31E11100 - Microeconomics: Pricing

Pauli Murto

Aalto University

Lecture 3: Monopoly and price discrimination
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Objectives for the next couple of weeks

- So far we have discussed linear pricing for a homogenous product with many sellers
- Typically sellers have some market power
- More instruments are then available for the seller:
  - Different price for different individuals, or different market segments
  - Different versions with different prices
  - Different unit price for different quantities
  - Time varying pricing
  - Bundling of different products together
- We will analyze such strategies for a monopoly seller

Taxonomy of price discrimination

- Traditionally, price discrimination practices are classified as follows
  - First degree price discrimination, or *personalized pricing*: each buyer gets an individual offer
  - Second degree price discrimination, or *menu pricing*: consumers choose freely from a menu of offers
  - Third degree price discrimination, or *group pricing*: seller can identify different market segments and price them separately
- How are new technologies changing viability of various forms of price discrimination?
- In this lecture we will consider first and third degree price discrimination (since they are conceptually very similar)
- Second-degree price discrimination is conceptually different, since it relies on self-selection by consumers (next lecture)

Framework: Pricing in Monopoly

- The setup is a single firm setting its price in a given market
  - Interpretations: true monopoly
    - Natural monopoly
    - Legal monopoly (patent, copyright, etc.)
    - A unique product
  - One large firm with a competitive fringe of small firms
    - Small firms' reactions can be interpreted as part of the demand curve
    - No game theory needed this time
- We start by analyzing linear prices, then consider non-linear prices and price discrimination

Optimal Linear Price

- Large number of buyers represented by demand curve

$$q = d(p),$$

where $d'(p) < 0$.

- A single seller produces the good with cost function $c(q)$ for producing q units of the good.
- Monopolist chooses the price, and quantity is given by the demand curve.
- Prices are linear so that revenue is pq.
- The monopolist chooses p to maximize revenue net of cost.

Optimal Linear Price

- Monopolist's problem is

$$\max_{p,q\geq 0} pq - c(q)$$
$$\text{subject to } q = d(p).$$

- Substituting the constraint into the objective function gives:

$$\max_{p} pd(p) - c(d(p)).$$

- Notice that this objective function is not always concave. Hence you should check all points at which the first-order condition holds and also the point where p is high enough to make $q = 0$ and pick the point that results in the highest profit.
- First-order condition:

$$pd'(p) + d(p) - c'(d(p))\, d'(p) = 0.$$

- Dividing through by $d'(p)$, and rearranging yields:

$$\frac{p - c'(d(p))}{p} = -\frac{d(p)}{pd'(p)}.$$

- Writing $\varepsilon_p = -\frac{pd'(p)}{d(p)}$ for the price elasticity of demand and $q = d(p)$ for the amount demanded, we have:

$$\frac{p - c'(q)}{p} = \frac{1}{\varepsilon_p}.$$

- In words, the percentage markup of the optimal monopoly price over marginal cost is the inverse of the elasticity of demand in the market.
- Less elastic demand leads to higher markup
- What are examples of markets with inelastic demand? Implications for multi-product firms?

Example of a not so good principle: cost-plus pricing

- For concreteness, assume a cost function with a fixed cost $F = 2000$ and variable cost $60q$. Hence $c(q) = 2000 + 60q$ if $q > 0$ and $c(0) = 0$.
- The demand function is given by

$$q = 200 - p.$$

- Optimal monopoly price is $p^* = 130$, corresponding sales $q^* = 70$, and optimal profit is 2900. (check that you can derive these)
- Suppose next that the firm considers cost-plus pricing.
- This form of pricing is widely used across many industries.
- Its supposed virtue is that pricing decision are based on hard engineering data from production side rather than softer demand estimates.

- Suppose the capacity at the plant is at 100.
- Cost plus pricing takes the form of setting a margin requirement for the sales price.
- Suppose that the goal is to have a 100% margin on production costs.
- How to set the price? How to allocate the fixed cost?
- Variable cost is 60, at full capacity, average fixed cost is 20 per unit.
- Hence cost-plus pricing demands a price of

$$(1+\alpha)(60+20)=160$$

  since at the required 100% margin, $\alpha=1$.
- Realized demand is then 40 and total profit is 2000.
- But now the production department reports that the true fixed cost per unit sold is 50 since production is well below capacity.
- If the price is raised in response to this report to 220, the demand disappears altogether and the firm makes a loss of 2000.

- As this example illustrates: knowing your demand is the key to profits
- But firms do not often have accurate information about the elasticity of demand for their product.
- Nevertheless, they can perform thought experiments along the lines:
  - Suppose we increase price by x per cent.
  - What is the maximal drop in sales volume that makes this increase still profitable?
  - Do we consider a price response of that magnitude is likely?
- Room for consumer surveys.
- Room for experimenting with different prices to find out more about the demand function.
- Models on how to price when also learning about the demand curve are beyond the scope of this course.

Personalized Prices or First-Degree Price Discrimination

- Recall that the market demand is obtained by summing together all individual demand functions:

$$d(p) = \sum_{i=1}^{I} d_i(p),$$

where $d_i(p)$ is the individual demand function of buyer i.

- Suppose now that the seller knows all the $d_i(p)$ and can set individual prices p_i for each buyer.
- Let $\varepsilon_{P,i}$ be the price elasticity of the individual demand of buyer i.
- Optimal pricing is given by:

$$\frac{p_i - c'(q)}{p_i} = \frac{1}{\varepsilon_{P,i}}.$$

- Notice that the marginal cost depends on the aggregate demand.

- Special case: Unit Demands
  - Good is sold in discrete units.
  - Each buyer gets a utility v_i from the first unit, no additional utility from further units.
  - Without loss of generality, rename the buyers so that $v_1 \geq v_2 \geq \ldots \geq v_I$.
  - If each unit costs c to produce, sell to all buyers with $v_i \geq c$.
  - If n units cost $c(n)$ to produce, then sell to the first n^* buyers, where

$$n^* = \max\{n : v_n \geq c(n) - c(n-1)\}.$$

  - Interpretation?
  - For $i \leq n^*$, set $p_i = v_i$.
- With unit demands, monopolist extracts all consumer surplus in the market.
- This can be easily modeled also by assuming a continuum of consumers, with reservation value distributed over an interval on real line

- With more general individual demands, the consumers do get some consumer surplus with linear individual prices.
- But what if the monopolist can use a two-part tariff for each consumer:
  - Suppose the monopolist can use two-part tariffs:

$$
\begin{aligned}
p_i(q_i) &= f_i + p_i q_i \text{ if } q_i > 0, \\
p_i(0) &= 0 \text{ if } q_i = 0.
\end{aligned}
$$

  - f_i is the fixed purchase fee of i.
  - p_i is the linear individual price for i.
  - Why is this helpful for the monopolist? How should the f_i and p_i be set?
  - The principle: choose p_i to maximize total surplus, and use f_i to exctract the consmer's surplus

- Do such two-part tariffs exist in reality?
  - Think about utilities, phone call plans, ...
- With two-part tariffs, the Pareto-efficient market outcome is obtained.
  - Caution: This does not necessarily hold in markets with free entry of sellers.
- Extreme distributional asymmetry. Sellers get all, buyers nothing.
  - This is Pareto-efficient, but is this a good societal outcome?
  - Relies on the seller's perfect knowledge of the preferences of the buyers.
  - Is this realistic?

- What about arbitrage, e.g. resale between buyers?
  - Always a question for models of price discrimination.
- Technological progress might make the model more relevant.
  - Collect information on individual buyers through loyalty cards, social media etc.
  - Tailor price offers available only on their loyalty card.
  - You can even experiment relatively cheaply by issuing coupons (price discounts) and observing the demand reactions.
  - Combined with statistical analysis of all data in the database of the selling firm, this is a potentially successful pricing tool.

Third-Degree Price Discrimination or Group Pricing

- What if the monopolist can identify different group and use separate price for each group?
  - Student/Pensioner/Disabled/Unemployed/Military Service discounts.
  - Geographically separate markets (e.g. countries)
  - What about differential insurance premiums based on sex/age etc.?
- Key assumption: membership in a market segment cannot be manipulated
- This is called third-degree price discrimination
- Can be thought of as a less extreme form of personalized pricing

- N market segments.
- Each with a demand curve $d_n(p)$.
- Since the markets are separate, optimal pricing formula is as before:

$$\frac{p_n - c'(q)}{p_n} = \frac{1}{\varepsilon_{P,n}}.$$

- Implications are then clear: set higher prices for the segments with less elastic demand,
  - What does this mean in terms of the examples listed above?
- What is the value for the seller of this form of price discrimination? What happens to the profit if there is more precise information available (i.e. finer grouping is possible)?
- What are the welfare consequences of this model?
- Let us next examine the effect of group pricing on welfare
  - Sum of consumer surpluses and profit.

- Assume for simplicity that marginal cost is constant at c and there are no fixed costs.
- Assume N market segments and let us compare group pricing to uniform pricing (i.e. the case where $p_n = p$ for all $n \in \{1, ..., N\}$).
- Let $q_n = d_n(p_n)$.
- Consumer surplus in market segment n at prices p_n is denoted by:

$$CS_n(p_n) = \int_{p_n}^{\infty} d_n(z)\, dz$$

- Notice that this is the area under the usual demand curve up to demand level q_n net of the payment $p_n q_n$.
- Total consumer surplus is then $\Sigma_n CS_n(p_n)$.

- If the monopolist had to set a single price for all markets that would be p^*.
- Let $q_n^* = D_n(p^*)$.
- Profit is then $\Sigma_n (p^* - c)\, q_n^*$. Consumer surplus is $\Sigma_n CS_n(p^*)$.
- Let $\Delta q_n = q_n - q_n^*$.
- Difference in total surplus between the discrimination and no discrimination cases is:

$$\Delta W = (\Sigma_n CS_n(p_n) - \Sigma_n CS_n(p^*)) + (\Sigma_n (p_n - c)\, q_n - \Sigma_n (p^* - c)\, q_n^*).$$

- We attempt to find bounds for ΔW
- $CS_n(p_n)$ is a convex function of p_n (can you see why?)
- But then $CS_n(p_n) - CS_n(p^*) \geq CS_n'(p^*)(p_n - p^*)$.
- Since $CS_n'(p^*) = -q_n^*$, we have:

$$\Delta W \geq \Sigma_n (p_n - c) \Delta q_n.$$

- Similarly: $CS_n(p^*) - CS_n(p_n) \geq CS_n'(p_n)(p^* - p_n)$ and thus:

$$\Delta W \leq \Sigma_n (p^* - c) \Delta q_n = (p^* - c)(\Sigma_n \Delta q_n).$$

- Hence an increase in the aggregate supply is a necessary but not sufficient condition for an increase in the total welfare.

Discussion and direction for the next lecture

- Both of the above models rely on the seller's ability to assign buyers to right categories
  - In the first-degree case, individual identification
  - In the third-degree case, identification at the level of the segment
- When is grouping of consumers feasible?
- What if the buyer can manipulate the classification?
  - Second-degree price discrimination or menu pricing
  - Buyers self-select
  - For the next lecture

Further readings

- A review of the economics of price discrimination: Armstrong (2006): "Recent developments in the economics of price discrimination", Advances in Economics and Econometrics: Theory and Applications. Ninth World Congress of the Econometric Society (contains also a lot of analysis of oligopoly that we do not cover in this course).
- For an example of empirical work on international price discrimination, see e.g. Goldberg and Verboven (2001): "The evolution of price dispersion in the European car market", Review of Economic Studies.
- Recent advances in the theory of group pricing include Aguirre, Cowan, and Vickers (2010): "Monopoly Price Discrimination and Demand Curvature", American Economic Review and Bergemann, Brooks and Morris (2015): "The Limits of Price Discrimination", American Economic Review.