Ch 8. Three-phase systems

• Lecture outcomes (what you are supposed to learn):
  
  – Generation of three-phase voltages
  
  – Connection of three-phase circuits
  
  – Wye-Delta transformation
  
  – Power of three-phase connected loads
Introduction

- High power equipments are built as three-phase systems.

- Three-phase systems can produce rotating field without special control.

- Three phase generator produce more power than single phase one with the same volume.

- Three-phase systems are more reliable. They can deliver power even if one phase fails.
Schematic structure of Power systems

- Power plants
- International links
- Transmission lines
- Substations and Transformer
- Load centers
- Industrial and Residential loads

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Generation of three-phase voltages (kolmivaihejärjestelmä)

- Simple three-phase generator

\[ e = nB\omega \]
Generation of three-phase voltages

- 3 single-phase circuits at different phase angle!

\[
v_{aa'} = e_a = V_{\text{max}} \cos(\omega t)
\]

\[
v_{bb'} = e_b = V_{\text{max}} \cos(\omega t - 120^\circ)
\]

\[
v_{cc'} = e_c = V_{\text{max}} \cos(\omega t - 240^\circ) = V_{\text{max}} \cos(\omega t + 120^\circ)
\]
The voltage of each phase are called phase voltages

The phase voltages are written in complex form

$$
\vec{V}_{aa'} = \frac{V_{\text{max}}}{\sqrt{2}} \angle 0^\circ = V \angle 0^\circ
$$

$$
\vec{V}_{bb'} = \frac{V_{\text{max}}}{\sqrt{2}} \angle -120^\circ = V \angle -120^\circ
$$

$$
\vec{V}_{cc'} = \frac{V_{\text{max}}}{\sqrt{2}} \angle 120^\circ = V \angle 120^\circ
$$
Connecting the 3-phase voltages

- The potential difference is known but not the potentials!
Connecting the 3-phase voltages

Line-to-line voltage

Pääjännitte

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Connecting the source and load

- Only 3 wires are needed to connect the source and load
Wye connection (Y- tai tähtikytkentä)

- Three similar terminal of each coil connected to the same point called neutral or N
Wye connection (Y-tai tähtikytkentä)

- For balanced symmetrical three phase system:
  \[ V_{an} = V_{bn} = V_{cn} = V_{ph} \]

\[
\bar{V}_{an} = V_{ph} \angle 0^\circ
\]

\[
\bar{V}_{bn} = V_{ph} \angle -120^\circ
\]

\[
\bar{V}_{cn} = V_{ph} \angle 120^\circ
\]
Line-to-line voltages in Wye connection

\[
\begin{align*}
\vec{V}_{an} &= V_{ph} \angle 0^\circ \\
\vec{V}_{bn} &= V_{ph} \angle -120^\circ \\
\vec{V}_{cn} &= V_{ph} \angle 120^\circ \\
\vec{V}_{ab} &= \vec{V}_{an} - \vec{V}_{bn} \\
\vec{V}_{ab} &= V_{ph} \angle 0^\circ - V_{ph} \angle -120^\circ \\
&= \sqrt{3} V_{ph} \angle 30^\circ \\
\end{align*}
\]

- Magnitude of line-to-line voltage \( \vec{V}_{ab} \) is larger than the magnitude of phase voltages \( \vec{V}_{an} \) by a factor of \( \sqrt{3} \)
- Line-to-line voltage \( \vec{V}_{ab} \) leads \( \vec{V}_{an} \) by 30°
Line-to-line voltages in Wye connection

\[ V_{bc} = V_{bn} - V_{cn} \]
\[ V_{bc} = V_{ph} \angle -120^\circ - V_{ph} \angle 120^\circ \]
\[ = \sqrt{3} V_{ph} \angle -90^\circ \]

\[ V_{ll} = \sqrt{3} V_{ph} \]

\[ \vec{V}_{ca} = \vec{V}_{cn} - \vec{V}_{an} \]
\[ \vec{V}_{ca} = V_{ph} \angle 120^\circ - V_{ph} \angle 0^\circ \]
\[ = \sqrt{3} V_{ph} \angle 150^\circ \]

\[ \vec{V}_{ab} = \vec{V}_{an} - \vec{V}_{bn} \]
\[ \vec{V}_{ab} = V_{ph} \angle 0^\circ - V_{ph} \angle -120^\circ \]
\[ = \sqrt{3} V_{ph} \angle 30^\circ \]
Delta connection (kolmiokytkentä)

- The entrance terminal of one coil is connected to the end terminal of the next coil
Delta connection

- Absence of a neural point i.e. floating potentials
- Phase voltages are identical with line-to-line voltages

\[
\begin{align*}
\overline{V}_{aa'} &= \overline{V}_{ab} \\
\overline{V}_{bb'} &= \overline{V}_{bc} \\
\overline{V}_{cc'} &= \overline{V}_{ca}
\end{align*}
\]
Single and three phases loads

- Residential loads are usually single phase loads (230 V)
- Industrial and commercial loads are mostly three phases loads (400 V)
- Clustered residential areas are powered by three phases
- Single houses might be powered by single phase
- Neutral point is grounded to ensure that all loads are powered regardless of fluctuations in current
Wye connected load

- Load impedances connected to a common neutral point from one terminal
  \[ Z_{an} = Z_{bn} = Z_{cn} = Z \]
- The load is powered by a three phases source
Wye connected load

• Phase currents

\[
\bar{I}_a = \frac{\overline{V}_{an}}{Z} = \frac{V_{ph} \angle \theta}{Z \angle \varphi} = \frac{V_{ph}}{Z} \angle (\theta - \varphi)
\]

\[
\bar{I}_b = \frac{\overline{V}_{bn}}{Z} = \frac{V_{ph} \angle (\theta - 120)}{Z \angle \varphi} = \frac{V_{ph}}{Z} \angle (\theta - \varphi - 120)
\]

\[
\bar{I}_c = \frac{\overline{V}_{cn}}{Z} = \frac{V_{ph} \angle (\theta + 120)}{Z \angle \varphi} = \frac{V_{ph}}{Z} \angle (\theta - \varphi + 120)
\]

• Equal magnitudes, 120 phase shifts
Wye connected load

• Neutral current

\[ \bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c \]

\[ \bar{I}_n = \bar{I}_a + \bar{I}_a \angle -120 + \bar{I}_a \angle 120 = 0 \]

• If source and loads are balanced the neutral current is zero

• In transmission this means no need for a neutral line
Delta-connected load

- Loads connected between two transmission lines
- Voltage across the single load is the line-to-line voltage

\[
\bar{I}_{ab} = \bar{I}_a + \bar{I}_{ca} \\
\bar{I}_{bc} = \bar{I}_b + \bar{I}_{ab} \\
\bar{I}_{ca} = \bar{I}_c + \bar{I}_{bc}
\]

- Balanced load and source balanced currents

\[
\bar{I}_{ab} = \frac{\bar{V}_{ab}}{Z} \\
\bar{I}_{bc} = \frac{\bar{V}_{bc}}{Z} \\
\bar{I}_{ca} = \frac{\bar{V}_{ca}}{Z}
\]
If we choose $\overline{I}_{ab}$ as reference

\[
\overline{I}_{ab} = I \angle 0^\circ
\]

\[
\overline{I}_{bc} = I \angle -120^\circ
\]

\[
\overline{I}_{ca} = I \angle 120^\circ
\]

\[
\overline{I}_a = \sqrt{3}I \angle -30^\circ = \sqrt{3}\overline{I}_{ab} \angle -30^\circ
\]

\[
\overline{I}_b = \sqrt{3}I \angle -150^\circ = \sqrt{3}\overline{I}_{bc} \angle -30^\circ
\]

\[
\overline{I}_c = I \angle 90^\circ = \sqrt{3}\overline{I}_{ca} \angle -30^\circ
\]
Circuits with mixed connections

- In general circuits can be so that the source or the load or both are connected in Y-, Delta-, or any combination.
Circuits with mixed connections

- Such circuits require the load and source to be in the same connection.

Y-Delta transformation

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Y-Delta transformation

• Consider the impedance seen by the source terminals

\[
\bar{Z}_{ab} = \frac{2\bar{Z}_{\Delta}}{3\bar{Z}_{\Delta}} = \frac{2}{3} \bar{Z}_{\Delta}
\]

• Transformation valid in both directions

\[
\bar{Z}_{Y} = \frac{2\bar{Z}_{Y}}{3}
\]
Power of three phases system

- The power of a three phase load is the sum of the powers of each load (each phase)
- Phases related quantities are called phase quantities (phase current, phase voltage, phase power)

\[
P_{ph} = V_{ph} I_{ph} \cos(\theta)
\]
\[
Q_{ph} = V_{ph} I_{ph} \sin(\theta)
\]

\[
P = 3P_{ph} = 3V_{ph} I_{ph} \cos(\theta)
\]
\[
Q = 3Q_{ph} = 3V_{ph} I_{ph} \sin(\theta)
\]
Power of Y-connected three phases load

- Phase current equal to line current
- Phase voltage different

\[ P = 3P_{ph} = 3V_{ph} I_{ph} \cos(\theta) \]
\[ = \sqrt{3}V_{ll} I_{l} \cos(\theta) \]

\[ Q = 3Q_{ph} = 3V_{ph} I_{ph} \sin(\theta) \]
\[ = \sqrt{3}V_{ll} I_{l} \sin(\theta) \]

\[ I_{ph} = I_{l} \]
\[ V_{ph} = \frac{V_{ll}}{\sqrt{3}} \]
Power of Delta-connected load

- Phase voltage equal to line-to-line voltage
- Phase current different

\[ P = 3P_{ph} = 3V_{ph}I_{ph} \cos(\theta) \]

\[ = \sqrt{3}V_{ll}I_{l}\cos(\theta) \]

\[ Q = 3Q_{ph} = 3V_{ph}I_{ph} \sin(\theta) \]

\[ = \sqrt{3}V_{ll}I_{l}\sin(\theta) \]
Summary of the lecture

- Three different coils shifted by 120 deg in space generate balanced three voltages
- The coils can be connected in Y or Delta
- Loads also can be connected in Y or Delta
- Basic equations for single phase system holds also for three phases system (remember the phase shifts)
- In y-connection, line current equals phase current
- In Delta-connection line-to-line voltage equals phase voltage
- Remember to divide by $\sqrt{3}$ for the other quantities.