

- Back to modeling price dispersion
- We build a model with a (possibly large) number of buyers and sellers
- We then vary the degree of information that the buyers have.
- How is sellers' equilibrium pricing strategies affected?
- We start with exogenously given information and then ask the question oh how buyers would choose how much information they have.

Formal model

- Suppose an infinite number of idential sellers (for the working of this model does not matter, could be $N = 2$ or just as well).
- A large number of buyers, in equal measure (i.e. one buyer per each firm).
- Each identical seller sells identical goods at production cost c per unit of good.
- Buyers have unit demand with reservation value $v > c$ (i.e. demand one unit of the good as long as price is less than v).
- Firms set their prices simultaneously.
- Fraction α of the buyers sample at random a single price. Fraction $(1 - \alpha)$ sample randomly two prices.
- Buyers choose the lowest of the prices they observe (If the two prices are tied, both sellers are chosen with equal probability)

Game between sellers

- Set of players: the set of all sellers $i \in [0, 1]$.
- Strategies of the sellers: choose price $s_i \in [c, v]$.
- Strategy vector is represented by the cumulative distribution of prices $F(s)$.
    - This means that $\Pr\{s_j \leq s\} = F(s)$ for a randomly chosen j.
    - This summary is adequate since the buyers treat the sellers symmetrically.
    - A single firm cannot change this distribution.

  (With a finite number of sellers, we could model the same as a symmetric mixed strategy profile)

- Fraction β of the buyers visiting firm i have observed a single price offer
- By Bayes' rule:

$$\beta = \frac{\alpha}{2 - \alpha}.$$

- Expected payoff to seller i per customer when choosing s_i and other sellers price according to $F(s)$:

$$\beta(s_i - c) + (1-\beta)\left((1 - F(s_i)) + \frac{1}{2}p(s_i)\right)(s_i - c),$$

  where $p(s_i) = \Pr\{s_j = s_i\}$ for a randomly chosen j.
- Notice that we assume here risk-neutrality of the sellers.
- Simple cases:
  - If $\beta = 1$, then $s_i = v$ is optimal for all i.
  - If $\beta = 0$, then all buyers have seen at least two prices and the equilibria are as in the Bertrand game, and the only equilibrium is $s_i = c$ for all i.

Partial price information

- If $0 < \beta < 1$, then there are no single price equilibria in the market.
  - Suppose to the contrary that all firms were pricing at $s_j = s^*$ for some s^*.
  - If $s^* = c$, firm i gets a higher profit at $s_i = v$ than at $s_i = c$. Hence $s_i = c$ for all i is not an equilibrium.
  - If $s^* > c$, then by setting $s_i = s^*$, firm i gets expected profit (per customer):

$$\beta\left(s^* - c\right) + \left(1 - \beta\right)\frac{1}{2}\left(s^* - c\right) = \frac{1}{2}\left(1 + \beta\right)\left(s^* - c\right)$$

  - For any $s_i = s^* - \varepsilon$, (where $\varepsilon > 0$), firm i gets (per customer)

$$\left(s^* - \varepsilon - c\right).$$

  - This follows since the firm now sells to all buyers with probability 1.
  - Profit from $s_i = s^* - \varepsilon$ exceeds profit from $s_i = s^*$ if:

$$\varepsilon < \frac{1}{2}\left(1 - \beta\right)\left(s^* - c\right).$$

  - Hence $s_i = s^*$ for all i is not an equilibrium.

- One can show similarly that there are no equilibria where $p(s) > 0$ for some s.
    - The argument is basically the same undercutting argument.
- Hence we have in equilibrium that $p(s) = 0$ for all s.
- But then the expected profit for firm i from price s_i if the others price according to $F(s)$ is:

$$\beta(s_i - c) + (1 - \beta)((1 - F(s_i)))(s_i - c).$$

- A really basic and useful observation is that since the firms are similar, and since they all maximize their payoff, all prices chosen in equilibrium must yield the same expected profit. (Why?)
- Furthermore, the highest price chosen in equilibrium must be v. (Why?)

- The profit from choosing $s_i = v$ is

$$\beta (v - c) .$$

- The firm with the lowest price $\underline{s}$ sells with probability 1 to all buyers. By equal profit requirement, we get

$$\underline{s} - c = \beta (v - c) .$$

For prices between the highest and the lowest, we have:

$$\beta (s - c) + (1 - \beta) ((1 - F (s))) (s - c) = \beta (v - c) .$$

- Solving for $F (s)$ gives:

$$F (s) = 1 - \frac{\beta (v - s)}{(1 - \beta) (s - c)} .$$

- You can check that $F (\underline{s}) = 0$ and $F (v) = 1$ and so we have a distribution function on $[c + \beta (v - c) , v]$.

Analyzing the results

- Sanity check: Do the predictions of the model make sense?
- What are the effects of an increase in information (i.e. a decrease in β)?
  - First of all, as β decreases, the lowest price decreases.
  - Furthermore, since $\frac{-\beta}{1-\beta}$ is decreasing in β,

$$F(s) = 1 - \frac{\beta(v - s)}{(1-\beta)(s - c)}$$

  - is also decreasing in β for all s.
  - We say that a distribution G first-order stochastically dominates distribution F if for all s,

$$F(s) \geq G(s).$$

  - Hence we see that price distributions with less information first-order stochastically dominate distributions with more information.
  - Industry profit is simply $2\beta(v - c)$ and hence also decreasing in β.

Is buyers search behavior optimal?

- So far we just assumed that some buyers see a single price and some see two prices
- What is the benefit of observing two rather than one price?
- Expected payoff difference $\Delta(\beta)$ from sampling the prices of two rather than one firm when fraction β of each firm's buyers are uninformed is:

$$\Delta(\beta) := \mathbb{E}s_i - \mathbb{E}\min\{s_i, s_j\},$$

  where s_i and s_j are independent random draws from $F^{\beta}(s)$.
- Note that $\Delta(0) = \Delta(1) = 0$.
- For $0 < \beta < 1$, $\Delta(\beta) > 0$.

Modeling buyers' search behavior

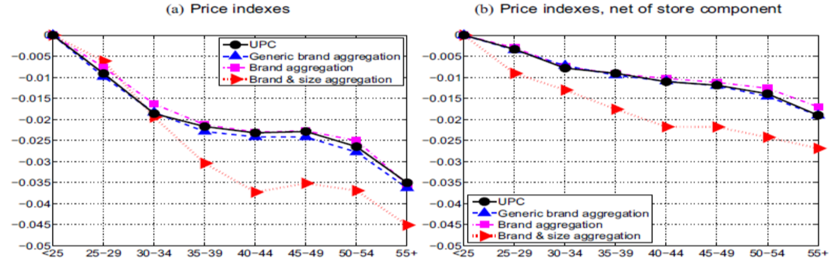
- Two possible models for buyers:
  - Fixed sample search: a buyer must decide at once whether to search for one or two (or perhaps more) prices
  - Sequential search: a buyer samples sequentially and decides after each sample whether to continue search or to buy at the lowest price found so far
- With fixed sample search, we can rationalize the behavior described above by a suitable cost structure
  - Given cost γ for each sampled price, if $\Delta(\beta) = \gamma$, each buyer indifferent between sampling one or two prices
  - Then, a mixed strategy where each buyer samples one price with probability $\alpha = 2\beta/(1+\beta)$ is an equilibrium
- The equilibrium description above remains hence valid even if consumers optimize their search sample
- But what about sequential search? Suppose next that the buyers see one price for free, but after observing it they can observe a second price at a cost γ

Diamond's Paradox

- If buyers see one price for free and then must choose whether to sample another price at cost $\gamma > 0$, then the unique equilibrium in the market is $s_i = v$ for all i, and no buyer pays the cost of becoming informed.
- To see this, notice that given the pricing decisions by the firms, additional price samples bring no benefit.
- Therefore no buyer becomes informed and the equilibrium is as with $\alpha = \beta = 1$ above.
- Moreover, this is the only possible equilibrium. Too see this:
  - Suppose there is an alternative equilibrium where some sellers price below v
  - Take the lowest $s' < v$ in the price support
  - No buyer who oberves price $s \in [s' + \gamma]$ will search for another price
  - So, why wouldn't a seller charging price to s' rather charge $s' + \varepsilon$?
- Striking result: arbitrarily small search costs result in monopoly profits to the firms.

Discussion

- This model shows that we can have price dispersion even with homogenous product, sellers, and buyers
- But with sequential search, this basic model leads to monopoly prices (Diamond' paradox)
- Introducing heterogeneity for sellers and buyers makes it easier to get price dispersion. One can show that price dispersion possible even with sequential search.
- Heterogeneities are natural for buyer side: opportunity costs of search are very different for different people (there are also "shoppers" who get positive utility from search)
- This is supported by empirical results that show that some buyers pay consistently much less for their shopping baskets than others



NOTES: Age refers to average age of household head(s).

FIGURE 9

AVERAGE PRICE INDEXES BY AGE

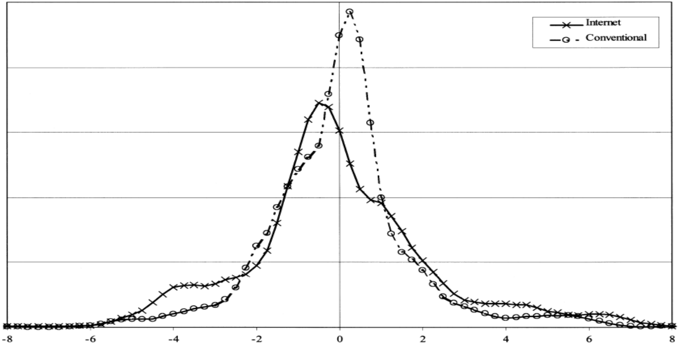
Discussion: on-line shopping and price dispersion

- Buying online should reduce search costs
- Does this mean that price dispersion disappears?
- Empirical findings so far suggest that price dispersion is not disappearing
- Also theory predictions are ambiguous. There are models that allow substantial price dispersion even with very low marginal search costs

Brynjolfsson and Smith (2000): Frictionless Commerce? A Comparison of Internet and Conventional Retailers, Management Science.

Price dispersion of books and CD:s similar in internet and convential retail outlets.

Figure A5 Kernel Density for De-Meaned Full Prices for Books (Epanechnikov Kernel)



Learning points from this model

- Using a game theoretic model, we can explain price dispersion as resulting from imperfect price information amongst buyers.
- The model predicts (reasonably) higher prices for markets with fewer informed buyers.
- The degree of price dispersion in the market is determined in equilibrium, and depends on the interaction between buyers and sellers:
  - Buyers' benefit of search
  - Seller's profitability of lowering prices
- Robustness of modeling: similar results obtain with richer models (see other literature)
- Scope of application: think about labor markets and wage dispersion, minimum wages etc.

Further readings

- The model framework of this lecture was a simplified version of Burdett and Judd (1983): "Equilibrium Price Dispersion", Econometrica.
- Other classical models of consumer search include:
  - Stigler (1961): "The Economics of Information", Journal of Political Economy.
  - Diamond (1971): "A model of price adjustment", Journal of Economic Theory.
  - Varian (1980): "A model of sales", American Economic Review.
- For a review of different models (and empirics as well), see Baye and Morgan (2005): "Information, Search, and Price Dispersion", Handbook on Economics and Information Systems.