31E11100 - Microeconomics: Pricing

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Lecture 10: Introduction to auctions
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Plan for the last three lectures of the course

- For the rest of this course, we will analyze auctions
- We will first introduce the topic with examples to demonstrate some of the key issues
- We then unify the analysis using more theory
  - Auctions as Bayesian games
  - Envelope formula and revenue equivalence theorem
- We will look at some key auction design elements such as:
  - Reservation price, handicaps, other instruments
- We will consider other practical auction design issues:
  - Collusion, entry
- Finally, we discuss common value auctions and information aggregation

Why use auctions?

- Suppose a seller has a single item to sell and a number of potential buyers. How to set the price?
- Why use an auction?
  - What is the right price? If too high, no one buys. If too low, excess demand.
  - Potential buyers know what they would pay, but why should they tell?
  - Auction is a mechanism for *price discovery*
  - At the same time, auction *induces competition* between buyers
- There are different possible auction formats
  - How to design an auction?

Most common auction formats

- Sealed bid auction
  - Seller asks for a single bid from each participant.
  - Highest bid wins and pays her bid.
  - Common in selling real estate and different commodities, or in procuring services (then lowest bid wins, and supplies a service at the bid price).
  - An important variant: second price auction.
    - Highest bidder wins but pays the second highest bid.
- Ascending price auction
  - Price starts low and raises gradually.
  - Bidders drop out.
  - The bidder who stays longest wins and pays the price where second last bidder drops out
  - Common for art, antique, company take-overs, ...
  - A variant: descending price auction (Dutch auction: price starts high and falls until someone buys - used in Dutch flower auctions).

Simple model

- A single object is for sale.
- Two bidders with values v_1 and v_2.
- Valuations v_1 and v_2 are independently drawn from uniform distribution $[0, 1]$.
- Valuations are private information of the bidders.
- Seller decides an auction format: ascending, second-price sealed bid, first-price sealed bid.
- We will later consider a more general value distribution, and N bidders instead of just 2.

Ascending auction

- Let us first consider the ascending auction.
- Price starts ascending from 0 and bidders indicate their willingness to buy by staying in the game.
- As soon as one bidder drops out (e.g. say "I give up"), the remaining bidder wins and pays the standing price.
- This is a game, where the strategy of each bidder is to decide when to "stop" (i.e. drop out).
- When should you stop?

Ascending auction

- The optimal strategy is: stay in the game until price hits your valuation.
- This strategy is optimal *irrespective of the strategy of the other player.* (Why?)
- Bidder with the highest valuation wins.
  - Outcome is efficient.
- Winner pays the second highest value.

Ascending auction

- What is the expected revenue by the seller?
  - Revenue is equal to the second highest valuation (i.e., with two bidders, the lowest valuation).
  - Hence, expected revenue is the expectation of the second highest value.
  - How to compute this? Derive the probability distribution for the second-highest valuation (*second order statistic*), and compute its expectation.
- Let $G(b)$ denote the cumulative distribution function (c.d.f.) of the second order statistic:

$$G(b) = 1 - (1 - b)^2$$

- Can you derive this? How to compute expected revenue from here?

Ascending auction

- With two bidders, expected revenue is $1/3$ (can you compute this?)
- Expected value of the winner is $2/3$ (why?)
- Hence, surplus is split equally between seller and winning bidder (on expectation)
- What if there are more bidders?
  - With 3 bidders, it is easy to show that expected revenue is $1/2$
  - Expected value of the winner is $3/4$
  - Hence, total surplus increases, but the share that goes to seller increases too

- This generalizes: as N increases, the seller gets a larger and larger share of the total surplus
  - With 10 bidders, expected price is $9/11$ and expected value of winner is $10/11$
- Is auction a good method for the seller? As a comparison, what would be the expected revenue if the seller just chooses optimal posted price? (i.e. posts a single price, and sells if at least one bidder has a value exceeding this)

Second price auction

- Let us next consider second-price sealed bid auction.
- Both bidders submit simultaenously a sealed bid (e.g. write it on a paper and submit to the seller).
- Bidder who submitted the highest bid wins, but pays the second highest bid.
- The strategy for each bidder is simply the bid.
- How should you bid?

Second price auction

- Claim: irrespective of the other bidder's strategy, it is optimal to bid one's valuation.
- Why?
  - Consider an alternative strategy (bid above/below your valuation).
  - Would such a deviation affect what you pay if you win?
  - Would such a deviation affect whether or not you win? If so, when? Would you be happy about that effect?
- As a result, in equilibrium every bidder bids their true value.
  - Bidder with the highest value wins.
  - Pays an amount equal to the the second highest value.
- This is exactly the same as in ascending auction: same winner, same price.
- Second price auction is an example of a "Vickrey"-auction, where bidding truthfully ones own valuation is optimal.

First price auction

- Finally, consider the first price sealed bid auction.
- As above, bidders submit bids simultaneously.
- Highest bid wins, but now the winner pays her own bid, i.e. the highest bid.
- Does this mean the this gives a higher revenue to the seller?
- Do the buyers behave similarly as in the second price auction?

First price auction

- Is it now optimal to pay your own bid?
  - Clearly you should bid less.
  - But how much less?
- Submitting a higher bid will
  - Increase chances of winning.
  - Decrease the surplus if winning (value minus price).
- Optimal bid will depend on what you think the other(s) will do (unlike with second price auction).
- We need to consider a full *equilibrium analysis.*

Bayesian Nash equilibrium

- A set of bidding strategies is a Bayesian Nash equilibrium if each bidder's strategy maximizes her expected payoff given the strategies of the other bidder(s).
- We will analyze this more thoroughly in the next lecture, but for now it sufficies to note that since each bidder know privately her valuation, a strategy must determine what a bidder bids as a function of her valuation.
- This is a game of incomplete information (each bidder knows privately her own value), hence the term *Bayesian* Nash equilibrium.
- Each bidder's equilibrium strategy must maximize her expected payoff accounting for the uncertainty about other bidders' values.

Finding the equilibrium bid function

- This example with two players and uniform value distributions can be solved easily by a simple trick (we will analyze the more general model later).
- Suppose bidder 2 uses bidding strategy $b_2(v_2) = \beta v_2$ for some $\beta > 0$.
- What is then the optimal bid for bidder 1? Suppose bidder 1 has value v_1, and consider payoff of bidding b:

$$\begin{aligned}
\pi(b; v_1) &= \Pr(win)(v_1 - b) \\
&= \Pr(\beta v_2 < b)(v_1 - b) \\
&= \Pr\left(v_2 < \frac{b}{\beta}\right)(v_1 - b) \\
&= \frac{b(v_1 - b)}{\beta}.
\end{aligned}$$

- This is maximized by choosing $b = \frac{1}{2} v_1$.

Finding the equilibrium bid function

- So, if bidder 2 uses a linear bidding strategy, the *best response* of bidder 1 is to use a linear bidding strategy $b_1(v_1) = \frac{1}{2}v_1$.
- Hence, if both bidders bid half of their value, they are both best-responding to each other.
- In other words, this is a Bayesian Nash equilibrium. In this equilibrium, both bidders use strategy

$$b_i(v_i) = \frac{1}{2}v_i,\ i = 1, 2.$$

Comparison between first price auction and ascending auction/second price auction

- How do the properties of the equilibrium correspond to ascending auction/second price auction?
- Bidder with the highest value wins here too: auction is efficient.
- How about expected revenue? Let us compute:
  - Remember, expected highest value is $\mathbb{E}\left(\max\left\{v_1, v_2\right\}\right) = \frac{2}{3}$
  - Therefore, expected price is $\mathbb{E}\left(\max\left\{b_1\left(v_1\right), b_2\left(v_2\right)\right\}\right) = \frac{1}{2}\frac{2}{3} = \frac{1}{3}$.
  - This is the same as with ascending auction/second price auction!
- Is this a coincidence?

Revenue equivalence theorem

- The equivalence of expected revenue in first price auction and ascending/second price auction is a manifestation of so called *Revenue equivalence theorem.*
- As we will see formally in the next lecture, it holds to any auction format where highest value bidder always wins.
- For example, the expected revenue would be the same in:
- Dutch auction
  - Price starts high and drops continuously until someone buys.
  - Strategically "equivalent" to a first price auction.
- All-pay auction
  - Bidders submit bids, high bidder wins, and everyone has to pay their own bid.
  - Winner pays on average less than in standard formats, but expected total payment is the same since also losers pay.
  - Not commonly seen as an auction format, but used as a stylized model of contests (e.g. political lobbying or R&D race).

Reserve price

- Is there any way for the seller to increase expected revenue?
- Suppose the seller sets a reserve price r, i.e. minumum accepted price.
- Is it a good idea?
  - Potential benefit: higher price.
  - Drawback: maybe no sale (if all bidders have value below r).
- Consider second-price auction with reserve price $r = \frac{1}{2}$ and compute expected revenue. Note:
  - if $\min\{v_1, v_2\} > r$, then price is $\min\{v_1, v_2\}$.
  - if $\min\{v_1, v_2\} < r < \max\{v_1, v_2\}$, then price is $p = r$.
  - if $\max\{v_1, v_2\} < r$, then there is no trade.

Reserve price

- Can you compute the expected revenue? (it is indeed higher than without reserve price)
- Can you show that $r = \frac{1}{2}$ is the optimal reserve price in this case?
- Revenue equivalence theorem does not hold here, because now the auction is not efficient! Reserve price means that sometimes there is no trade at all even when bidders have positive values for the object.
- There is a trade-off between efficiency and revenue.
- So, standard lesson about monopoly power applies in auctions too!
  - Monopolist distorts allocation (causes inefficiency) in order to transfer consumer surplus into profit.

Example: Common value auctions and winner's curse

- Consider an auction for a jar of coins (second-price, say).
- All bidders can see the jar and can visually assess the amount of money it contains
- How much would you bid?
- Suppose you bid your best estimate of value and win.
  - What does that tell about others' estimates?
  - What does that tell about your own estimate?
- Bidder's should take this into account in their bids to shield them from winner's curse
- Is this relevant in real life? Instead of jar of coins, think of
  - Oil field with unknown amount of oil.
  - Company going public in an IPO.
  - Any financial asset.
- This is an example of a *common value auction.* We will come back to this later..

Summary so far

- We looked at the most common auction formats in the simplest possible environment
  - Two symmetric bidders, private values drawn independently from uniform distribution
- We saw that although different auction rules induce different behavior, they may give rise to surprisingly similar outcomes in terms of efficiency and revenues (However: there are also differences that we have not talked about yet!)
- Auction design is an area in microeconomics with many important applications: spectrum auctions, treasury auctions, internet auctions, electricity markets, etc.etc.
- We will next:
  - Dig a bit deeper into the theory.
  - Look at more practical issues relevant for auction design.
  - Look at some applications.