31E11100 - Microeconomics: Pricing

Pauli Murto

Aalto University

Lecture 12: Topics on auction design
October 17, 2018

Plan for the leecture

- In this last lecture we will first discuss some specific auction design issues:
  - Asymmetric auctions and subsidies
  - How to guarantee participation of bidders?
  - How to prevent collusion by bidders?
- Then we will discuss common values auctions
  - Winners curse
  - Information aggregation in auctions

Reserve price

- We already saw in the previous lectures that seller can increase profits by using a reserve price
- This is a way for the seller to extract more information rents from the buyers
- Trade-off between efficiency and rent extraction
- Similar to standard monopoly pricing
- Other auction design instruments?

Bidder subsidies and set-asides

- In real auction it is common that seller treats some bidders preferentially. Why?
- Distributional reasons:
  - Government favoring domestic bidders, municipal favoring local producers in procurement, etc.
  - Favoring of small businesses by subsidies or restricting entry (exclusions, or set-asides)
- Competition, or other post-auction market reasons:
  - Make sure there is sufficient competition in the market after auction
- Is it possible to increase revenue by subsidies?
- Let us look at a specific example with asymmetric bidders

Example of bid subsidies

- Two bidders with private values v_1 and v_2.
- Suppose the bidders are ex-ante asymmetric in the following sense:
  - Valuations are independently drawn from

$$\begin{aligned} v_1 &\sim U[0,1], \\ v_2 &\sim U[0,2]. \end{aligned}$$

- Consider an ascending auction (or equivalently, second price auction)
  - Both bidders bid up to their values and the higher value bidder wins.
  - This is more likely to be bidder 2.
- What is the expected price?

Example of bid subsidies

- Consider two equally likely events:
  - Bidder 2 has value $v_2 > 1$
  - Bidder 2 has value $v_2 < 1$
- In the former case, bidder 2 wins and pays on expectation $1/2$
- In the latter case, each bidder as likely to win, and expected price $1/3$
- So, bidder 2 wins with probability $\frac{3}{4}$ and the expected revenue is $\frac{1}{2}\frac{1}{2} + \frac{1}{2}\frac{1}{3} = \frac{5}{12}$.

Example of bid subsidies

- Suppose the seller gives 50% discount to the weaker bidder (bidder 1)
- What is the optimal bidding strategy of bidder 1?
  - Bid up to $2v_1$
- Behavior of bidders is as if both bidders have values drawn uniformly from $[0, 2]$
- As a result, both bidders are as likely to win
- Expected "clock price" is now $\frac{2}{3}$
- But taking into account the subsidy payment, the expected revenue of the seller is

$$R = \frac{1}{2}\frac{2}{3} + \frac{1}{2}\frac{1}{3} = \frac{1}{2}.$$

Example of bid subsidies

- Effect of subsidies:
- With no subsidy
  - Strong bidder is more likely to win ($\frac{3}{4}$ against $\frac{1}{4}$)
  - Expected revenue is $\frac{5}{12}$
  - Auction is efficient: higher value bidder always wins
- With subsidy:
  - Both bidders equally likely to win
  - Expected revenue is $\frac{1}{2} > \frac{5}{12}$
  - Auction is inefficient
- Again: seller gives up on efficiency to increase revenue

Entry of bidders

- A common problem in organizing auctions: how to ensure there are enough bidders participating?
- More bidders guarantees more competition
- But if bidders expect tough competition, why would they participate if entry is costly?
- This is a typical problem for example in procurement auctions, where it takes some work and effort for the participants to prepare offers
- Asymmetries can also be problematic

Entry of bidders

- Take the same example as above. Two bidders with independently drawn valuations:

$$v_1 \sim U[0,1],$$
$$v_2 \sim U[0,2].$$

- Second price auction / ascending auction
- Ex-ante expected payoffs of the two bidders (before they learn their valuations):
  - Bidder 1 expects to get $\frac{1}{12}$ (why?)
  - Bidder 2 expects to get $\frac{1}{2}+\frac{1}{12}$ (why?)

Entry of bidders

- Suppose now that there is a cost of $\frac{1}{10}$ to enter
  - Think of this as the cost of learning how much you value the good (cost of inspecting the procurement contract, cost of learning the production cost of service, etc.)
- Given this, bidder 1 should not enter at all
- Therefore, bidder 2 is the only one to enter and bids zero!
- Not good for the seller...

How to promote entry of bidders in practice?

- Subside weaker bidders
  - Increase their payoff of entering, hence encourage entry
- Subsidize the entry costs directly
  - E.g. reimburse costs of preparing documentation for procurement contract offers
- Restrict the strong bidders from participating: set-asides
  - Excluding a strong incumbent may increase profits by inducing more competitive entry
- Auction format?
  - The possibility of very low price in the sealed bid auction may encourage weaker bidders to participate in the hope of stealing the auction from the stronger bidders
  - In contrast, in ascending price auction, the strong bidders can always respond to weaker bidders

Collusion

- Collusion occurs if bidders agree in advance or during the auction to let price settle at some low level.
  - This is illegal, but happens anyway.
- This occurs most naturally in situations, where there are multiple items for sale.
  - All bidders get a fair share, why raise price?
  - In extreme situations, incentives for price competition can be very low, even without formal collusion.
  - E.g. three similar objects, three bidders. Each bidder gets one, why raise prices?
  - Spectrum auctions?
- With a single object, collusion may rely on:
  - Side agreements: you win and share profits with me.
  - Intertemporal arrangement: you win today, I win tomorrow.

How to deter collusion?

- Tougher law enforcement?
- What about the auction format?
- Ascending auction
  - Suppose bidders 1 and 2 agree in advance that 1 should win.
  - What happens if bidder 2 deviates the agreement, and keeps on bidding as price increases?
  - Bidder 1 can bid back - makes deviation unprofitable and helps the collusion.
- Sealed bid auction
  - Again, suppose bidders 1 and 2 agree on bids such that bidder 2 wins.
  - But then bidder 1 can secretly outbid and steal the auction.
  - Deviation from agreement more tempting - makes it harder to sustain collusion.

Common value auctions

- So far we have considered models, where
  - each bidder's value depends on his/her own signal only (private values), and
  - signals are independently drawn
- Recall the example: how much would you bid for a jar of coins?
- Here the value of the object is common to all the bidders, but different bidders have a different estimate about the value
- Do you care about the estimates of the other bidders?

Winner's curse

- Winning means that all the other bidders were more pessimistic about the value than you.
- Winning is "bad news".
- Equilibrium bidding should take this into account.
- But how exactly?
- Do bidders take it into account in reality?
  - If not, then selling jars of coins is a money printing business
  - Experienced/inexperienced bidders

A simple model of common value auction

- Suppose that there is a common value v for the good, but its value is unknown.
- Formally, v is a random variable with some known probability distribution (e.g. Normally distributed)
- Both bidders observe a private signal that is correlated with the trure value v. For example, we might have

$$\begin{aligned} \theta_1 &= v + \varepsilon_1, \\ \theta_2 &= v + \varepsilon_2, \end{aligned}$$

  where ε_1 and ε_2 are some i.i.d. random variables (e.g. Normally distributed noise terms)
- Then a high signal indicates that it is likely that also v is high

A simple model of common value auction

- This model is often called mineral-rights model
  - think of v as the true value of an mineral right, such as oil field
- Note: θ_1 and θ_2 are independently drawn, *conditional on* v
- But because v is unkonwn, θ_1 and θ_2 are correlated with each other through v
- Signals provide information about v (but only imperfect):
  - The expected value for bidder i based on her own signal is $\mathbb{E}(v|\theta_1)$
  - The expected value based on both signals is $\mathbb{E}(v|\theta_1, \theta_2)$
- It is natural to assume that these are increasing in signal values (a high signal predicts a high value)

A simple model of common value auction

- Recall from the previous lecture, we can specify an auction environment by defining the utility for a bidder if he wins as $u_i(\theta_i, \theta_{-i})$.
- In this case, we have:

$$u_i(\theta_i, \theta_{-i}) = \mathbb{E}(v \,|\theta_1, \theta_2).$$

- Hence, the utility of winning depends on both signals
- Moreover, the signals are correlated
- Hence, this is an auction with *interdependent values* and *correlated signals*

How to bid in a second price common value auction?

- Assume second price auction format
- Suppose bidder 2 uses strategy $b_2(\theta_2)$
- Bidder 1 has signal θ_1. How to bid?
- Consider bidding some p, or slightly more or less:
  - Makes no difference if $b_2(\theta_2) << p$, or if $b_2(\theta_2) >> p$
  - Only matters if $b_2(\theta_2) \approx p$
- The only situation where b is "pivotal" is when $b_2(\theta_2) = p$, i.e. $\theta_2 = b_2^{-1}(p)$.

How to bid?

- If bidder 1 wins being pivotal, her expected value for the object is

$$\mathbb{E}\left(v\left|\theta_1, \theta_2 = b_2^{-1}(p)\right.\right)$$

- To be indifferent between winning and not means

$$p = \mathbb{E}\left(v\left|\theta_1, \theta_2 = b_2^{-1}(p)\right.\right).$$

- Bidding more or less than p would lead to expected loss, so a best response strategy $b_1(\theta_1)$ for bidder one is to bid $b_1(\theta_1)$ that satisfies:

$$b_1(\theta_1) = \mathbb{E}\left(v\left|\theta_1, \theta_2 = b_2^{-1}(b_1(\theta_1))\right.\right).$$

How to bid?

- Hence, a symmetric Bayesian equilibrim is given by $b(\theta)$ that satisfies:

$$b(\theta_i) = \mathbb{E}\left(v \,|\theta_i, \theta_{-i} = \theta_i\right).$$

- It is optimal to bid as if the other bidder has exactly the same signal as you
- This generalizes to a symmetric model with N bidders:

$$b(\theta_i) = \mathbb{E}\left(v \middle| \theta_i, \max_{-i}\{\theta_{-i}\} = \theta_i\right).$$

- In other words, you should bid as if you have the highest signal, and the second highest signal within all the bidders is the same as your signal
- How would you now bid for the jar of coins?

No regret property

- The strategy that we derived shields against the winner's curse
- Suppose that bidder 1 wins:

$$b(\theta_1) > b(\theta_2) \Longleftrightarrow \theta_1 > \theta_2$$

- Bidder 1 expected value post auction is $\mathbb{E}(v \mid \theta_1, \theta_2)$
- But her payment is $\mathbb{E}(v \mid \theta_2, \theta_2) < \mathbb{E}(v \mid \theta_1, \theta_2)$ (note: second price auction)
  - Bidder 1 is happy she won
- Bidder 2 expected value post auction is also $\mathbb{E}(v \mid \theta_1, \theta_2)$
- But to win, she should have outbid bidder 1, in which case she would have paid $\mathbb{E}(v \mid \theta_1, \theta_1) > \mathbb{E}(v \mid \theta_1, \theta_2)$
  - Bidder 2 is happy she lost!

Bidding in common value auctions

- The general idea in bidding in common value auctions: winning or losing conveys information about the information of the other bidders, so take this into account
- There is also a "loser's curse".
- Suppose that there are multiple identical objects for sale, say 10 bidders and 9 objects
- Suppose you lose. What does that tell about the value of the objects?

Winner's curse and IPO:s

- Winner's curse may have implications in other environments too
- Consider an initial public offering (IPO) of a company at price p:
  - All buyers have essentially the same value v for shares (unknown future trading price)
  - You should buy if you think $v > p$
  - If there is a lot of demand, then there is rationing (not every buyer gets shares)
  - What does it tell about other's information if you get shares?
  - Winner's curse?
- IPO:s are often underpriced. Why?

Revenue comparison between auction formats

- When signals are not independent, the Revenue equivalence theorem does not hold
- There is another principle called *Linkage Principle*, which allows for revenue comparison between different formats
- This important result is due Milgrom and Weber (1982): "A Theory of Auctions and Competitive Bidding", Econometrica.
- It turns out that second price auction is better for revenue than first price auction.
- The linkage principle also suggests that it is typically beneficial for the seller to release additional information about the object for sale (if she has any)

Information aggregation in common value auctions

- Where do asset prices come from?
- One view: prices reflect all the information that the traders have about asset values
- But how does price get to reflect that information?
- To investigate this question, we can model a financial market using an auction model
- The question is: can equilibrium price in an auction *aggregate* the bidders' information?

Information aggregation in common value auctions

- What is information aggregation?
- Suppose the value of an asset is v
- N bidders have an independent signal $\theta_i = v + \varepsilon_i$
- If N is large, then the median signal gives a very precise estimate of v:

$$Median\,(\theta_i) = v + median\,(\varepsilon_i) \approx v$$

  if for example $\varepsilon_i \sim N\left(0, \sigma^2\right)$
- "Wisdom of the crowds"
- But can the price in an auction aggregate information?
- If there is only one object, then not likely.

Information aggregation in common value auctions

- Assume a common value auction, with N bidders and K identical objects (think of N as a very large number)
- For simplicity, assume $K = N/2$
- Think of this as a market for an asset (K units, e.g. shares, and N bidders)
- The value of the asset is v and each bidder has a signal $\theta_i = v + \varepsilon_i$
- Auction format is a generalization of second price auction: $K + 1^{st}$ price auction
- Equilibrium bidding function can be shown to be

$$b(\theta_i) = \mathbb{E}\left(v \,\middle|\, \theta_i \text{ ties with the } K + 1^{st} \text{ highest signal}\right).$$

- Intuitively: bid as if you were just pivotal

Information aggregation in common value auctions

- But then

$$\begin{aligned} b(\theta_i) &\approx \mathbb{E}(v \,|\theta_i \text{ is median signal}) \\ &= \mathbb{E}(v \,|v + median(\varepsilon_i) = \theta_i) \\ &= \theta_i \end{aligned}$$

- Price will be $b\left(\theta^{(K+1)}\right)$, where $\theta^{(K+1)}$ is the $K + 1^{st}$ highest signal
- So the auction price will be approximately the median signal, and hence aggregates information!
- This model is a very simplified version of Pesendorfer and Swinkels (1997): "The loser's curse and information aggregation in common value auctions", Econometrica.

Conclusions

- We first looked briefly at some auction design issues
  - Asymmetries, subsidies, bidder exclusions
  - Ensuring sufficient participation
  - Collusion
  - These are all important considerations in realistic applications
- Then we analyzed common value auctions. Important issues:
  - Winning (or losing) reveals information about others' estimates
  - Taking into account winner's curse requires caution in bidding
  - Auction price can aggregate information

Further readings

- A broad (but a bit old by now) survey on auctions is Klemperer (2002): "Auction Theory: A Guide to the Literature", Journal of Economic Surveys.
- An empirical analysis of collusion in auctions: Asker (2010): "A Study of the Internal Organization of a Bidding Cartel", American Economic Review.
- For an interesting account of applying auction theory in an important application, see Binmore and Klemperer (2002): "The Biggest Auction Ever: The Sale of the British 3G Telecom Licences", Economic Journal.

- For on-line auction applications, see e.g.
  - Edelman, Ostrovsky, Schwarz (2007): "Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords", American Economic Review.
  - Varian (2009): "Online Ad Auctions", American Economic Review (Papers and Proceedings)
  - Varian and Harris (2014): "The VCG Auction in Theory and Practice", American Economic Review (Papers and Proceedings).