Microeconomics: Pricing
31E11100
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Final Examination
Please answer all four questions.

1. Determine if the following claims are true or false. Provide short explanations for your answers and explain the terms in italics.

   (a) Whenever the "*Law of One Price*" fails in a given market, some firms must be making rents (higher profits than others) in that market.

   (b) When a monopolist engages in *third-degree price discrimination*, some segment of the market pays a higher price than in the *uniform price* case.

   (c) In a model with differentiated qualities and *second-degree price discrimination*, high valuation buyers consume goods of higher quality than the low value buyers and therefore they get a higher consumer surplus than in the case where the monopolist must offer the same quality to all buyers.

   (d) Since the winning bidder in a *second-price auction* pays the second highest bid, the seller gets a smaller revenue in second-price auction than in a *first-price auction.*

2. The inverse demand function for rooms in the only hotel of a small town is given in two seasons $i \in \{s, w\}$ by

$$p^s = 10 - q^s,$$
$$p^w = 10 - 2q^w.$$

Here q^i is the number of rooms occupied in season i and p^i is the market clearing price for that occupancy. Let k denote the available capacity of hotel rooms available. Let fk denote the cost of building capacity for k rooms. Capacity is built once and it lasts for both seasons. Let $c^i q^i$ denote the cost of servicing occupants in q^i rooms in season i. Notice that the cost of servicing rooms depends on the season. Since you cannot house guests in more rooms than the capacity, you must have $q^i \leq k$.

(a) Write the revenue and the costs of the hotel over the two seasons as a function of (k, q^s, q^w).

(b) Find the optimal choice of (k, q^s, q^w) to maximize the hotel's profit. (Hint: When is the capacity constraint binding in both seasons?)

3. Consider a model where a monopolistic seller offers a service for sale at different dates. Assume that buyers differ in their valuations in the following sense: High valuation buyers have a willingness to pay of θ^H for immediate delivery of the service, low valuation buyers have a valuations θ^L for immediate delivery. Let q denote the delay in delivery time and assume the following payoff function:

$$v(\theta, q) = \begin{cases} 2\theta - \theta q, & \text{if } q \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Let the cost of the seller as a function of delay q be $c(q) = (2-q)^2$ for $0 \leq q \leq 2$ and $c(q) = 0$ for $q > 2$.

(a) What are the first best delays for the two types of buyers? I.e. how would the seller choose $\widehat{q}^H$ and $\widehat{q}^L$ for θ^H and θ^L respectively if she could see the type of the buyer? What would the profit maximizing (nonlinear) prices $\widehat{t}^H$ and $\widehat{t}^L$ corresponding to those delays be?

(b) Would the menu $\{(\widehat{q}^H, \widehat{t}^H), (\widehat{q}^L, \widehat{t}^L)\}$ be incentive compatible if the seller does not see θ? Suppose that fraction λ of the buyers are of type θ^H and $(1-\lambda)$ are of type θ^L.

(c) For concreteness, take $\theta^H = 3$ and $\theta^L = 1$ and solve for the profit maximizing incentive compatible menu of delays and (nonlinear) prices for the seller.

4. A seller auctions an indivisible object to one of three potential buyers. Suppose that the valuations θ_i of the bidders are privately known, statistically independent, and drawn from the uniform distribution on $[0, 1]$ so that the cumulative distribution function $F(\theta_i) = \theta_i$.

   (a) Assume that the bidders have private values and the auction is a second-price auction. Explain what is meant by a second-price auction and argue that bidding the true valuation is a dominant strategy in this auction.

   (b) Suppose that the bidder submitting the highest bid wins the object and pays the third highest bid. Is it still optimal to bid your true value?

   (c) Assume next that after the initial second-price auction, one of the bidders, say bidder 3, has the option of participating in a second auction (also second-price) where an identical object is auctioned between bidders 3,4, and 5. Suppose that bidder 3 has value for only one unit of the good so that she bids 0 (and loses) in the second auction if she won the first. The valuations θ_4 and θ_5 are again private information and independently drawn from the uniform distribution. Discuss how bidder 3 should bid in the first auction. In particular, should she bid her true valuation in the first auction?