Microeconomics: Pricing
31E11100
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Final Examination
Please answer all four questions.

1. Determine if the following claims are true or false. Provide short explanations for your answers and explain the terms in italics.

   (a) When a monopolist engages in *third-degree price discrimination*, then total consumer surplus in the market falls relative to the *uniform price* case.

   (b) In a model with differentiated qualities and *second-degree price discrimination*, buyers with low valuations benefit from the existence of buyers with higher valuations.

   (c) If the buyers of a multiproduct monopolist have independent valuations for the products, then the monopolist makes the optimal profit by selling each product sperarately at its monopoly price.

   (d) When a seller has no use for an indivisible good by herself, she can get the highest possible sales revenue for it by auctioning the good in a *second-price auction.*

   (e) A monopolist selling *durable goods* over time can always make a better profit by renting the goods rather than selling them.

2. Consider a market with two segmentss s and r where the inverse demand functions for segments s, r are given by:

$$p^s = \alpha^s - 2q^s, p^r = \alpha^r - q^r.$$

Assume that the cost function is linear:

$$c(q^s, q^r) = cq^s + cq^r,$$

and $c < \alpha^r < \alpha^s$.

(a) For the case where the seller cannot distinguish between buyers from the different segments, compute the optimal uniform price.

(b) How much would the seller be willing to invest in a technology that allows to separate the two market segments and to set a different price for each.

(c) Give real life examples that fit this model description.

3. A monopolist sells a product in a marketmarket where the buyers differ in their willingness to pay. Suppose that the buyers have logarithmic utility functions where q denotes the quantity of the good that they consume and t denotes the price they have to pay for these q units.

$$u(\theta, q, t) = \theta \ln(q+1) - t.$$

Assume that the cost of production is linear so that the marginal is constant:

$$c(q) = cq, \text{ with } 0 < c < 1.$$

(a) Suppose first that the seller knows the buyer's valuation and solve the optimal q and t (as functions of θ).

(b) Suppose there are two types of buyers: $\theta^H = 3$, $\theta^L = 1$. Let λ denote the fraction of buyers of type θ^H. Write down the IC and IR constraints for both types of buyers. How large is the information rent of type θ^H (as a function of q^L)?

(c) Which of the IC and IR constraints bind? Solve the optimal menu $\{(q^H, t^H), (q^L, t^L)\}$. For what values of λ will the monopolist sell positive amounts to θ^L?

(d) Suppose that the monopolist must set a single linear price p in the market (so that $t = pq$). What is the optimal linear price p for $\lambda = \frac{1}{2}$?

4. A seller auctions an indivisible object to one of three potential buyers. Suppose that the private valuations θ_i of the bidders are privately known, statistically independent, and drawn from the uniform distribution on $[0, 1]$ so that the cumulative distribution function $F(\theta_i) = \theta_i$.

   (a) Assume that the auction is a second-price auction. Explain what is meant by a second-price auction and argue that bidding the true valuation is a dominant strategy in this auction.

   (b) Analyze a first-price auction. Argue that this auction does not have a solution in dominant strategies and find a symmetric bidding equilibrium for the game.

   (c) Assume next that players 2 and 3 can collude in the sense that they can disclose their valuations to each other and form their bids to maximize their joint surplus (i.e. the payoff from winning the auction and getting the valuation $\max\{v_2, v_3\}$ net of the price paid in the auction). How does this collusion opportunity change the optimal bids of the colluding bidders in a second-price auction? What about first-price auction? How does your answer depend on whether bidder 1 knows that 2 and 3 collude or not?