Chapter 7

DC-to-AC Converters

“Introduction to Modern Power Electronics”, 3rd Ed., John Wiley 2015
by
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CONTENT

Voltage-source inverter supplied from a diode rectifier

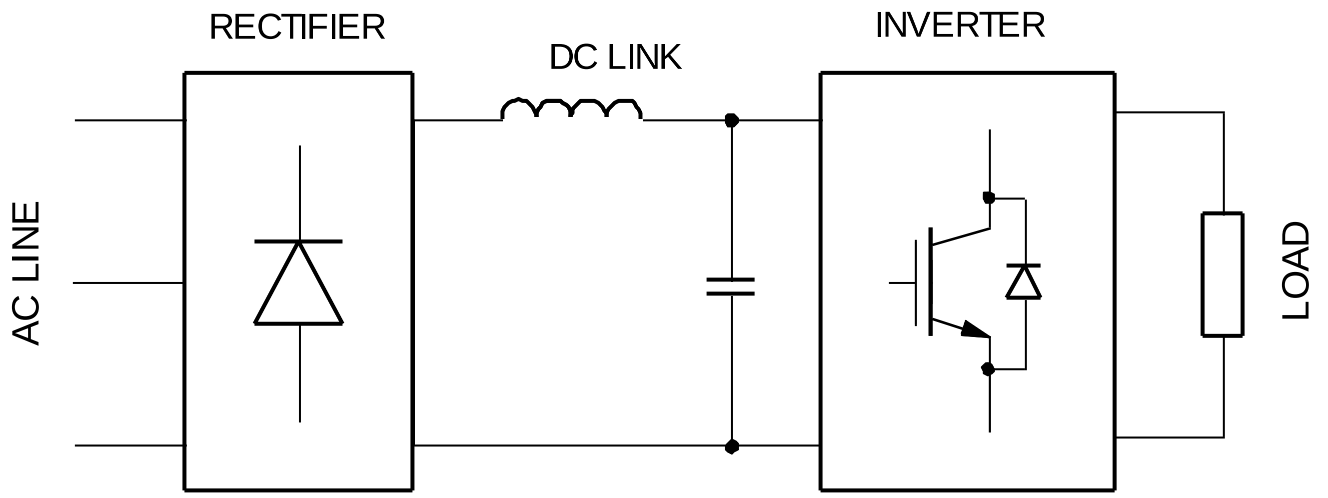


Fig. 7.1

Single-phase voltage-source inverter

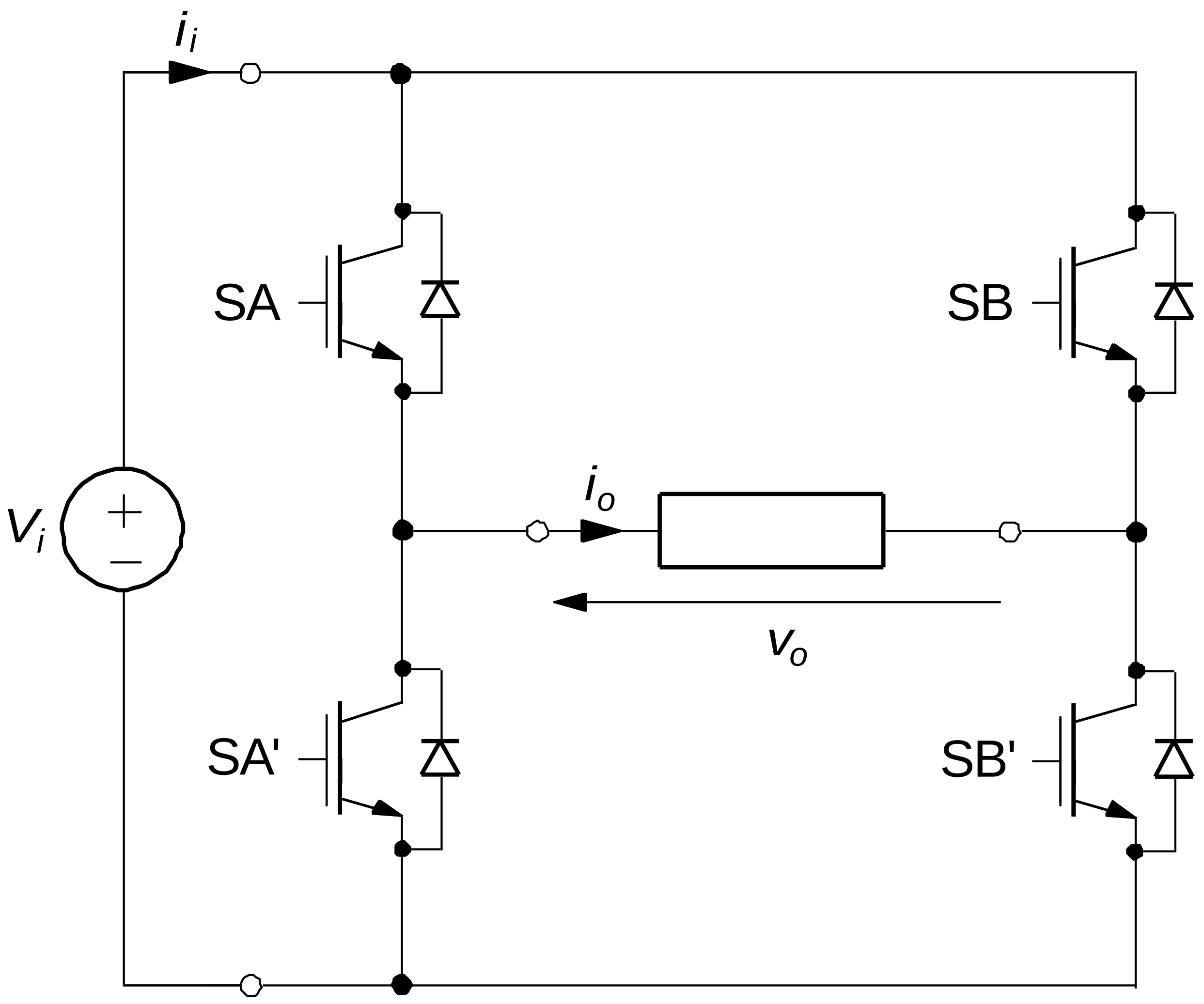


Fig. 7.2

Each inverter leg can assume two states only: either the upper (common-anode) switch is on and the lower (common-cathode) switch is off, or the other way around. Thus, two switching variables *a* and *b* can be assigned to the inverter legs, and defined as

$$a = \begin{cases} 0\ if\ \mathrm{SA}\ is\ \mathrm{OFF}\ and\ \mathrm{SA}'is\ \mathrm{ON} \\ 1\ if\ \mathrm{SA}\ is\ \mathrm{ON}\ and\ \mathrm{SA}'is\ \mathrm{OFF} \end{cases}$$

$$b = \begin{cases} 0\ if\ \mathrm{SB}\ is\ \mathrm{OFF}\ and\ \mathrm{SB}'is\ \mathrm{ON} \\ 1\ if\ \mathrm{SB}\ is\ \mathrm{ON}\ and\ \mathrm{SB}'is\ \mathrm{OFF} \end{cases}$$

An inverter state is designated as ab_2. Four states are possible. The output voltage, v_o, of the inverter can be expressed as

$$v_o = V_i(a - b)$$

and the voltage can assume three values only: V_i, 0, and -V_i, corresponding to state 2, states 0 or 3, and state 1, respectively.

The basic version of the so-called *square-wave* operation mode of the inverter is described by the following control law:

$$a = \begin{cases} 1\ for\ 0 < \omega t \le \pi \\ 0\ otherwise \end{cases}$$

$$b = \begin{cases} 1\ for\ \ \pi < \omega t \le 2\pi \\ 0\ otherwise \end{cases}$$

where ω is the fundamental output radian frequency of the inverter. Only states 1 and 2 are used.

States, switching variables, and waveforms of output voltage and current in a single-phase VSI in the basic square-wave mode

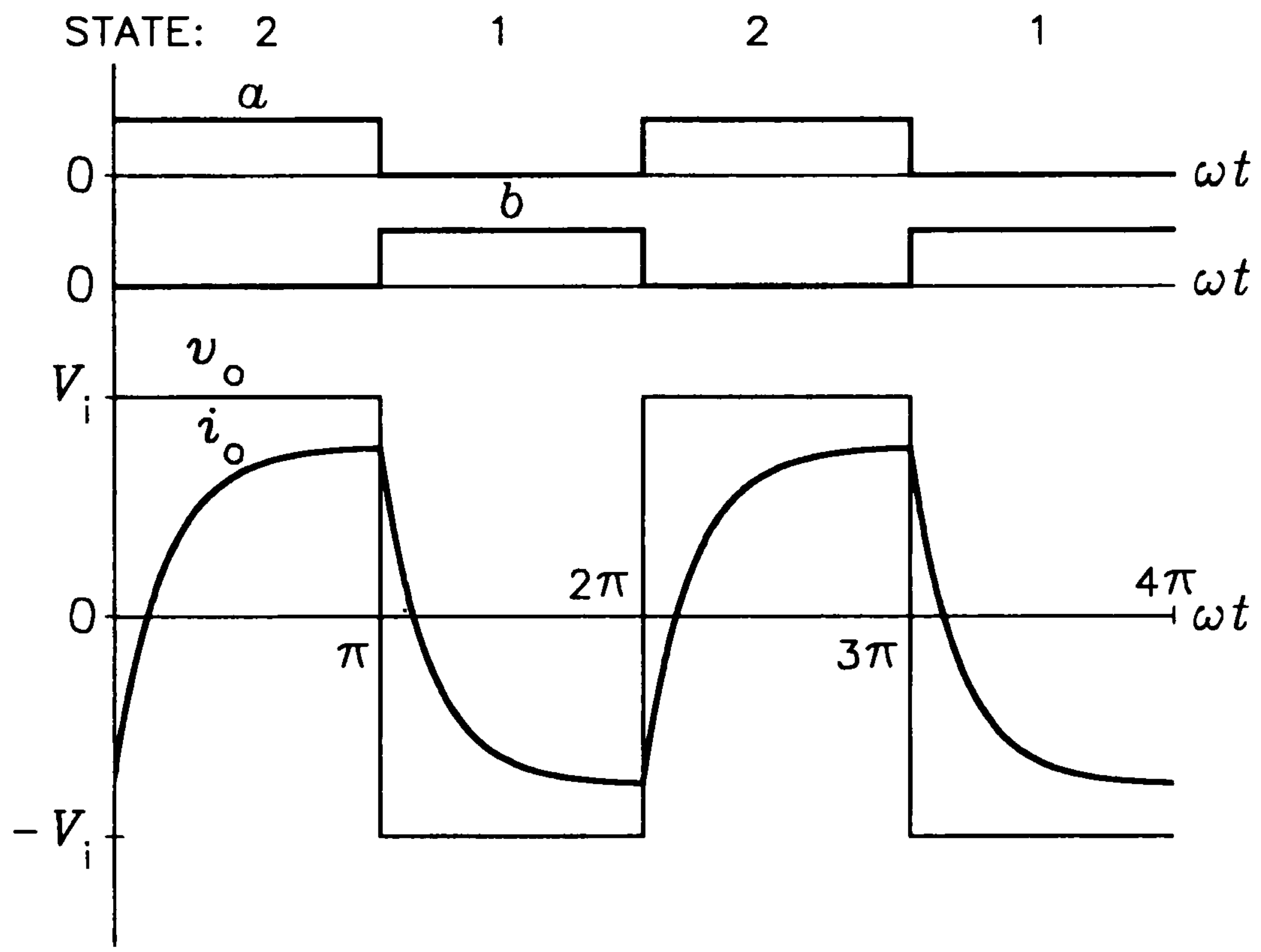


Fig. 7.4

Phase-shift control

- The total harmonic distortion of the output voltage can be minimized by interspersing states 1 and 2 with states 0 and 3 lasting in the ωt domain 0.81 rad (46.5^o) each, as shown in Figure 7.5.
- Then, in comparison with the basic square-wave operation, the fundamental output voltage decreases by 8%, to 0.828V_i, but the total harmonic distortion is reduced by as much as 40%, to 0.29. The control law yielding the optimal square-wave mode is

- $a = \begin{cases} 1 \; for \; \alpha_d < \omega t \leq \pi + \alpha_d \\ 0 \; otherwise \end{cases}$
- $b = \begin{cases} 1 \; for \;\; \pi + \alpha_d < \omega t \leq 2\pi - \alpha_d \\ 0 \; otherwise \end{cases}$ where α_d = 0.405 rad (23.2^o)

- Fundamental component can be adjusted by using different phase-shifts but of course THD is not then minimized

States, switching variables, and waveforms of output voltage and current in a single-phase VSI in the optimal square-wave mode

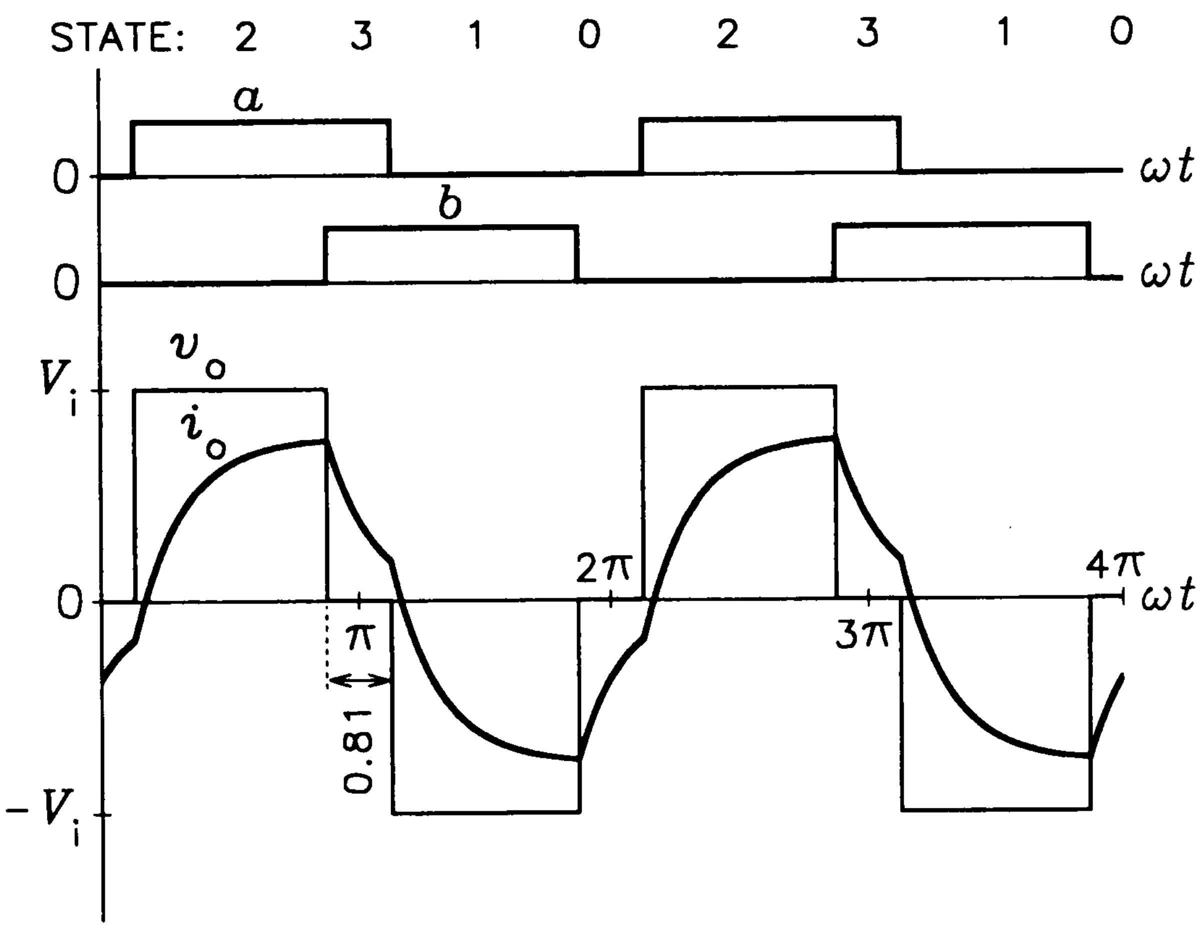


Fig. 7.5

Pulse width modulation

- The quality of operation of the inverter can be improved further by pulse width modulation
- The single-phase inverter has only one output voltage, so the space vector PWM technique is not applicable
- Here, for illustration purposes, a simple PWM strategy based on a sinusoidal modulating function, $F(m,\omega t) = m\sin(\omega t)$, is assumed

- As in all PWM converters, the operating time is a sequence of short switching cycles. Denoting the duty ratios of switching variables *a* and *b* in the n^{th} switching cycle by d_{an} and d_{bn}, the control law is

$$d_{an} = \frac{1}{2}[1 + F(m, \alpha_n)]$$
$$d_{bn} = \frac{1}{2}[1 - F(m, \alpha_n)]$$

- where *m* denotes the modulation index and α_n is the phase angle of the output voltage in the center of the switching cycle

Waveforms of output voltage and current in a single-phase VSI in the PWM mode, N = 10: (a) m = 1, (b) m = 0.5

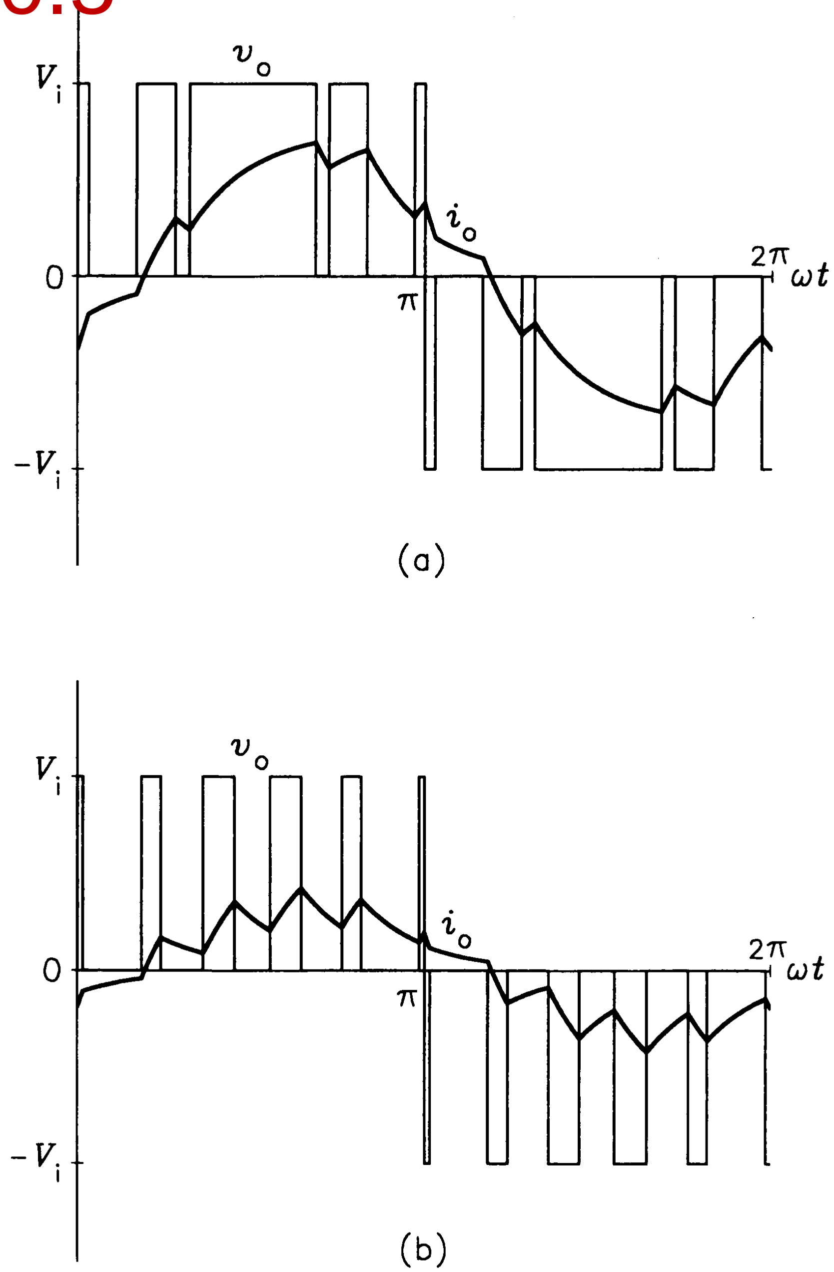


Fig. 7.6

Waveforms of output voltage and current in a single-phase VSI in the PWM mode, N = 20: (a) m = 1, (b) m = 0.5

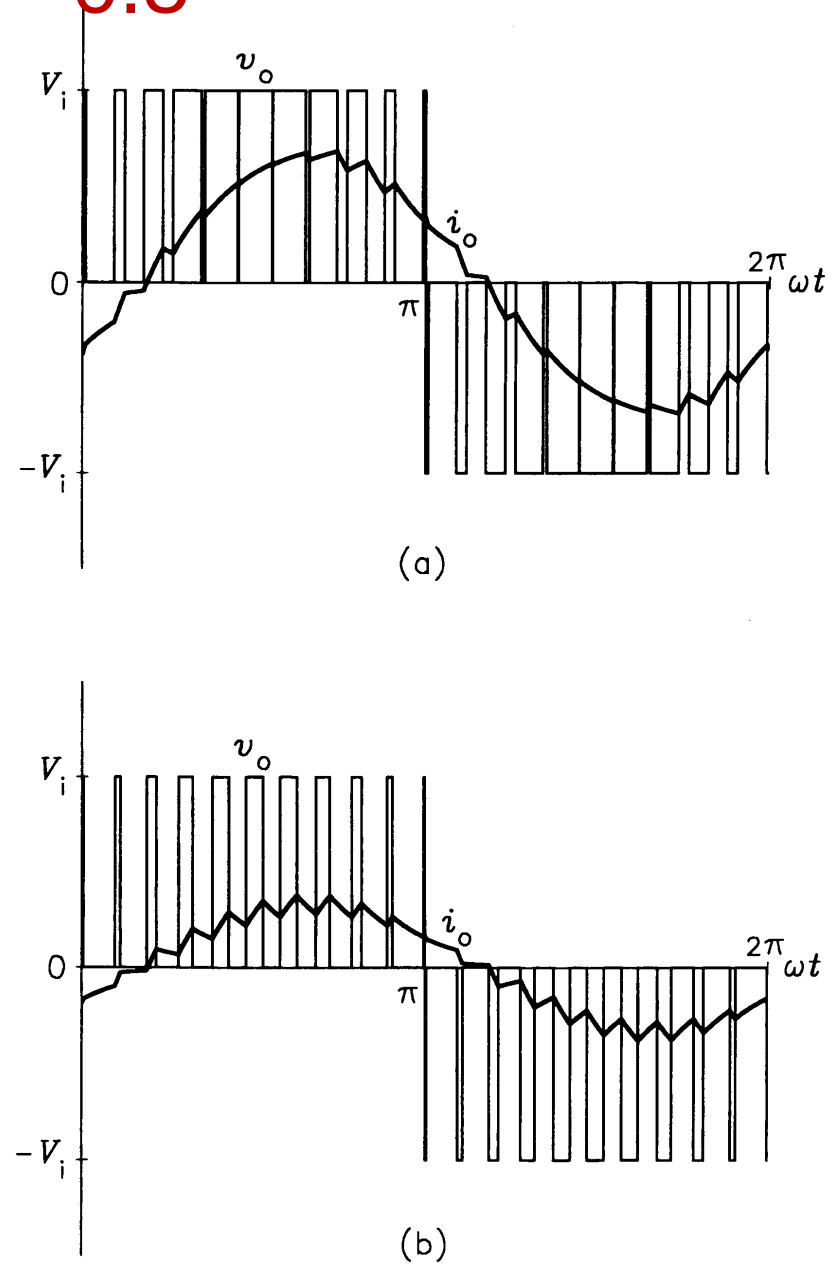


Fig. 7.7

Harmonic spectra of output voltage in a single-phase VSI: (a) basic square-wave mode, (b) optimal square-wave mode

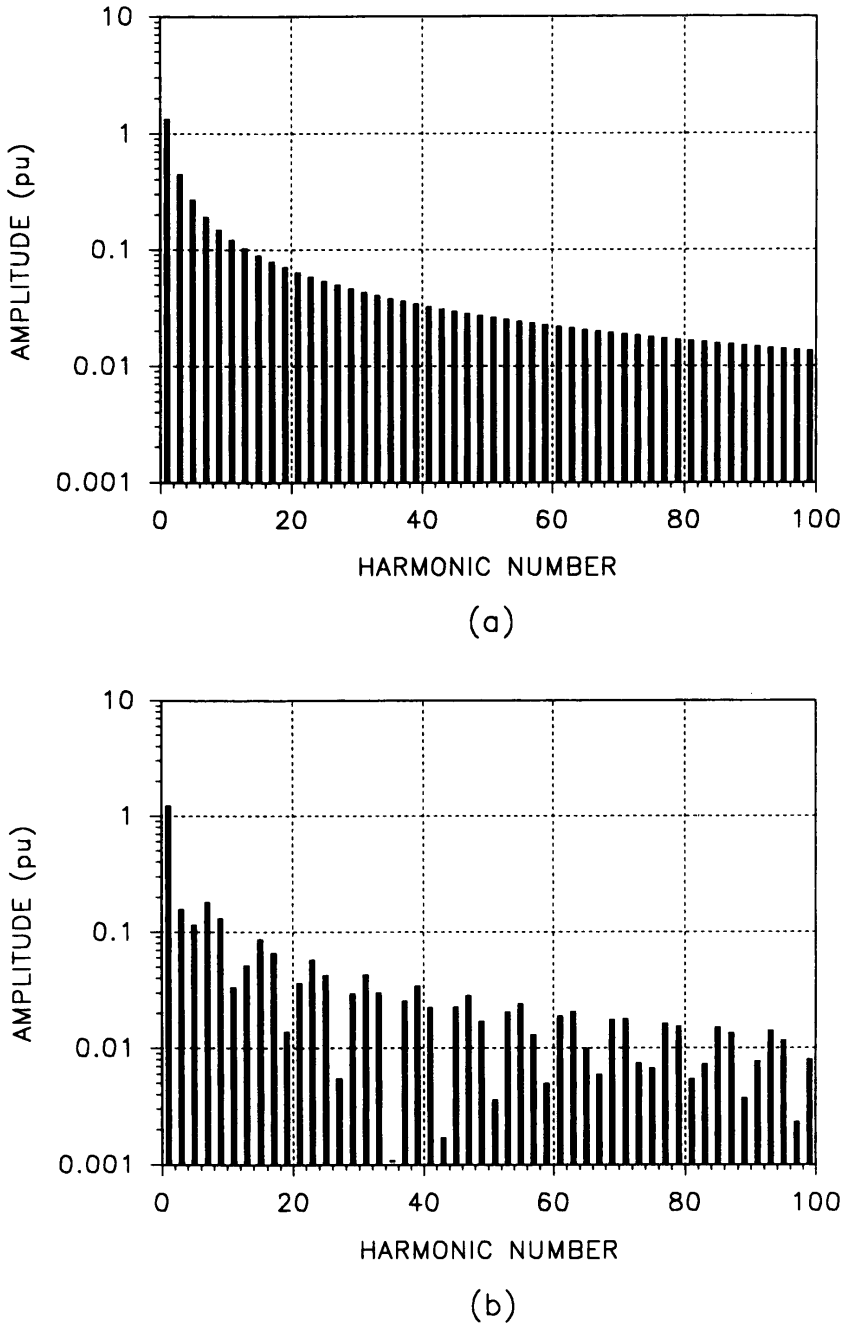


Fig. 7.8

Harmonic spectra of output voltage in a single-phase VSI in the PWM mode (m = 1): (a) N = 10, (b) N = 20

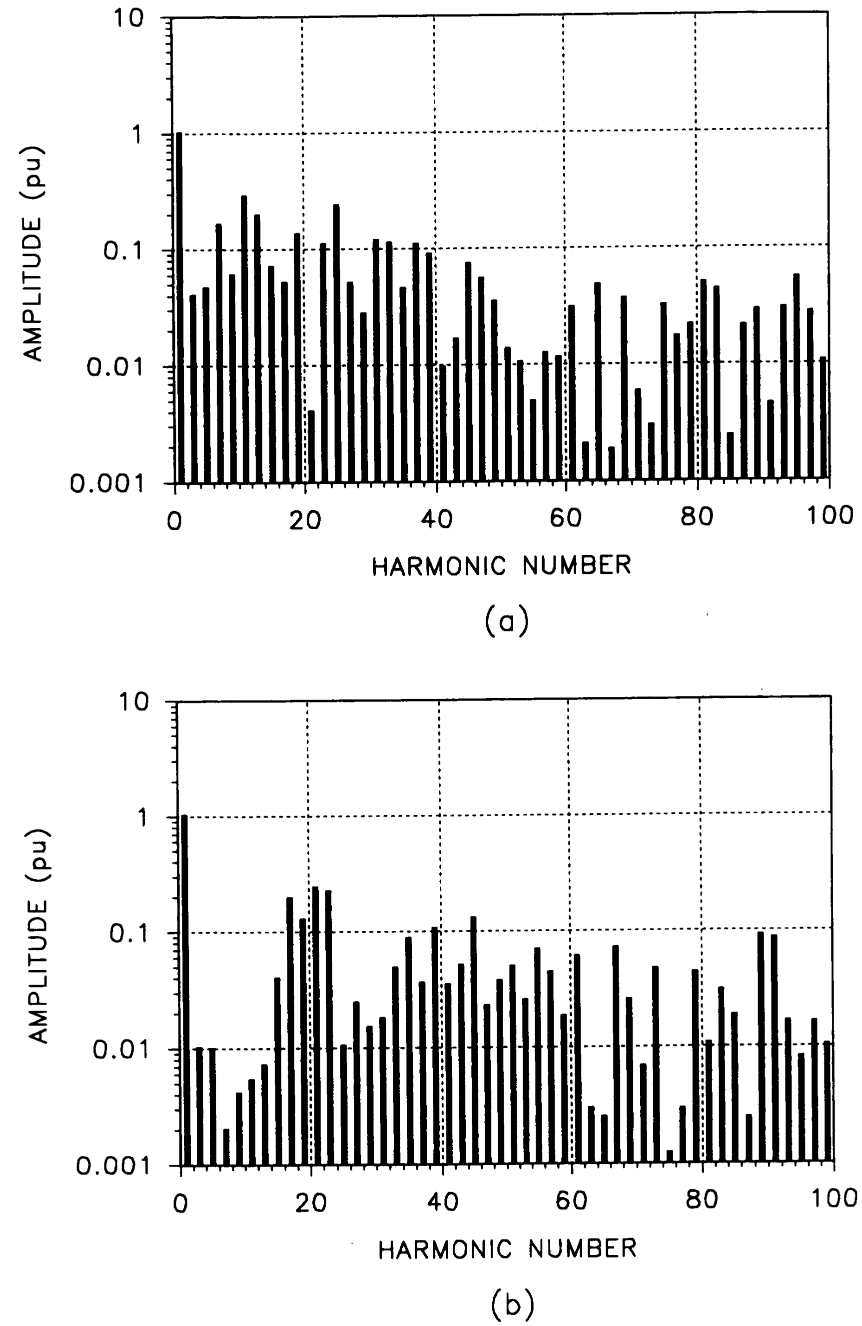


Fig. 7.9

Harmonic spectra of output current in a single-phase VSI: (a) basic square-wave mode, (b) optimal square-wave mode

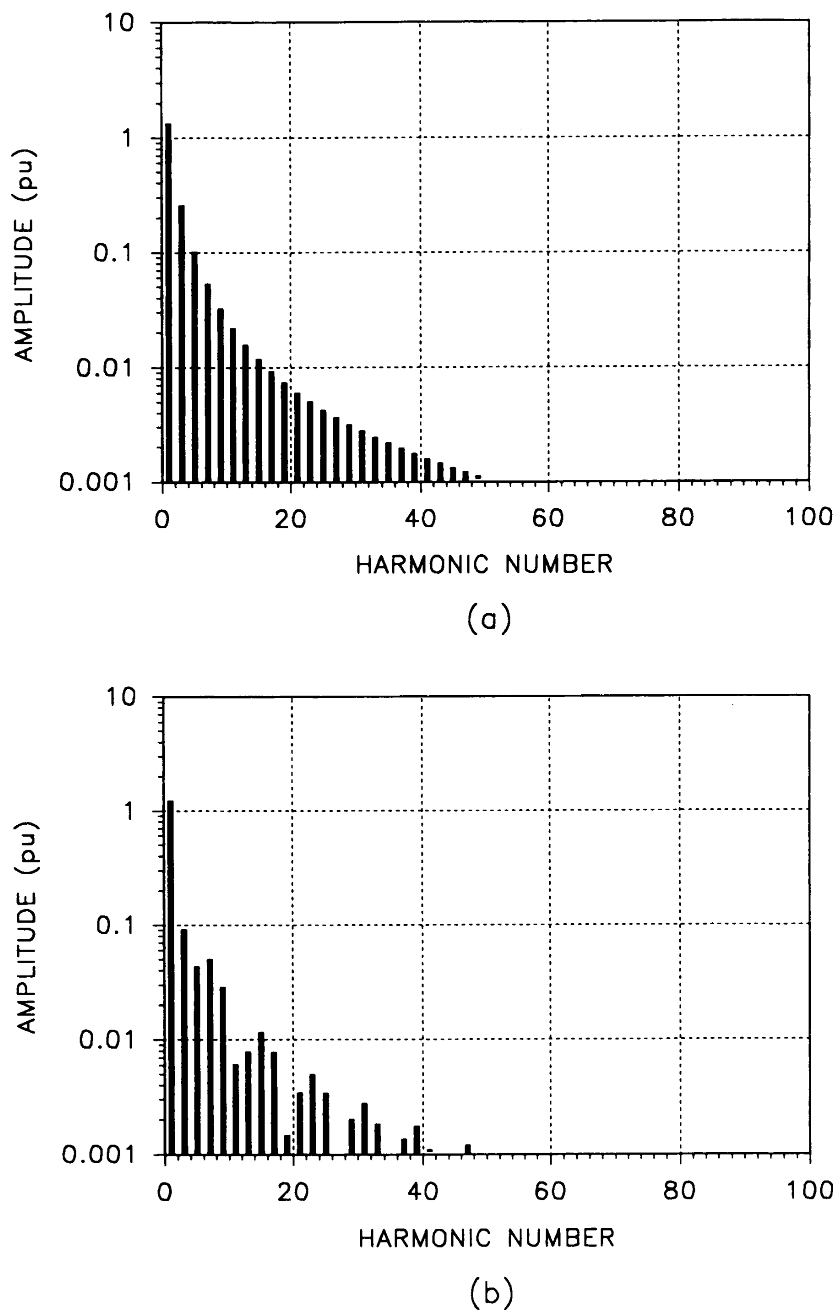


Fig. 7.10

Harmonic spectra of output current in a single-phase VSI in the PWM mode (m = 1):
(a) N = 10, (b) N = 20

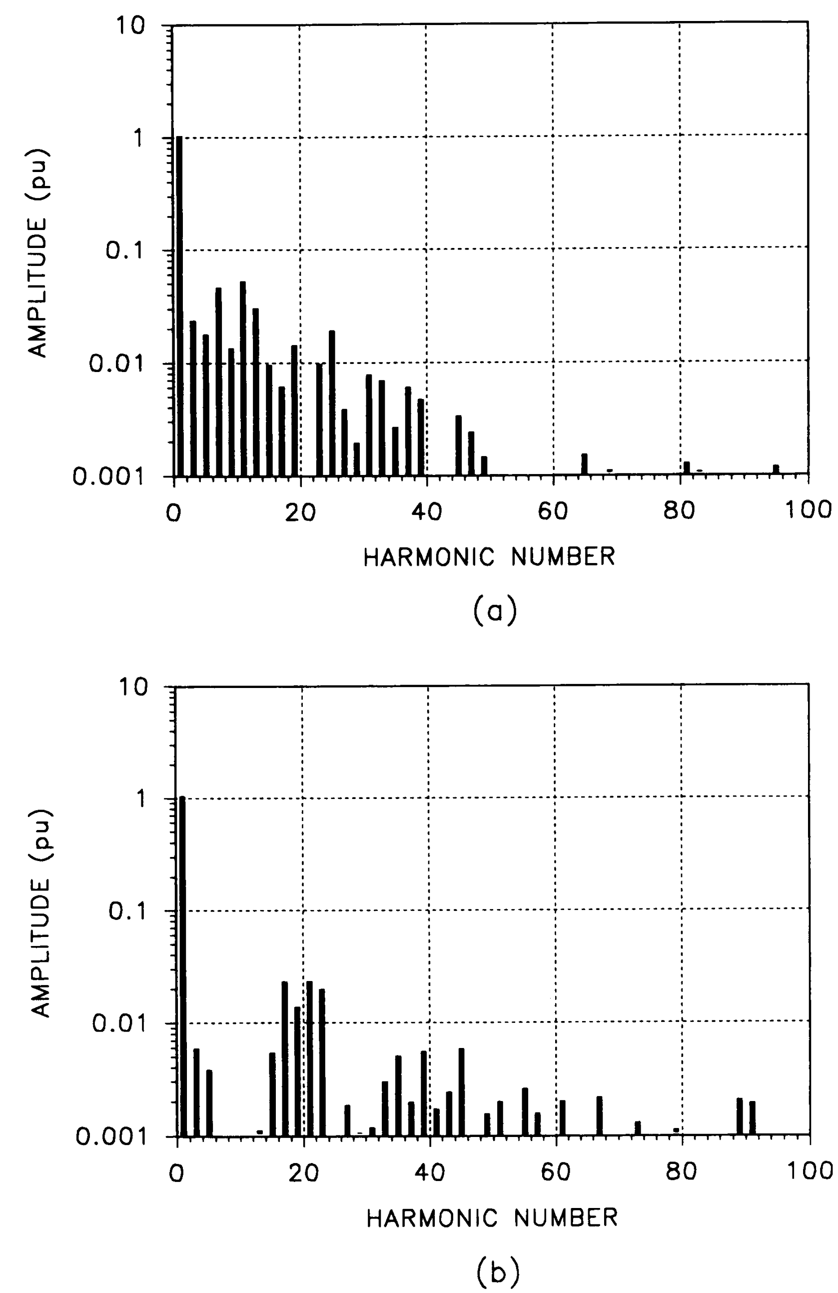


Fig. 7.11

Synthesis of a Sinusoidal Output by PWM

- High frequency triangle (switching frequency f_s) and sinusoidal modulating function (output frequency f_o) are compared
- Bipolar switching
- Harmonics are in multiples of switching frequency plus/minus multiples of output frequency
- m_a is here amplitude of the sinusoidal function and $m_f = f_s/f_o$

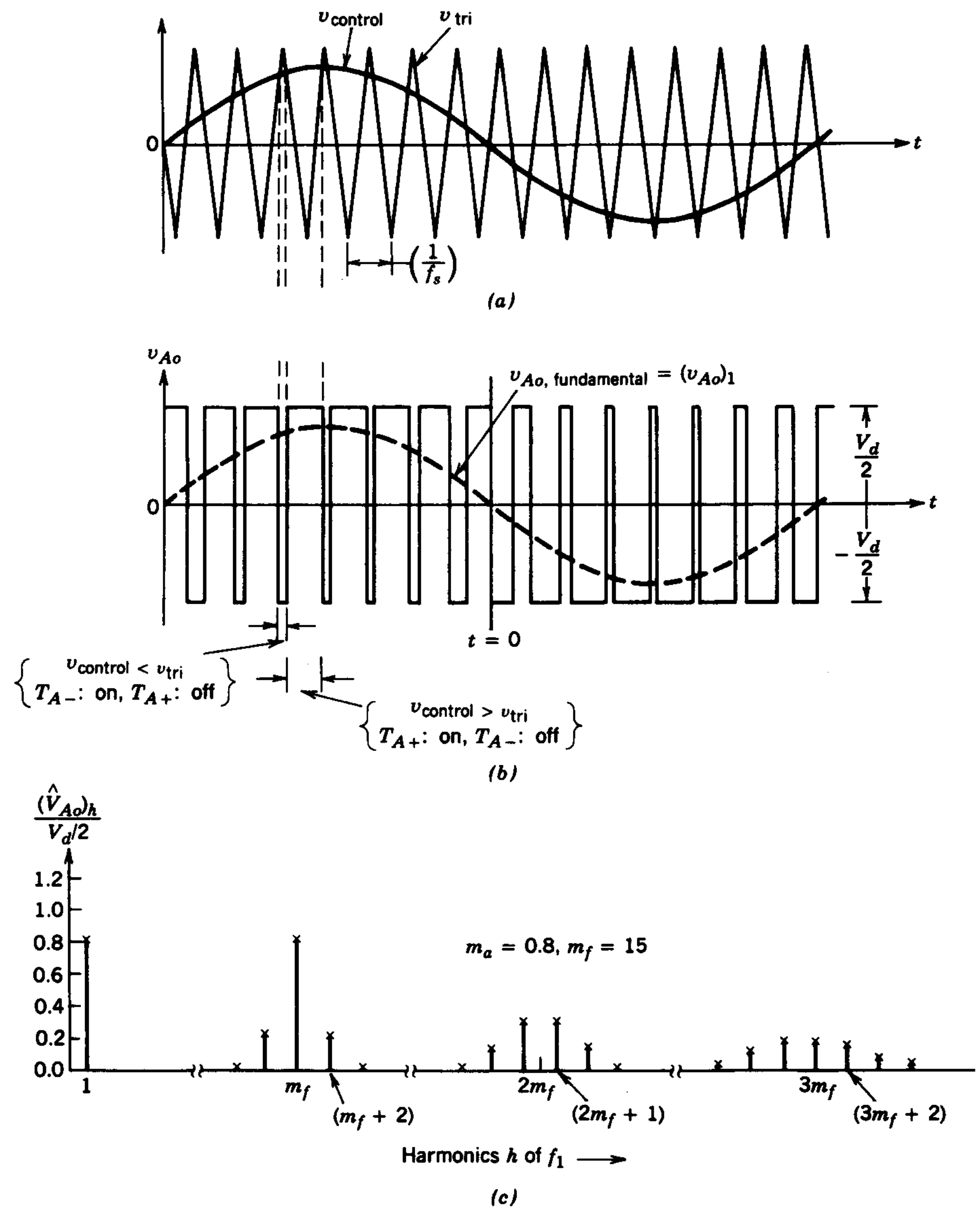


Figure 8-5 Pulse-width modulation.

Details of a Switching Time Period

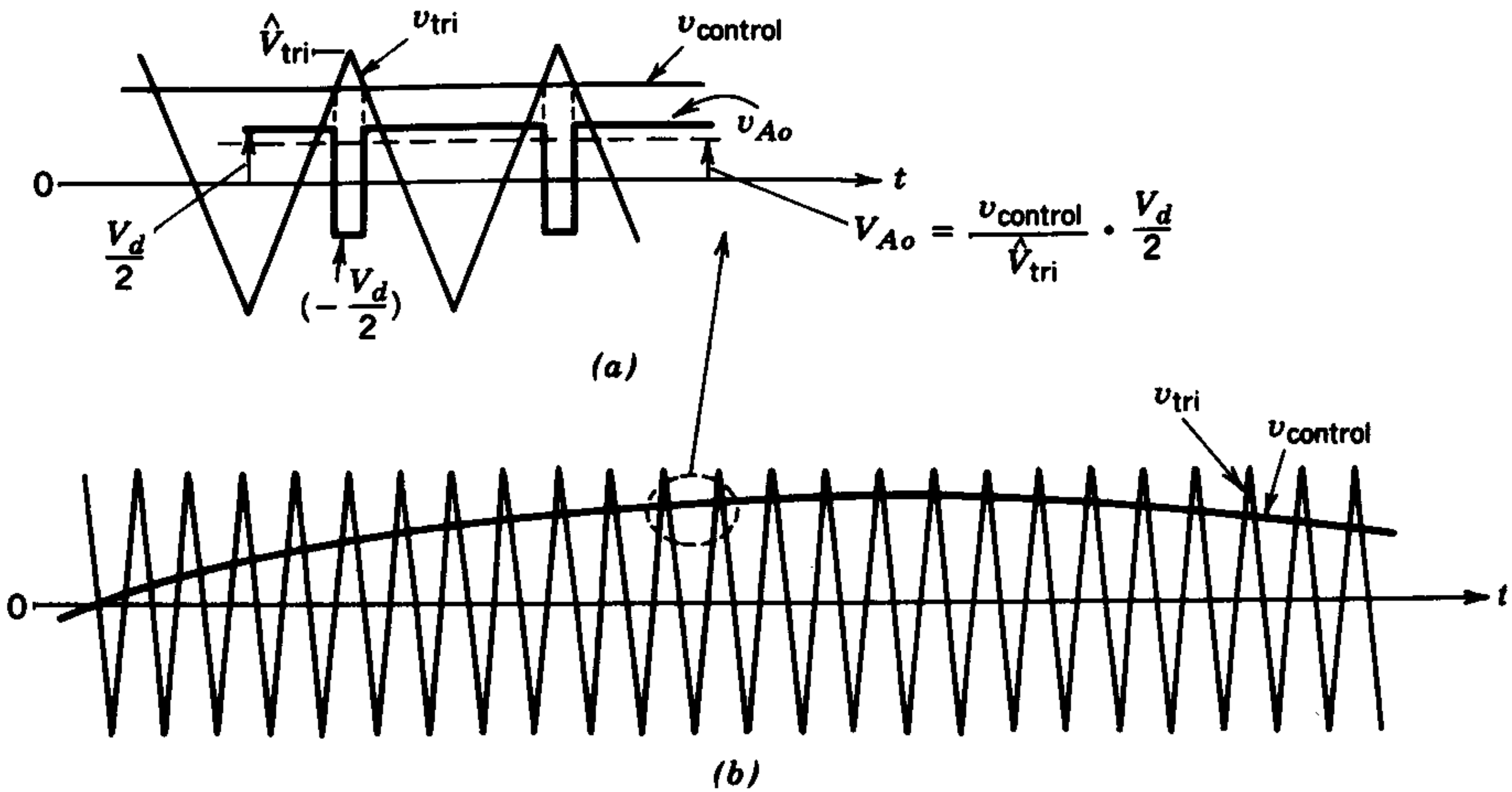


Figure 8-6 Sinusoidal PWM.

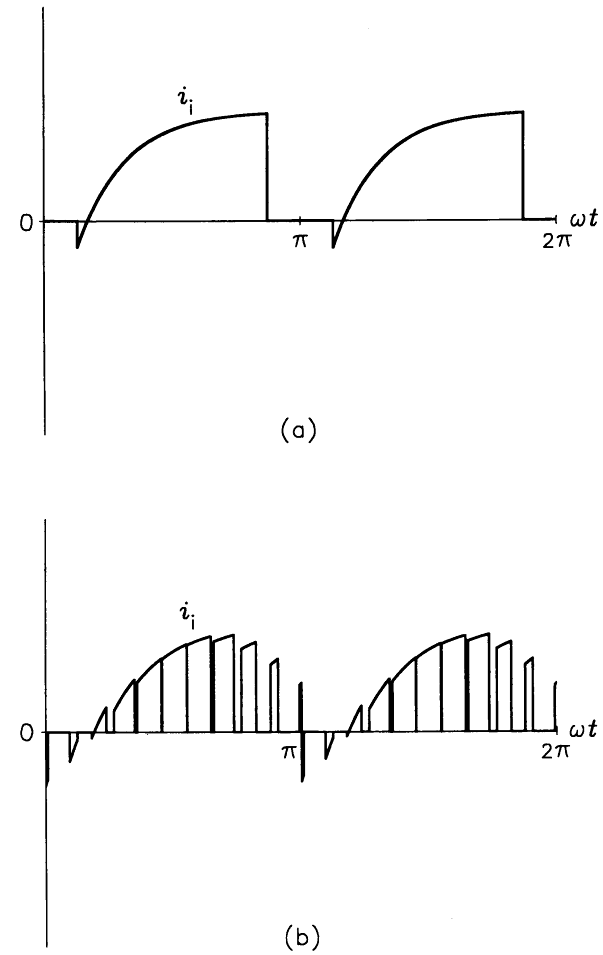
- Control voltage can be assumed constant during a switching time-period

Input current in a single-phase VSI: (a) optimal square-wave mode, (b) PWM mode (m = 1, N = 20)

- Input current of the inverter, the current from the dc-bus is given by

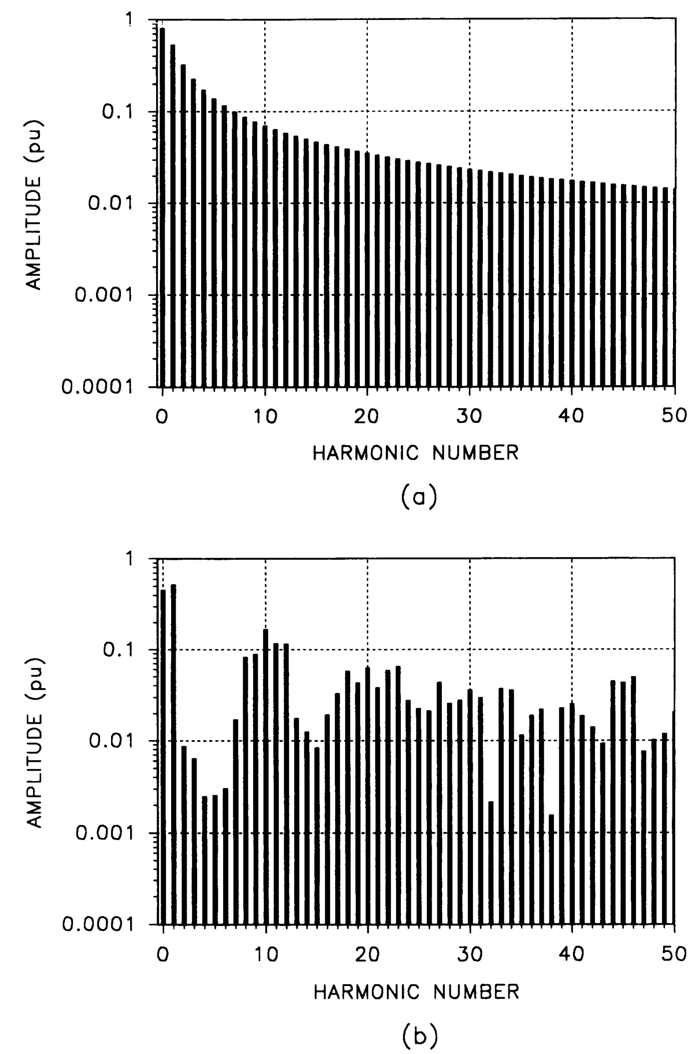
$$i_i = (a - b)\, i_o$$

- As a and b has values 0 and 1, current is equal to $-i_o$, 0 or i_o



(a) optimal square-wave mode, (b) PWM mode (m = 1, N = 20)

Fig. 7.12



Harmonic spectra : (a) optimal square-wave mode, (b) PWM mode (m = 1, N = 20)

Fig. 7.13

Analysis assuming Fictitious Filters

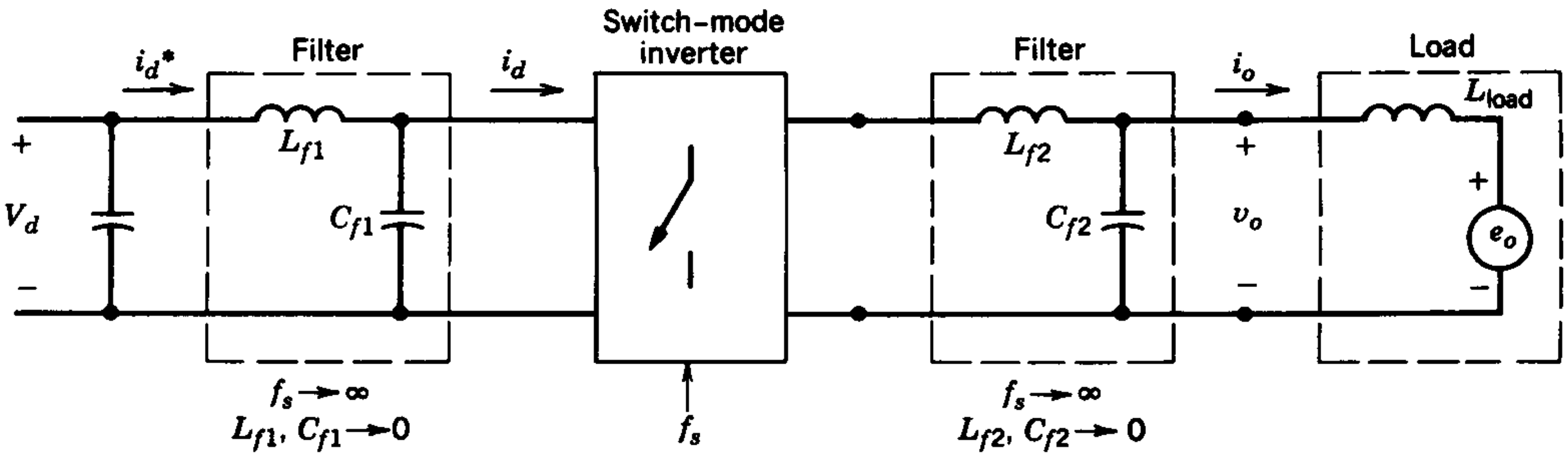


Figure 8-13 Inverter with "fictitious" filters.

- Small fictitious filters eliminate the switching-frequency related ripple

DC-Side Current

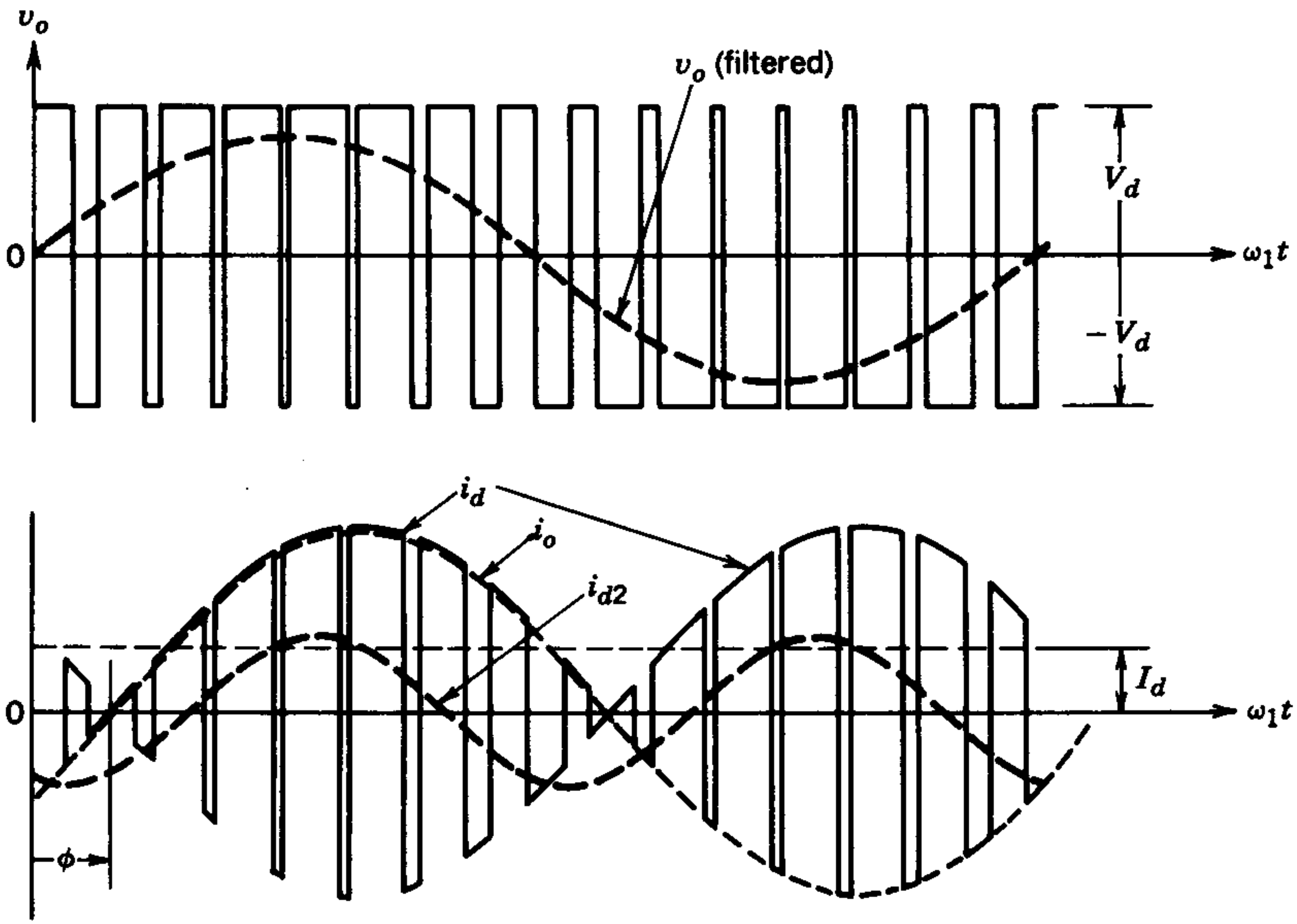


Figure 8-14 The dc-side current in a single-phase inverter with PWM bipolar voltage switching.

- Bi-Polar Voltage switching

DC-bus crrent

- No losses and output current is sinusoidal

$$p_{in} = \sqrt{2}U_s \left|\sin \omega t\right| \left|\sqrt{2}I_s \sin \omega t\right| = U_s I_s - U_s I_s \cos 2\omega t$$

  - In an ideal single-phase system instantaneous power contains a dc-component and a component with twice the frequency
- Dc voltage u_d can be assumed constant

$$p_d = U_d i_d$$

- Large switching frequency or ideal filtering
  - Instantaneous powers are equal, $p_{in} = p_d$

$$i_d = I_d + i_C = \frac{U_s I_s}{U_d} - \frac{U_s I_s}{U_d} \cos 2\omega t$$

Ripple in the dc-bus voltage

- As shown $$i_d = I_d + i_C = \frac{U_s I_s}{U_d} - \frac{U_s I_s}{U_d}\cos 2\omega t \qquad I_d = I_{load} = \frac{U_s I_s}{U_d}$$
- Capacitor current $$i_C = -\frac{U_s I_s}{U_d}\cos 2\omega t = -I_d \cos 2\omega t$$
- Capacitor voltage ripple $$u_{d,ripple} = \frac{1}{C_d}\int i_C dt = -\frac{I_d}{2\omega C_d}\cos 2\omega t$$
- In a single-phase inverter, there is always 2*output frequency component in the dc-bus voltage and its value can be reduced by increasing the capacitor size but ripple never disappears totally

Output Waveforms: Unipolar Voltage Switching

- Instead of one modulating function we are using two, which are complements
- Unipolar output voltage
- Harmonic components around the switching frequency are absent

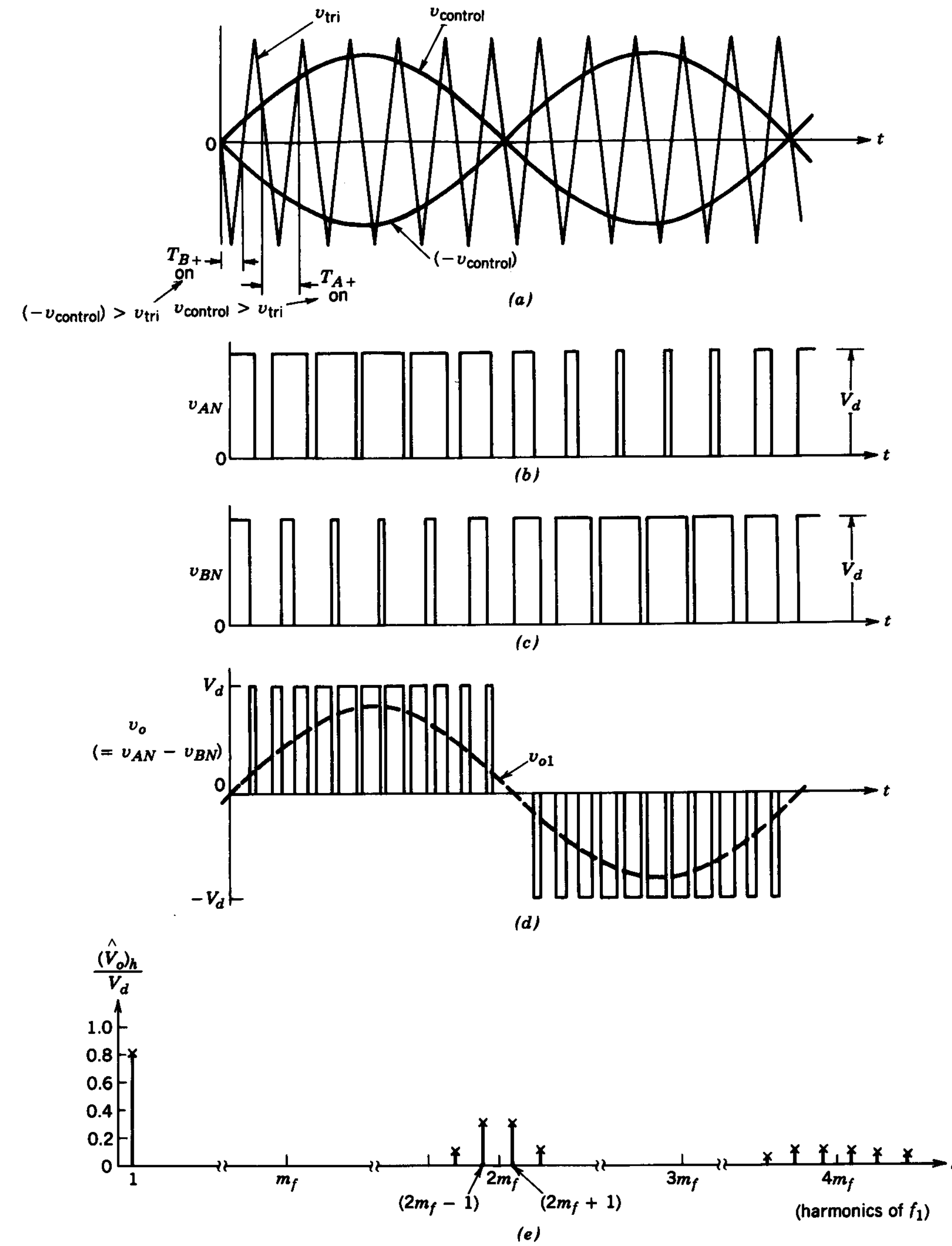


Figure 8-15 PWM with unipolar voltage switching (single phase).

DC-Side Current in a Single-Phase Inverter

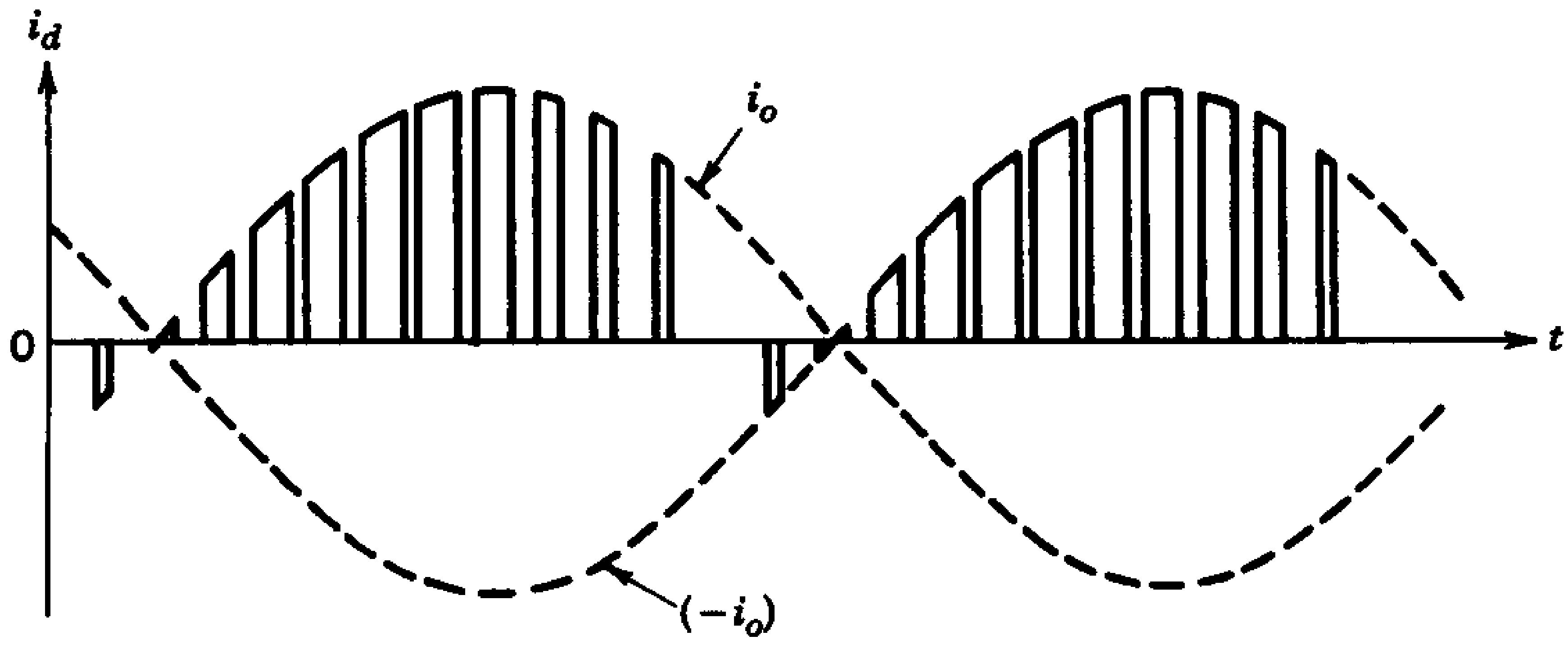


Figure 8-16 The dc-side current in a single-phase inverter with PWM unipolar voltage switching.

- Unipolar voltage switching

Three-phase voltage-source inverter

- When taking three single-phase inverters and connecting the load to star (or delta) we are achieving the shown three-phase converter
- Individual legs can be connected either + or –
- There are two reference points, star point of the load N and the minus bar of the dc-bus, also marked as n

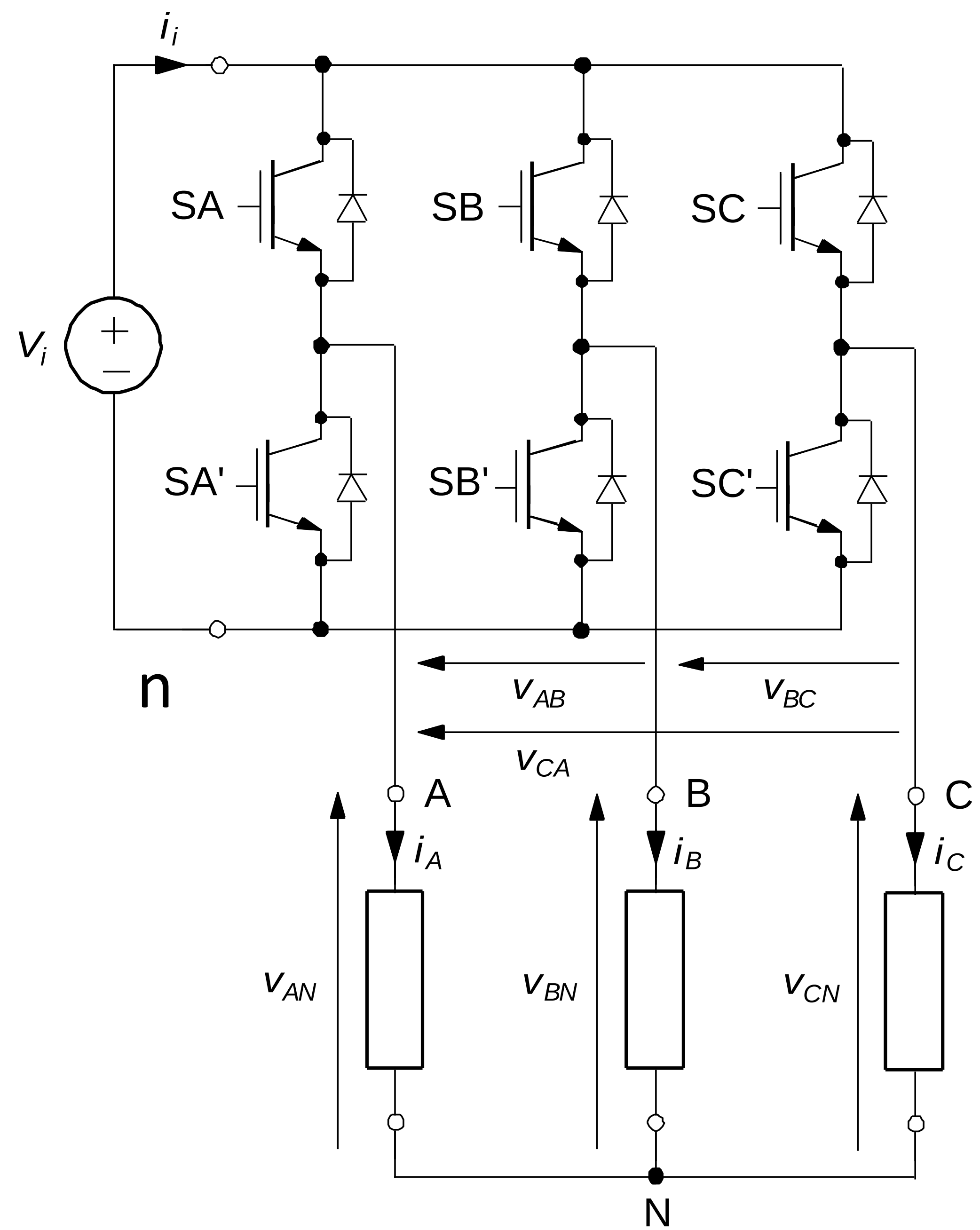


Fig. 7.14

It is easy to show that in the three-phase inverter the instantaneous line to line output voltages, v_{AB}, v_{BC}, and v_{CA}, are given by

$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = V_i \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

In a balanced three-phase system, the instantaneous line-to-neutral output voltages, v_{AN}, v_{BN}, and v_{CN}, can be expressed as

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix}$$

which yields

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{V_i}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

The line-to-line voltages can only assume three values, 0 and $\pm V_i$, while the line-to-neutral voltages can assume five values, 0, $\pm V_i/3$, and $\pm 2V_i/3$.

If the 5 – 4 – 6 – 2 – 3 – 1 – ... sequence of states is imposed, each state lasting one-sixth of the desired period of the output voltage, the individual line-to-line and line-to-neutral voltages acquire waveforms shown in Figure 7.15. This is the square-wave mode of operation, in which each switch of the inverter is turned on and off once within the cycle of output voltage. The peak value, $V_{LL,1,p}$, of the fundamental line-to-line output voltage equals approximately 1.1 V_i and that, $V_{LN,1,p}$, of the line-to-neutral voltage, 0.64 V_i. Both voltages have the same total harmonic distortion, *THD,* of 0.31. As in the square-wave single-phase inverter, the magnitude control of the output voltage must be realized on the dc supply side.

TABLE 7.1 States and Voltages of the Three-Phase Voltage-Source Inverter

State	*abc*	v_{AB}/V_i	v_{BC}/V_i	v_{CA}/V_i	v_{AN}/V_i	v_{BN}/V_i	v_{CN}/V_i
0	000	0	0	0	0	0	0
1	001	0	-1	1	-1/3	-1/3	2/3
2	010	-1	1	0	-1/3	2/3	-1/3
3	011	-1	0	1	-2/3	1/3	1/3
4	100	1	0	-1	2/3	-1/3	-1/3
5	101	1	-1	0	1/3	-2/3	1/3
6	110	0	1	-1	1/3	1/3	-2/3
7	111	0	0	0	0	0	0

Switching variables and waveforms of output voltages in a three-phase VSI in the square-wave mode

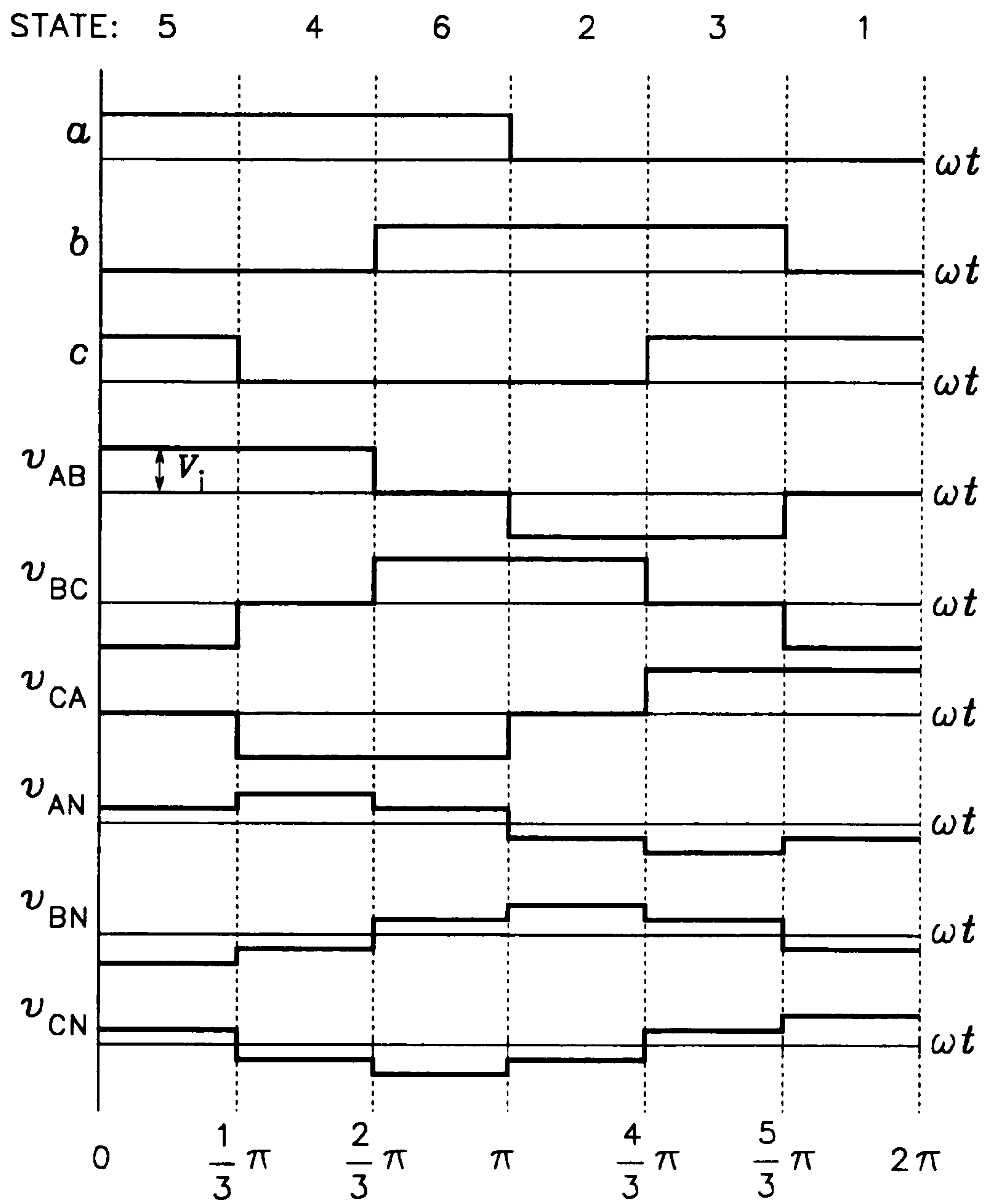
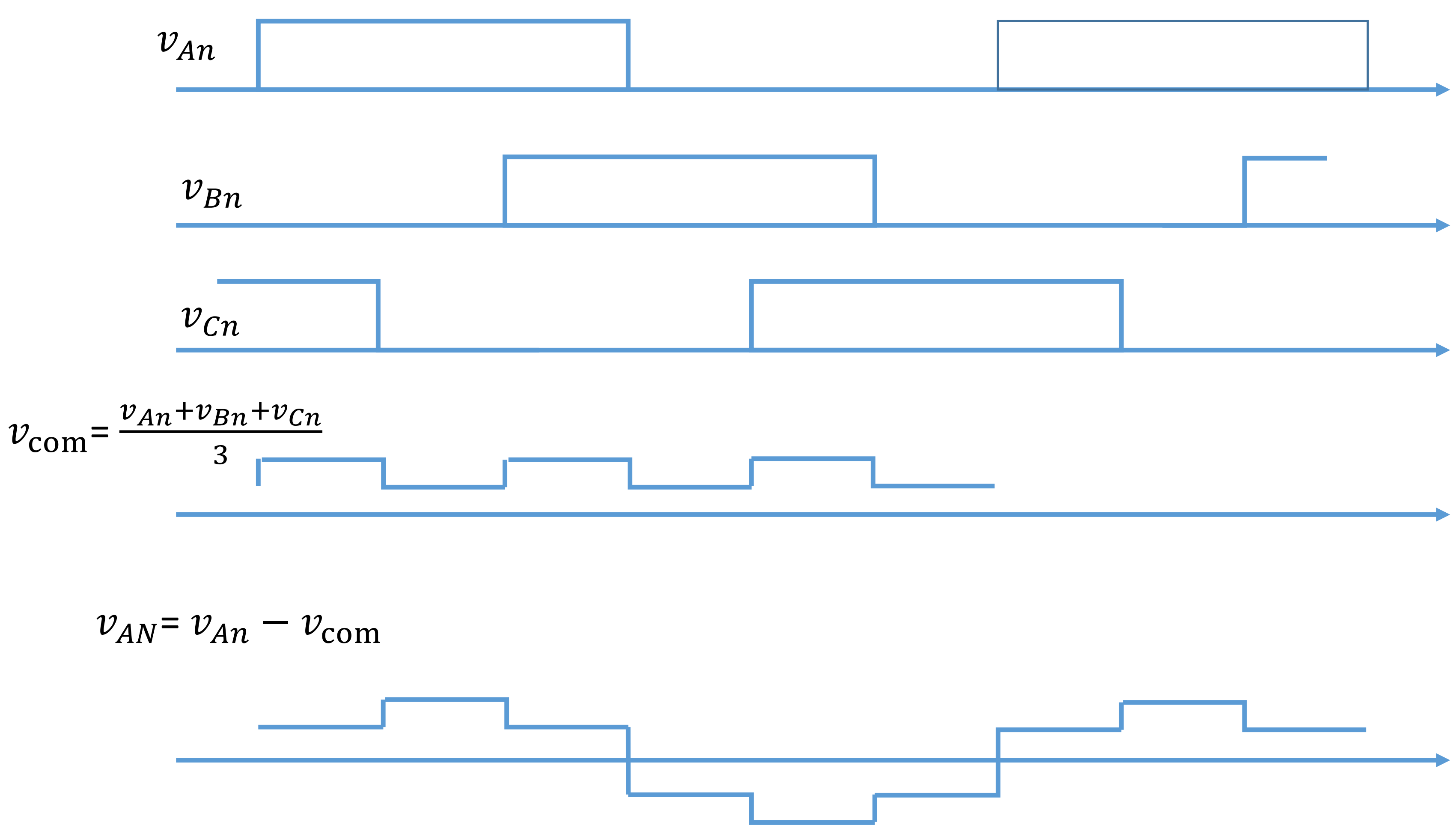


Fig. 7.15

Common mode voltage

- As the output voltage has only two choices, + or – the sum of all leg voltages, Van, vBn, vCn cannot be zero
- Figure shows the same leg voltages are in the previous slide
- Common mode voltage has values 1/3 and 2/3 of the dc
- Phase voltage can also be obtained by subtracting the common mode voltage and result is the same as in the previous slide



Waveforms of output voltage (line-to-neutral) and current in a three-phase VSI in the square-wave mode (RL load)

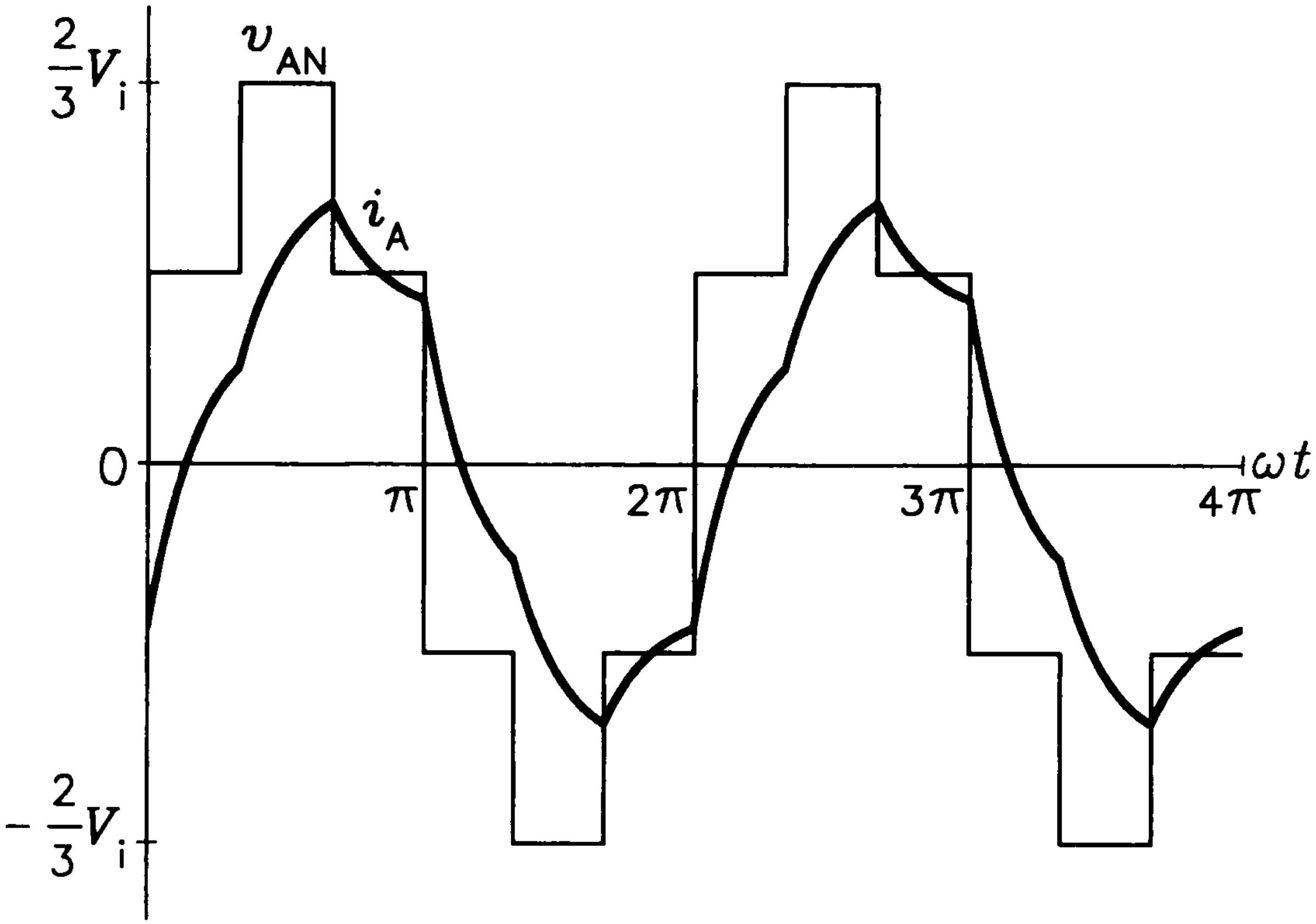


Fig. 7.16

Waveform of input current in a three-phase VSI in the square-wave mode

- Input current or current taken from the dc bus depends on the position of the switches and currents of all three phases

$$i_i = a i_A + b i_B + c i_C$$

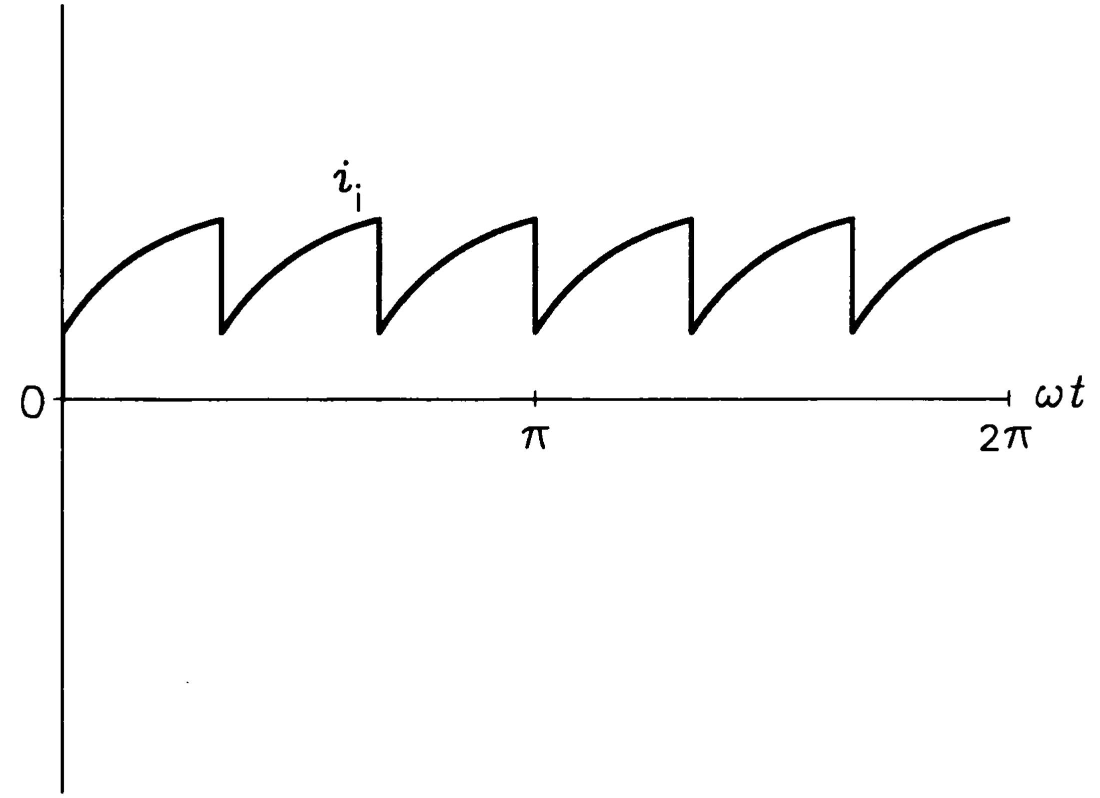
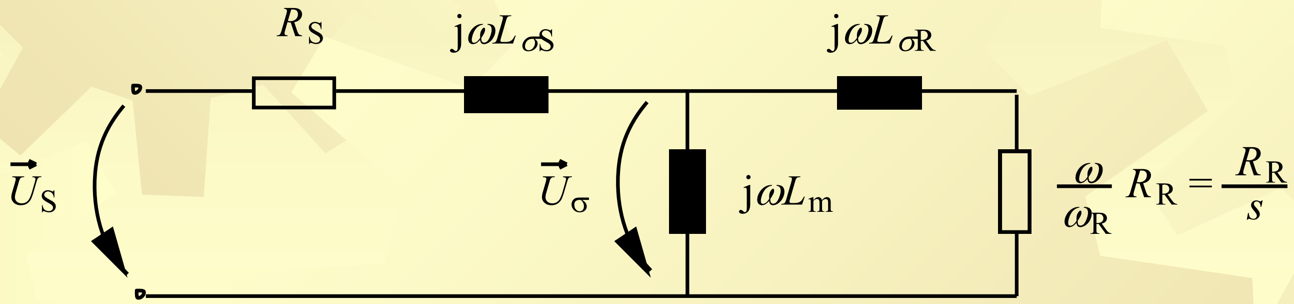


Fig. 7.17

Output current waveform

- Voltage-sourced inverter, VSI
  - Inverter is a voltage source => output voltage waveform
  - Output current depends on the impedance of the load
    - Fundamental component and harmonics can be analysed separately in linear circuits, superposition principle
- Current-sourced inverter, CSI
  - Output current is defined by the inverter
  - Output voltage depends on load

Induction machine as a load How the impedance behaves in frequency domain?



Equivalent circuit of an induction machine

- f = supply frequency
- $\omega = 2\pi f$ angular frequency
- ω_m = rotor mechanical speed
- $\omega_M = p\ \omega_m$ rotor electrical speed
- $\omega_R = \omega - \omega_M = s\omega$ angular frequency of induced currents in the rotor
- p = number of poles
- s = slip

High frequency impedance

- Harmonic *n* of voltage, angular frequency ω_n and ω_{Rn} $\omega_n = n\omega$

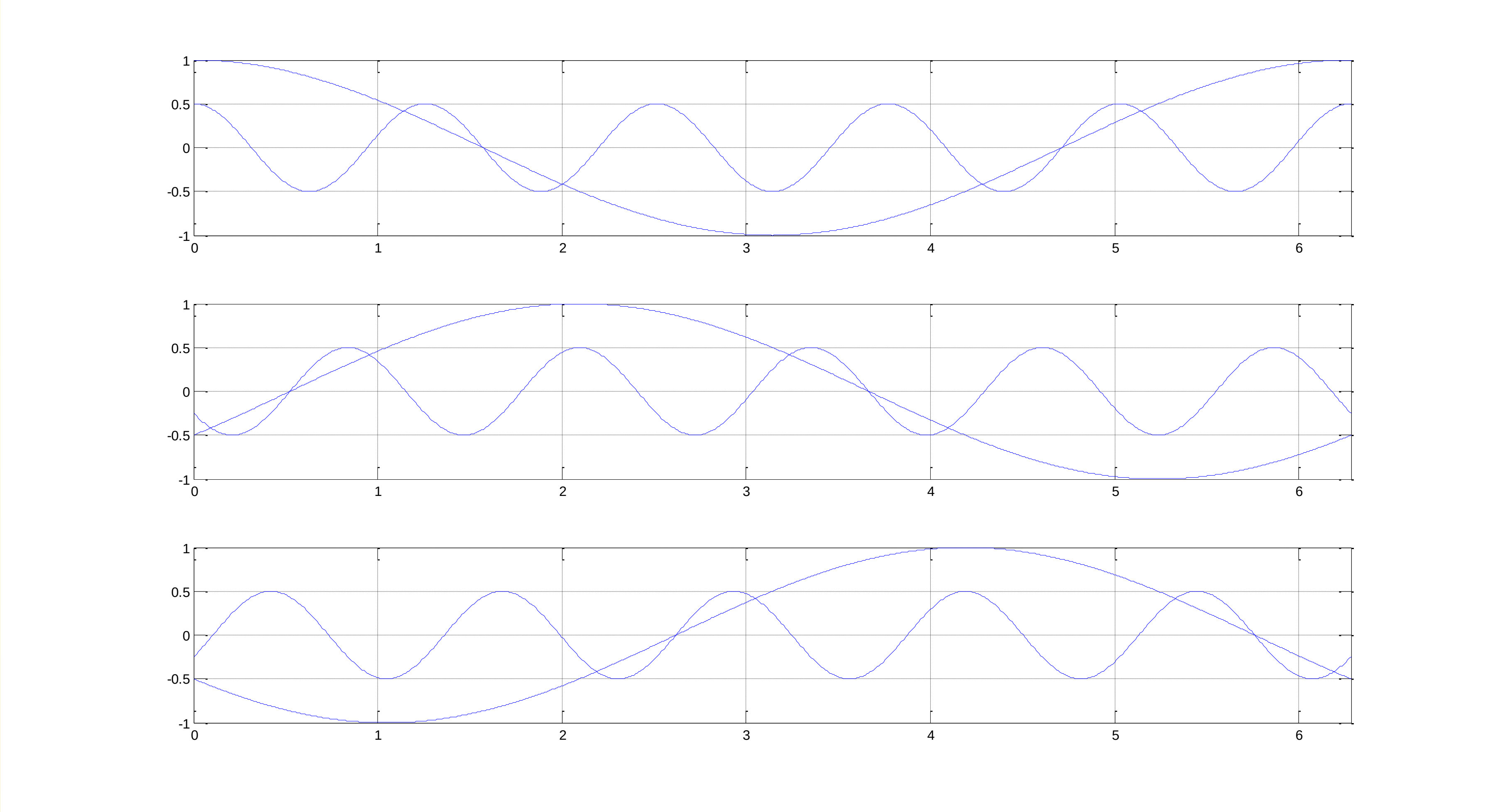
$$\omega_{Rn} = n\omega \pm (\omega - \omega_R) = (n \pm 1)\omega \mp \omega_R \approx (n \pm 1)\omega$$

  - Plus sign e.g with 5th harmonic
  - Minus sign e.g. with 7th harmonic
- Magnetising reactance
  - High when compared to rotor resistance and reactance
  - Can be neglected
- High frequency impedance can be approximated with the leakage inductances of the machine

$$\vec{Z}_n = R_s + R_R \frac{n}{n \pm 1} + \mathrm{j}n\omega(L_{\sigma S} + L_{\sigma R}) \approx \mathrm{j}n\omega(L_{\sigma S} + L_{\sigma R})$$

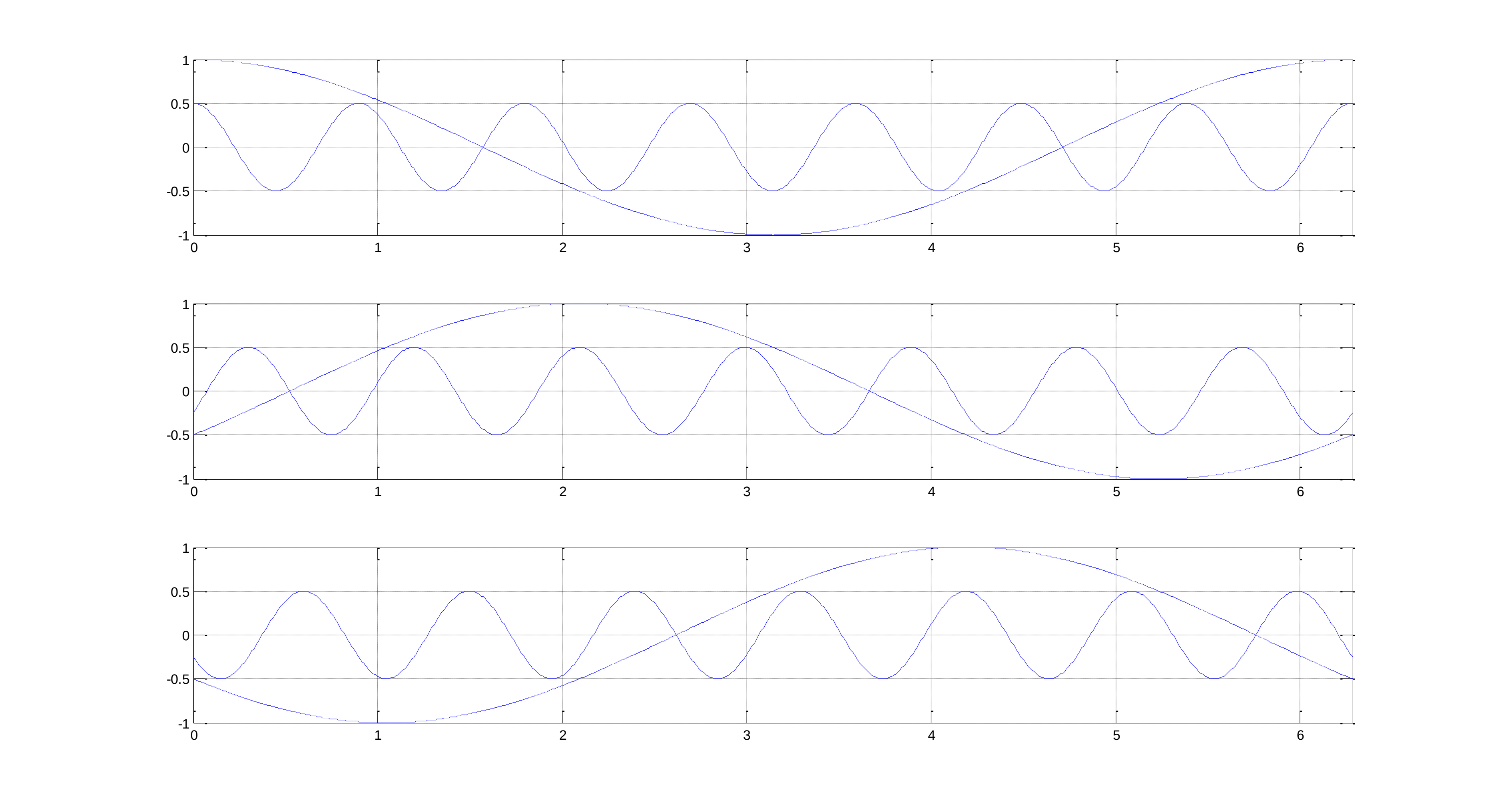
5th harmonic in a three phase system

- Fundamental component, phase sequence is A, B, C
- As seen in the figure below, at 5th harmonic phase sequence is, A, C, B, and not A, B, C, which means that system is reversed
- => 5th harmonic system rotates in reverse direction when compared to the fundamental



7th harmonic

- Fundamental, phases A, B, C
- At 7th harmonic phases, A, B, C, same as fudamental
- => 7th harmonic system rotates in the same direction



Harmonic current

- When induction machine is at stand still connected to nominal supply U_N, starting current I_s is

$$Z_k = \frac{U_N}{I_s} \approx \mathrm{j}\omega_N \left(L_{\sigma S} + L_{\sigma R}\right)$$

- High frequency impedance can be estimated with starting impedance

$$Z_n \approx n Z_k \frac{\omega}{\omega_N}$$

- Harmonic current

$$I_n = \frac{U_n}{Z_n} = \frac{U_n}{n} \frac{I_s}{U_N} \frac{\omega_N}{\omega} = \frac{U_n}{\frac{\omega}{\omega_N} U_N} \frac{I_s}{n}$$

  - $(\omega/\omega_N)\, U_N$ is the wanted output voltage U_1 at frequency ω
- Harmonic current when compared to the starting current

$$\frac{I_n}{I_s} = \frac{U_n}{n\, U_1}$$

- We can calculate relative harmonic current components from the voltage waveform!

Example

- 5th voltage harmonic is 10%
  - $I_5 = 0,1/5^* I_s = 0,02 I_s$
  - Starting current of the motor is $I_s = 5 \ldots 7 I_N$
  - $I_5 = 0,1 \ldots 0,14 I_N$ fifth harmonic is therefore larger than the corresponding voltage harmonic
- Harmonic currents are the higher
  - the higher the starting current of the motor is
  - i.e. the smaller leakage inductances are
- Motors with high power
  - Leakage inductances are getting smaller
  - Harmonic currents are higher than in smaller machines

Current harmonics in CSI

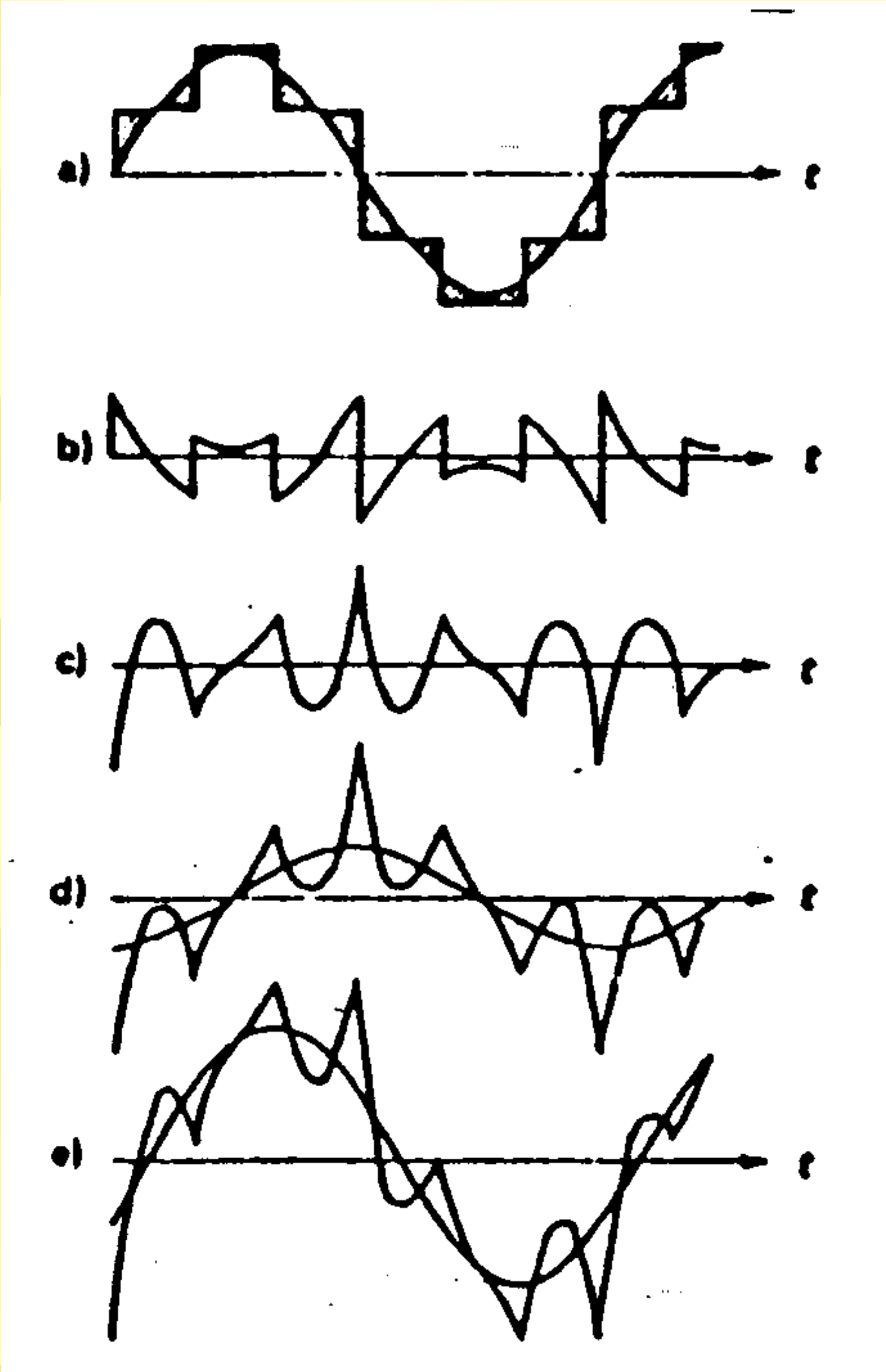
- Output current is 120° wide positive and negative pulses separated by 60° zero
  - Harmonics $I_n = \frac{I_1}{n}$, $n = 6k \pm 1,\ k = 1,\ 2, ...$
  - Harmonics depend on the fundamental component, not on the structure (impedances) of the machine
- VSI
  - Current harmonics depend on motor leakage inductance and not on the motor load

Current in induction machine

- a) output voltage and fundamental component
- b) harmonics of the voltage
- c) harmonics of the current
  - integral of voltage harmonics

$$\tilde{i} = \frac{\int (u - u_1) dt}{L_{\sigma S} + L_{\sigma R}} = \frac{\int \sum_{n=2}^{\infty} u_n(t) dt}{L_{\sigma S} + L_{\sigma R}}$$

- d) current at no load
- e) current at load



Side effects of harmonics

- In the inverter
  - Peak values of current are higher, higher current rating for the power semiconductor devices needed
- Motor
  - RMS value of current includes harmonics
  - Fundamental current component is reduced and torque production is lower
  - More losses, magnetising and winding losses
  - Motor rating must be higher than with sinusoidal current

Torque harmonics

- Torque harmonics caused by current harmonics are often small but not in six step (square wave) operation
- Fundamental component of airgap flux and current harmonics
  - Are causing torque harmonics
  - 5th current harmonic rotates in reverse direction compared to the fundamental => speed difference six
  - 7th current harmonic rotates in same direction as the fundamental => speed difference six
  - Both are producing 6th torque harmonic
  - Small inertia
    - Angular speed starts to change, oscillate
  - Also mechanical resonances possible

Switching variables and waveforms of output voltages in a three-phase VSI in the PWM mode

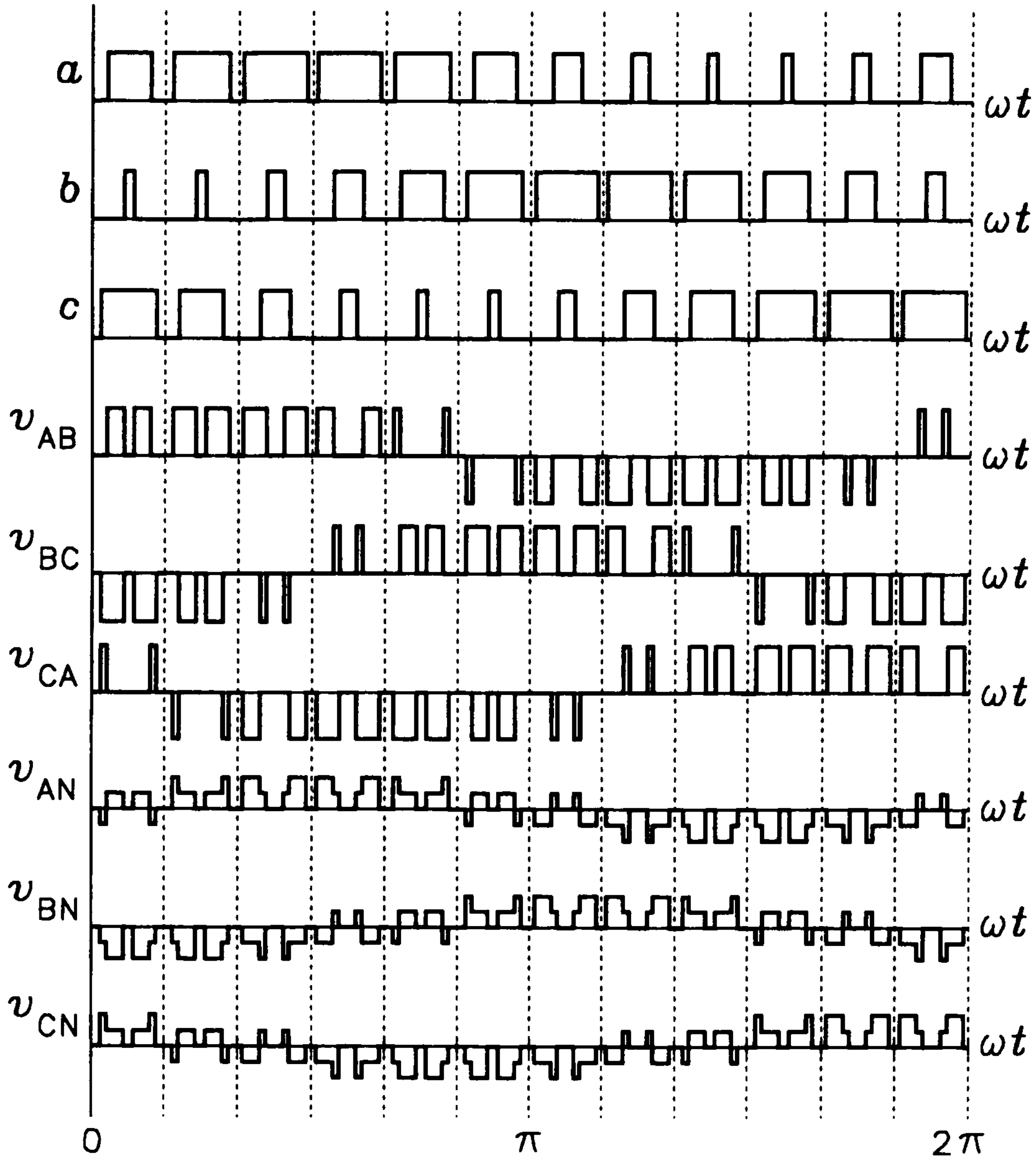


Fig. 7.18

Waveforms of output voltage and current in an RL load of a three-phase VSI in the PWM mode:
(a) load angle of 30^o, (b) load angle 60^o

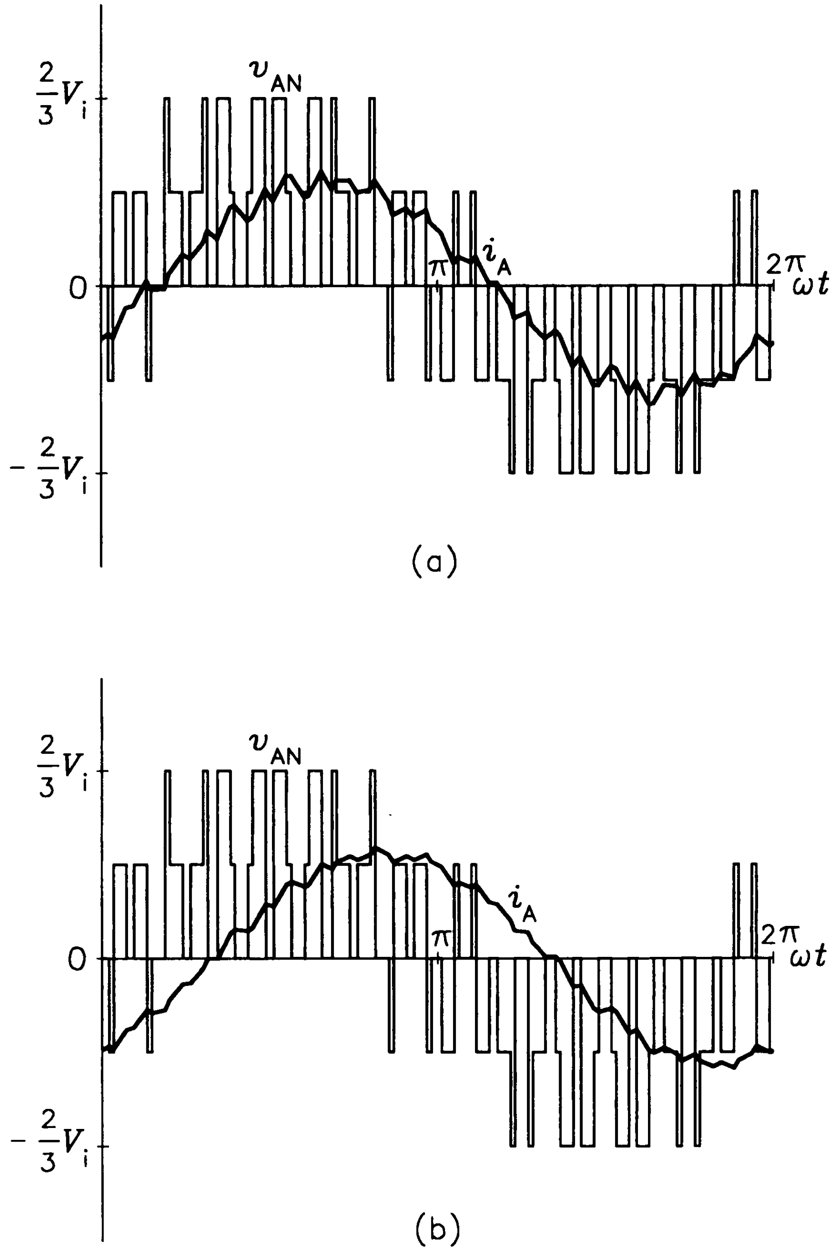


Fig. 7.19

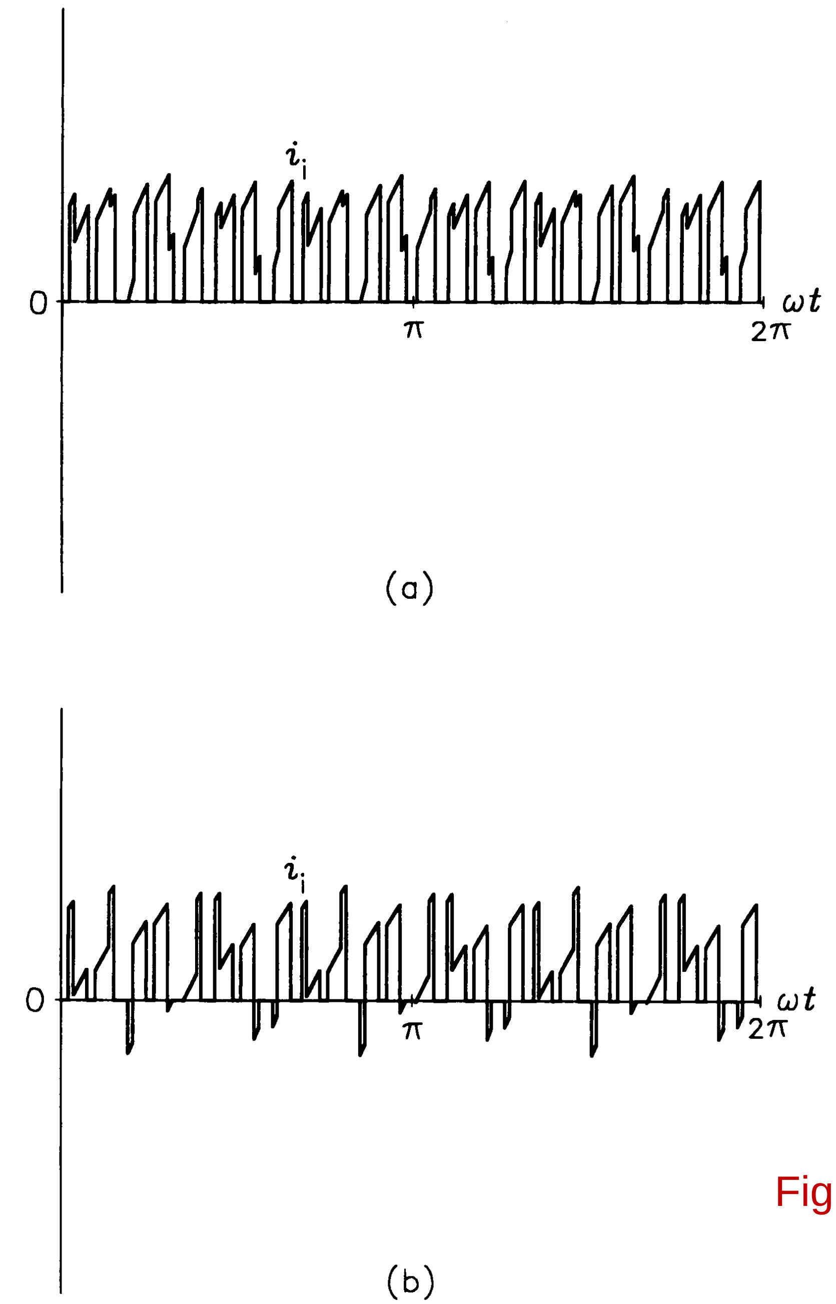


Fig. 7.20

Input current: (a) load angle of 30^o, (b) load angle 60^o

The principle of the so-called carrier-comparison method is illustrated in Figure 7.21. Reference waveforms, r_A, r_B, and r_C, given by

$$r_A(\omega t) = F(m, \omega t)$$
$$r_B(\omega t) = F\left(m, \omega t - \frac{2}{3}\pi\right)$$
$$r_C(\omega t) = F\left(m, \omega t - \frac{4}{3}\pi\right)$$

where $F(m, \omega t)$ denotes the modulating function employed, are compared with a unity-amplitude triangular waveform *y*. Values of the switching variables, *a*, *b*, and *c*, change from 0 to 1 and from 1 to 0 at every sequential intersection of the carrier and respective reference waveforms.

The sinusoidal modulating function, $F(m, \omega t) = m\sin(\omega t)$, is simple, but the voltage gain of the inverter can significantly be increased using a non-sinusoidal modulating function, many of which were developed over the years. All those functions consist of a fundamental and triple harmonics, which are not reflected in the three-phase output voltages and currents of the inverter. The popular *third-harmonic modulating function*, shown in Figure 7.22 with $m = 1$, is given by

$$F(m, \omega t) = \frac{2}{\sqrt{3}} m \left[\sin(\omega t) + \frac{1}{6}\sin(3\omega t)\right]$$

having thus only the fundamental and third harmonic. At $m = 1$, the fundamental equals $2/\sqrt{3} \approx 1.15$, which represents a 15% increase in the voltage gain at no changes to the inverter.

Carrier-comparison PWM technique (N = 12, m = 0.75)

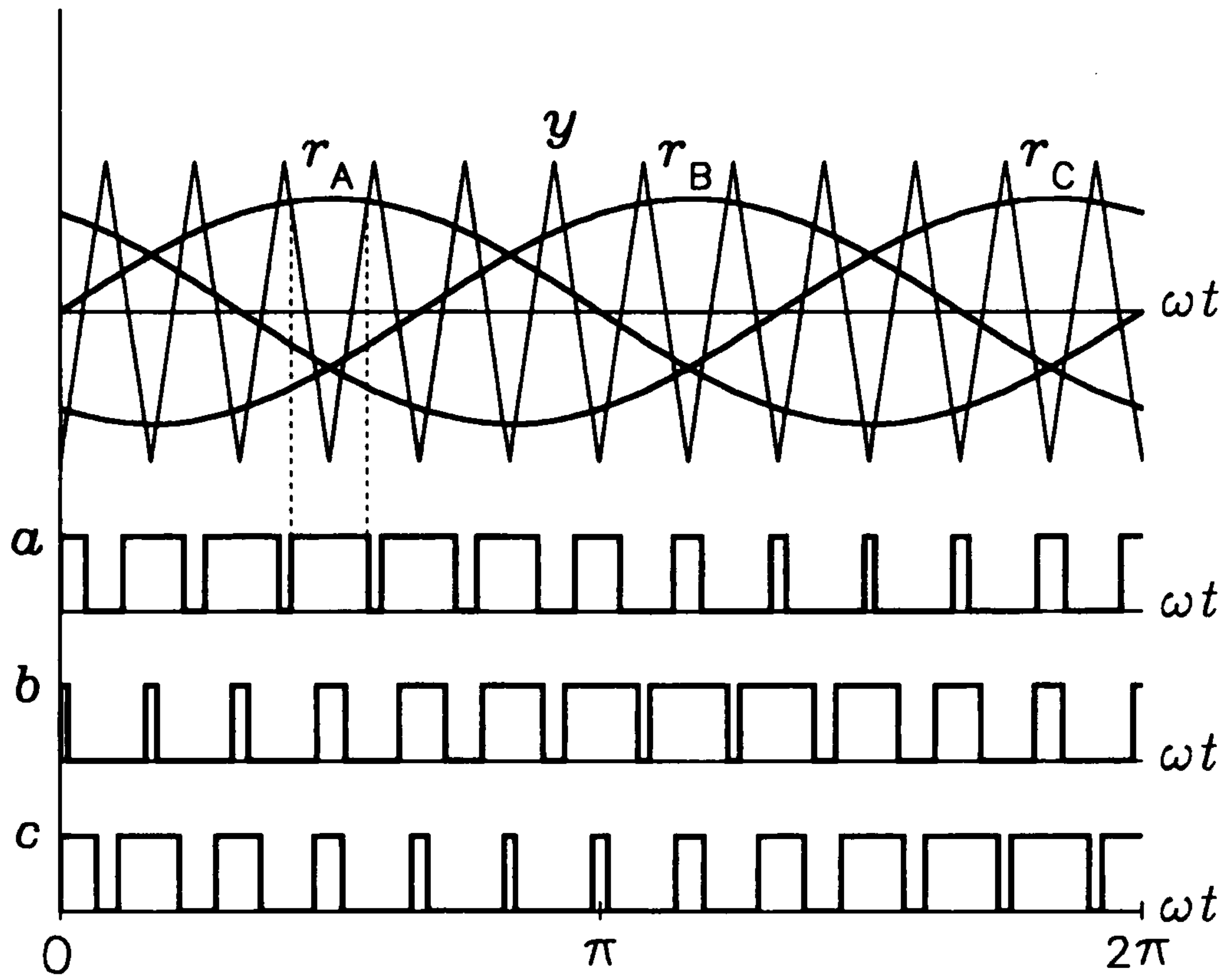


Fig. 7.21

Third-harmonic modulating function
and its components at m = 1

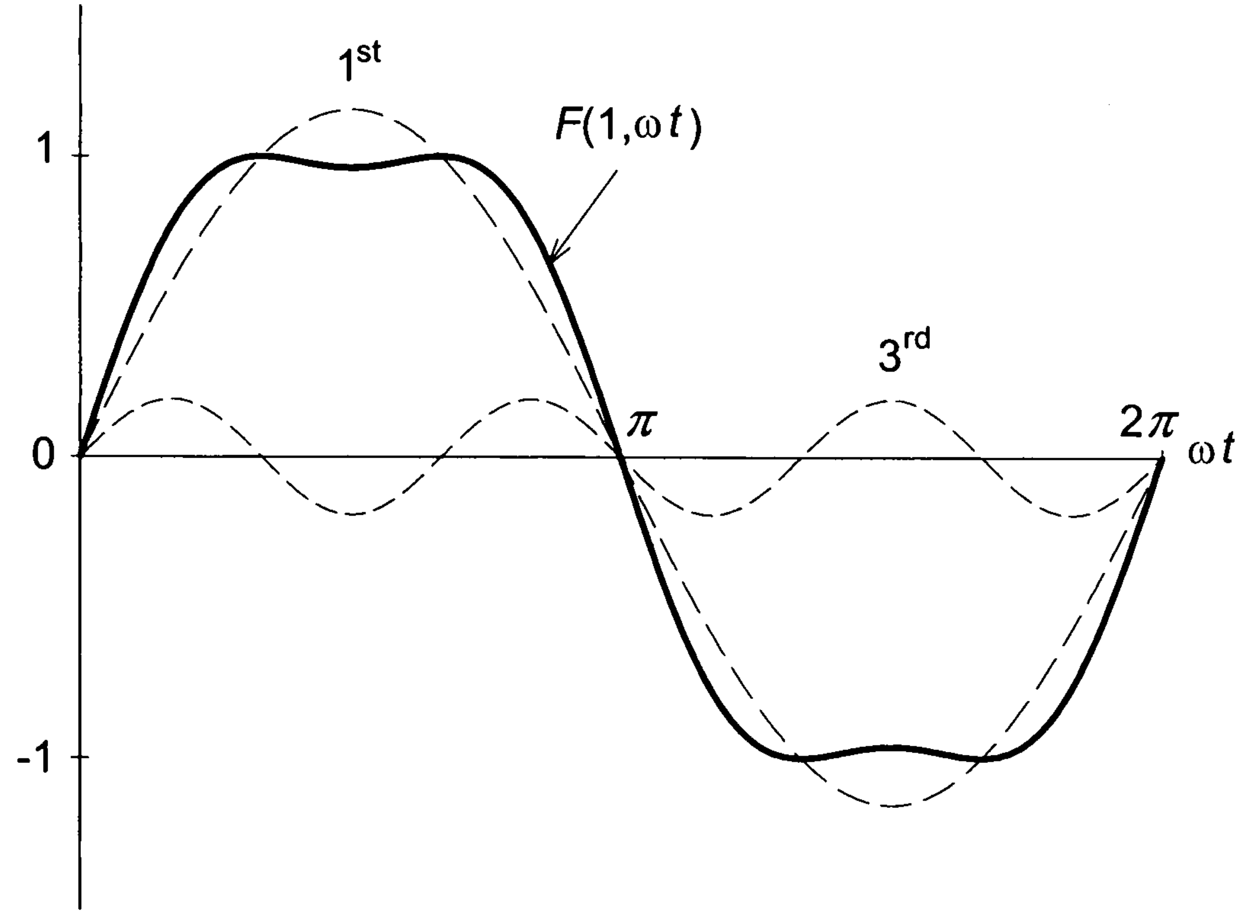


Fig. 7.22

Overmodulation

- When m_a < 1, linea area
  - Harmonics around multiples of switching frequency + output frequency
  - Output voltage is not reaching its maximum
- Overmodulation, m_a > 1
  - Nonlinear
  - Also lower frequency harmonics
  - Output voltage depends also on frequency ratio of output/switching, m_f

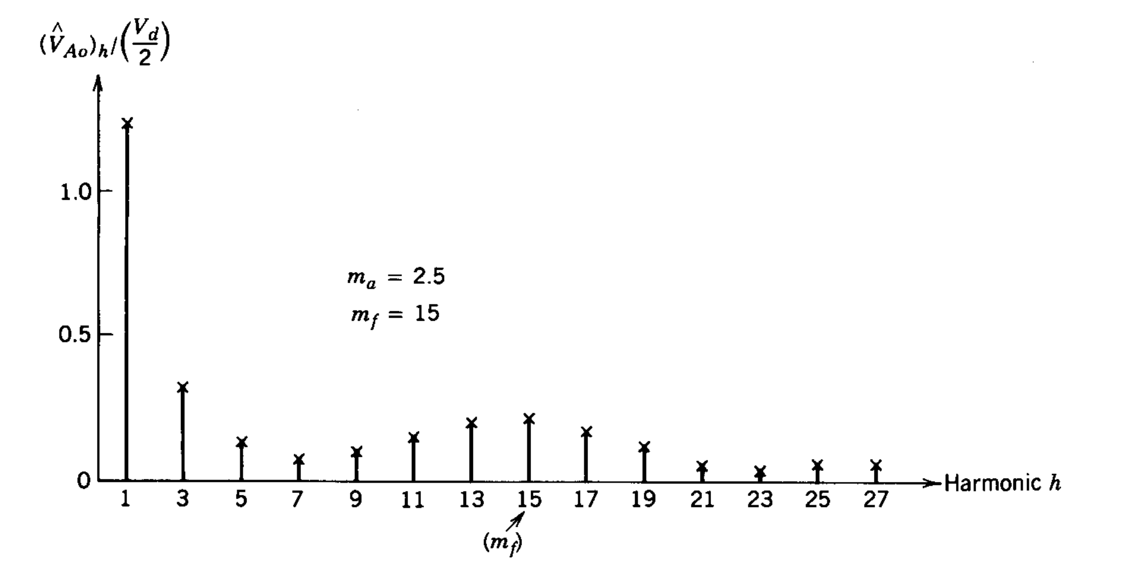


Figure 8-7 Harmonics due to overmodulation; drawn for m_a = 2.5 and m_f = 15.

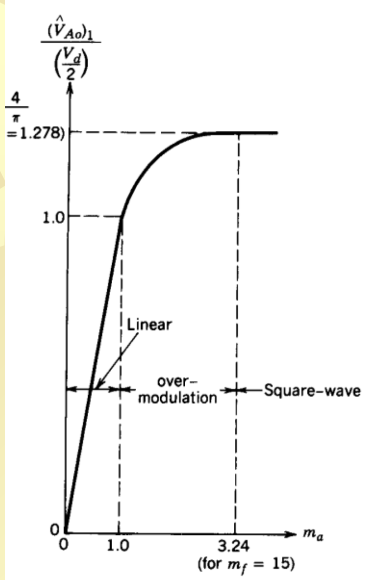
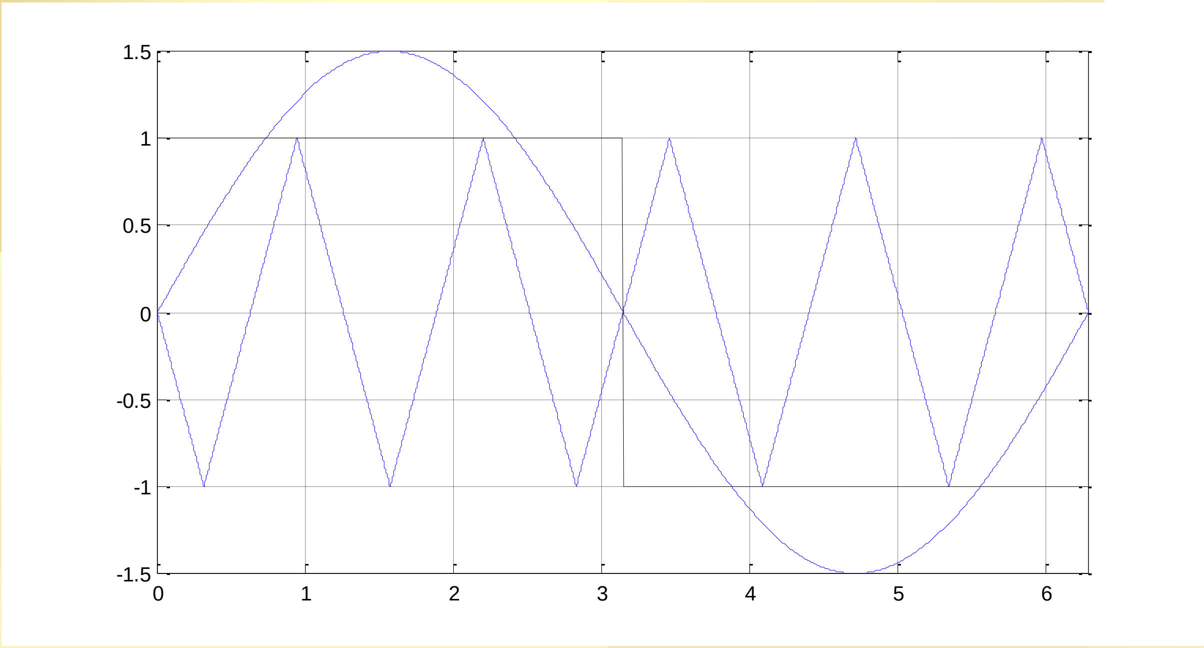


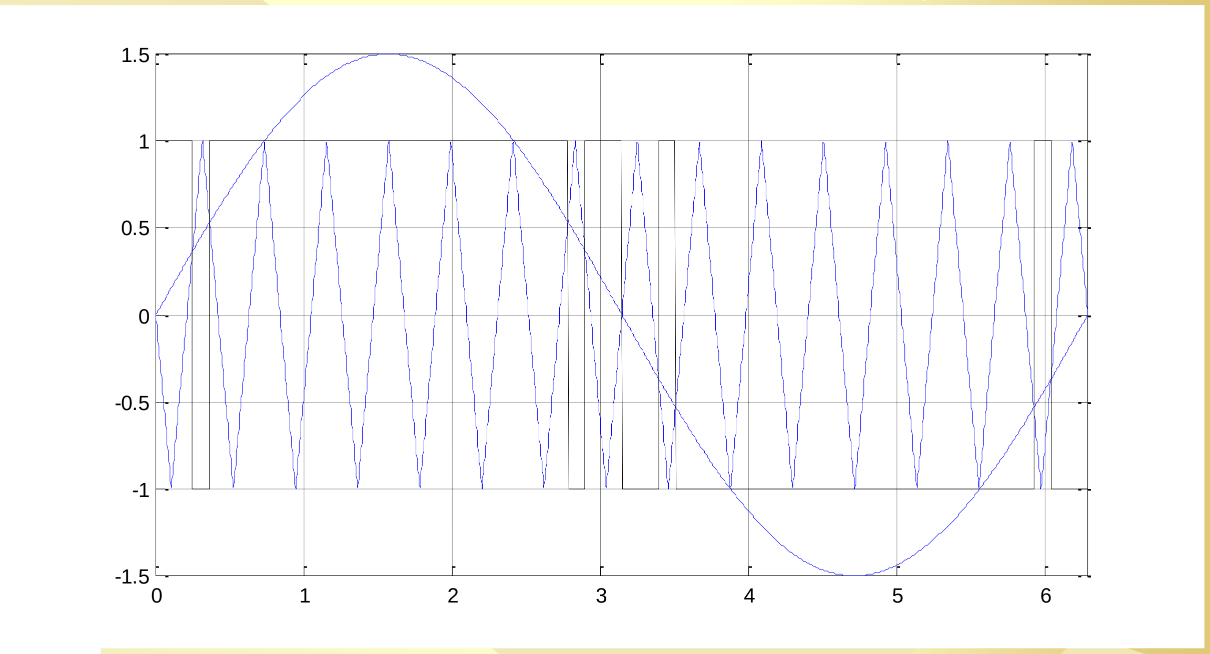
Figure 8-8 Voltage control by varying m_a.

Effect of switching frequency

- Same amplitude of reference but with lower fs only one pulse/half cycle
  - Fundamental of output is higher
  - Harmonics at lower frequencies and amplitudes higher



m_a = 1,5 m_f = fo/fs = 5

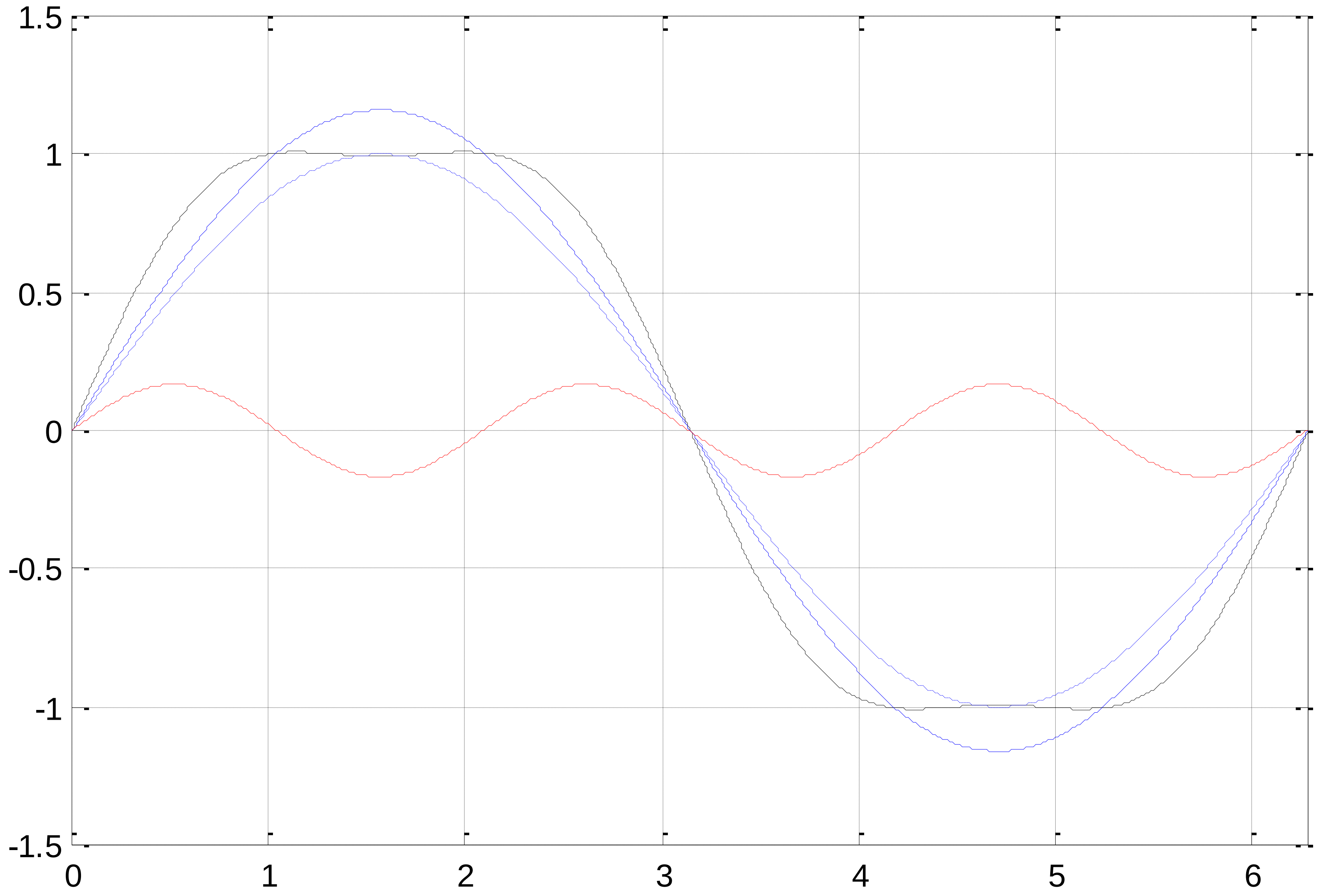


m_a = 1,5 m_f = 15

Adding third harmonic

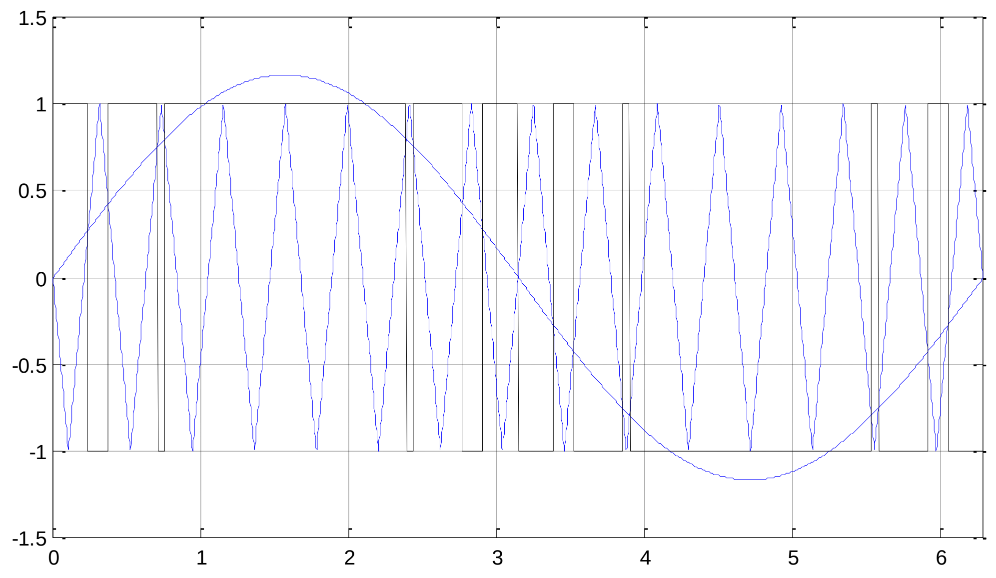
- No effect in load
  - Eliminated in line-to-line voltages
  - In wye or delta connection loads have no path for zero-sequence current
- Amplitude of third harmonic can be 16.7 %
  - Flattens the sinusoid at its peak
  - Modulator is not saturating
- If added amplitude were higher than 16.7 % the result would be higher than 1 at $\pi/3$

Modulating function when 16,7 % third harmonic added

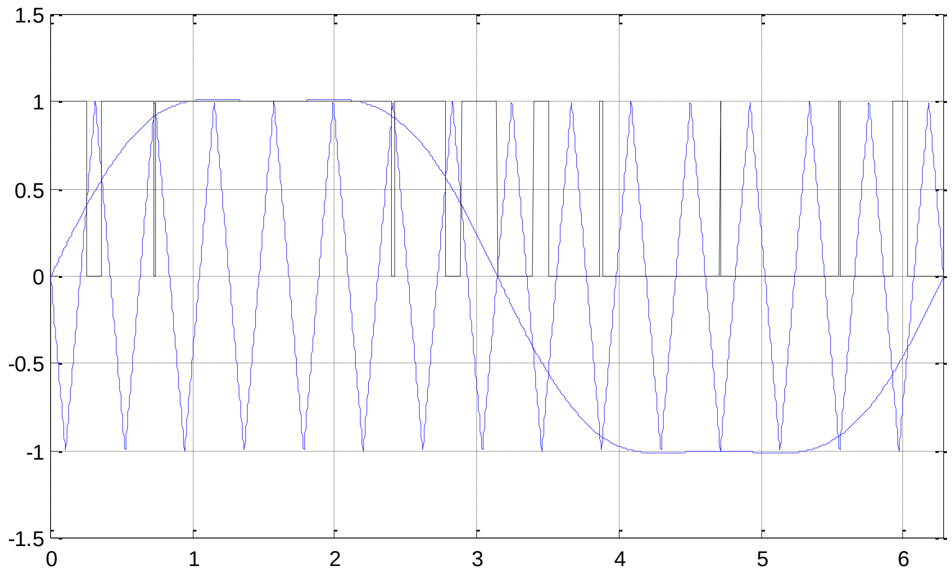


$m_a = 1{,}167 \qquad m_f = 15$

- Adding third harmonic prevents pulses from merging together
  - Modulator is linear in 0 - 1,167



No third hamonic added



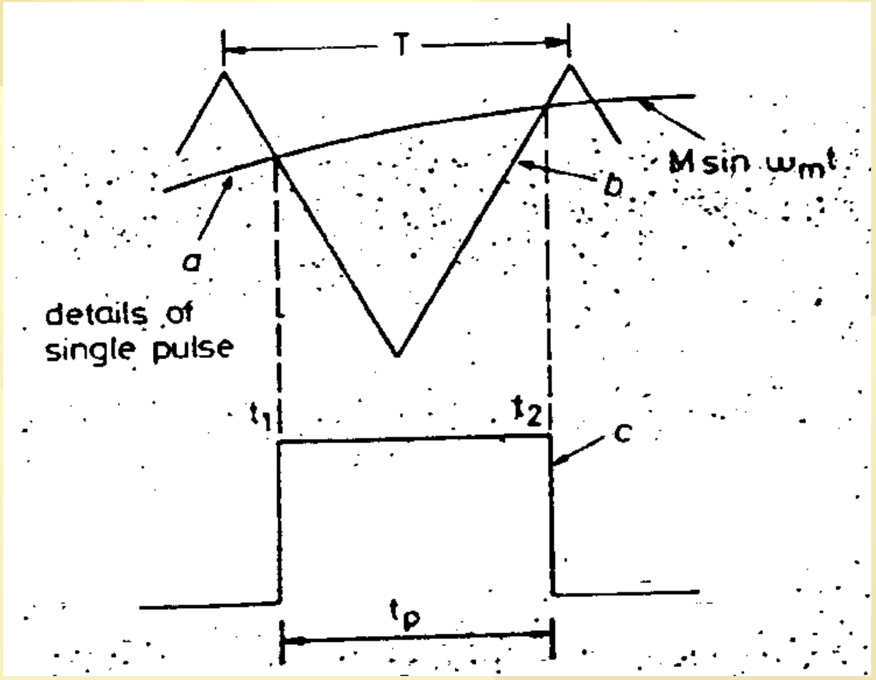
Added third harmonics is 16,7 %

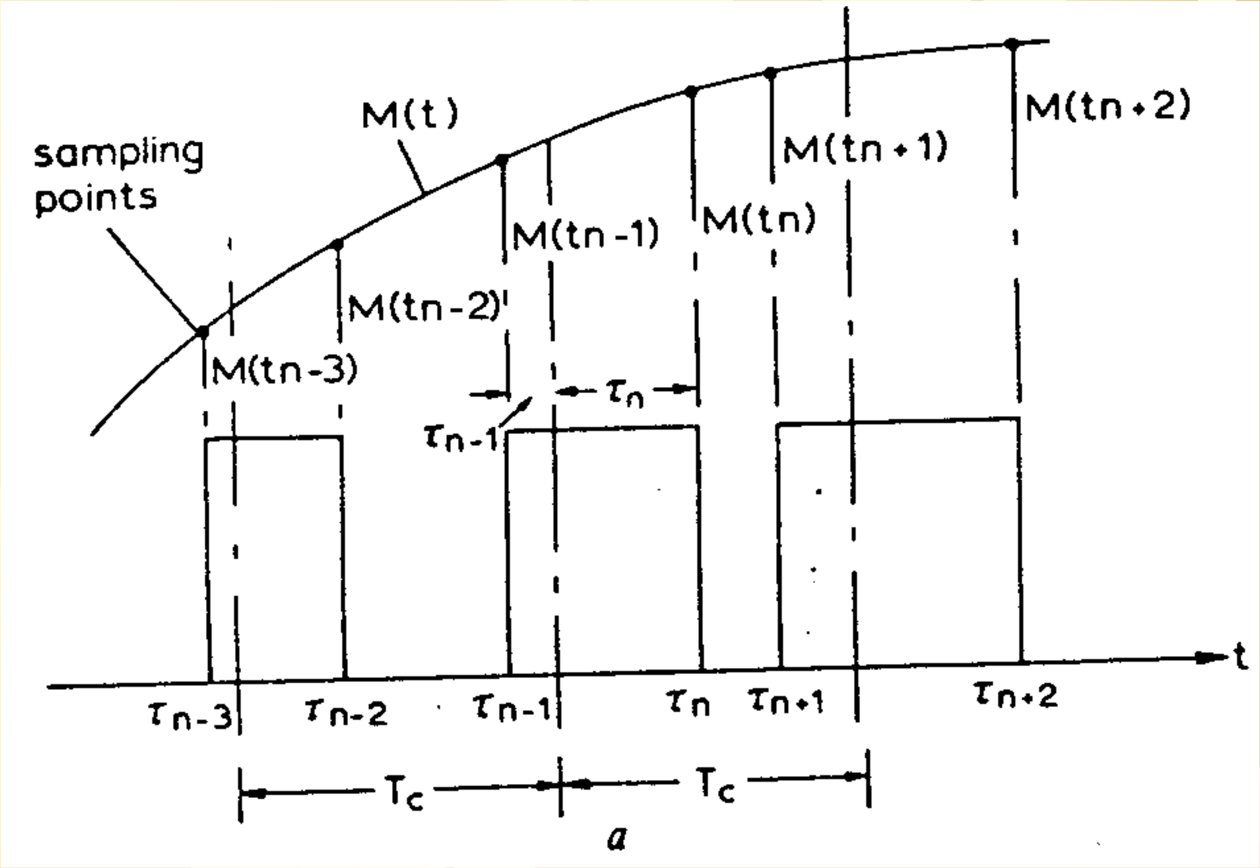
Digital modulation

- Previous methods were based on analog technologies
  - Ideal sinusoid easy to realise
  - Often called **natural sampling**
- Digital electronics
  - Everything is replaced by sampled signals
  - Symmetrical sampling (also called uniform, regular)
  - Asymmetrical sampling

Natural sampling, analog

- Comparison is done on both edges of the pulse, leading and trailing
- Result contains much information





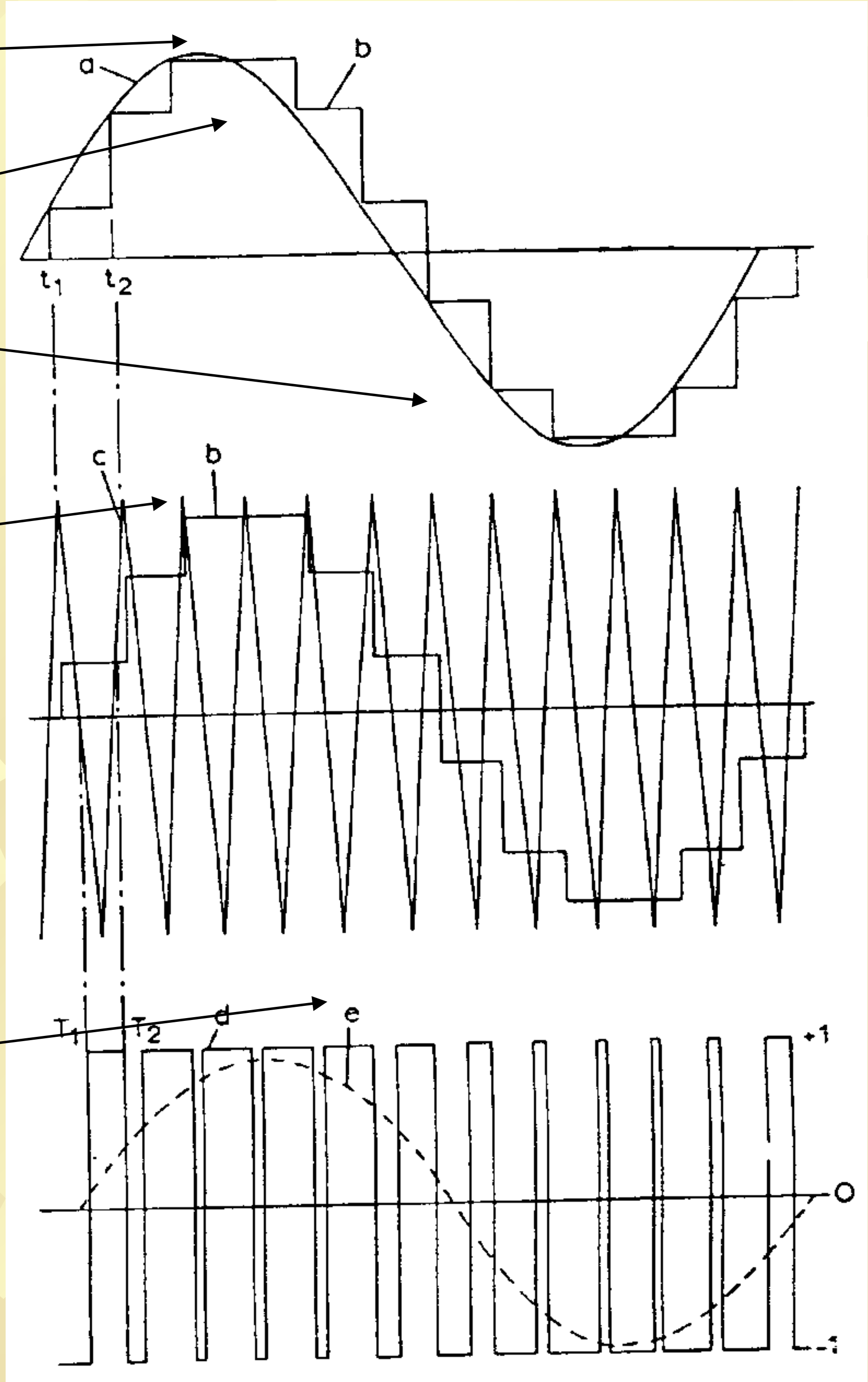
Symmetrical sampling

Modulationg function

Sampled modulating

Carrier

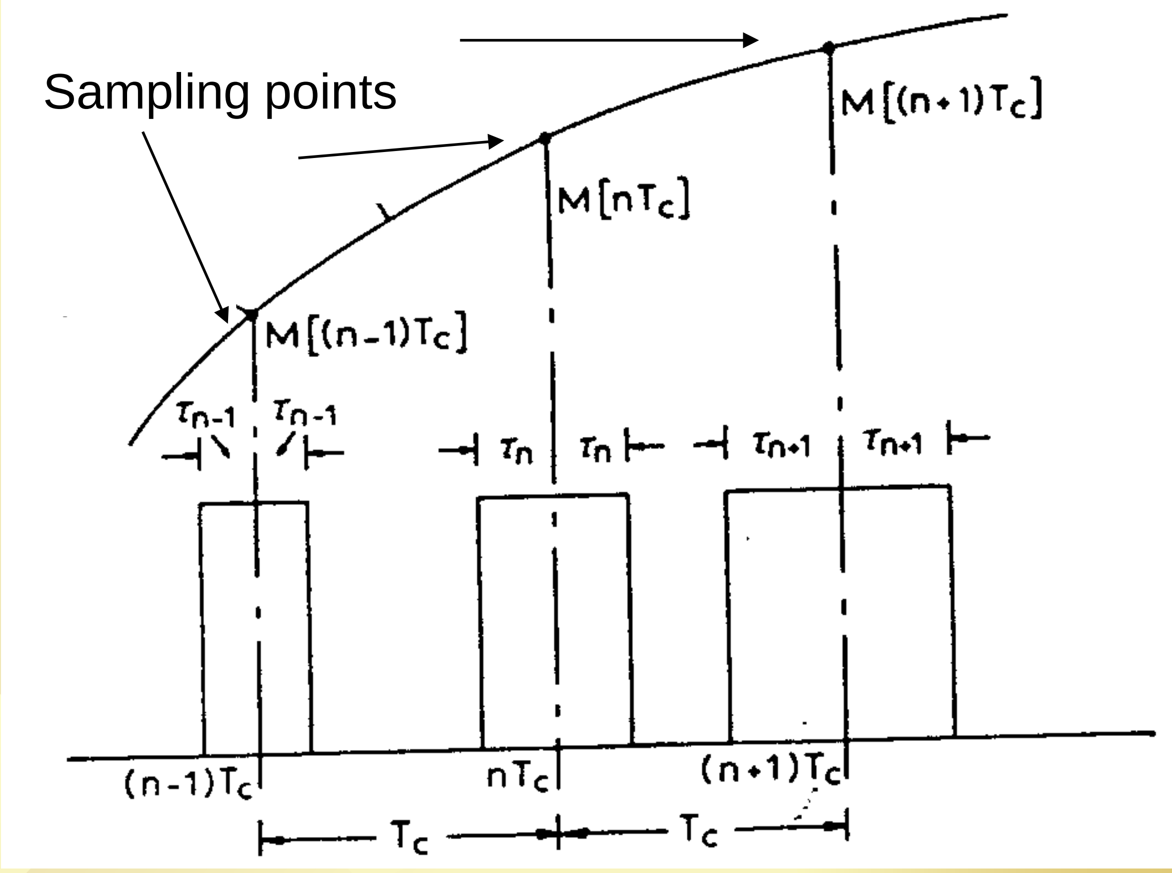
Fundamental of output



Symmetrical sampling

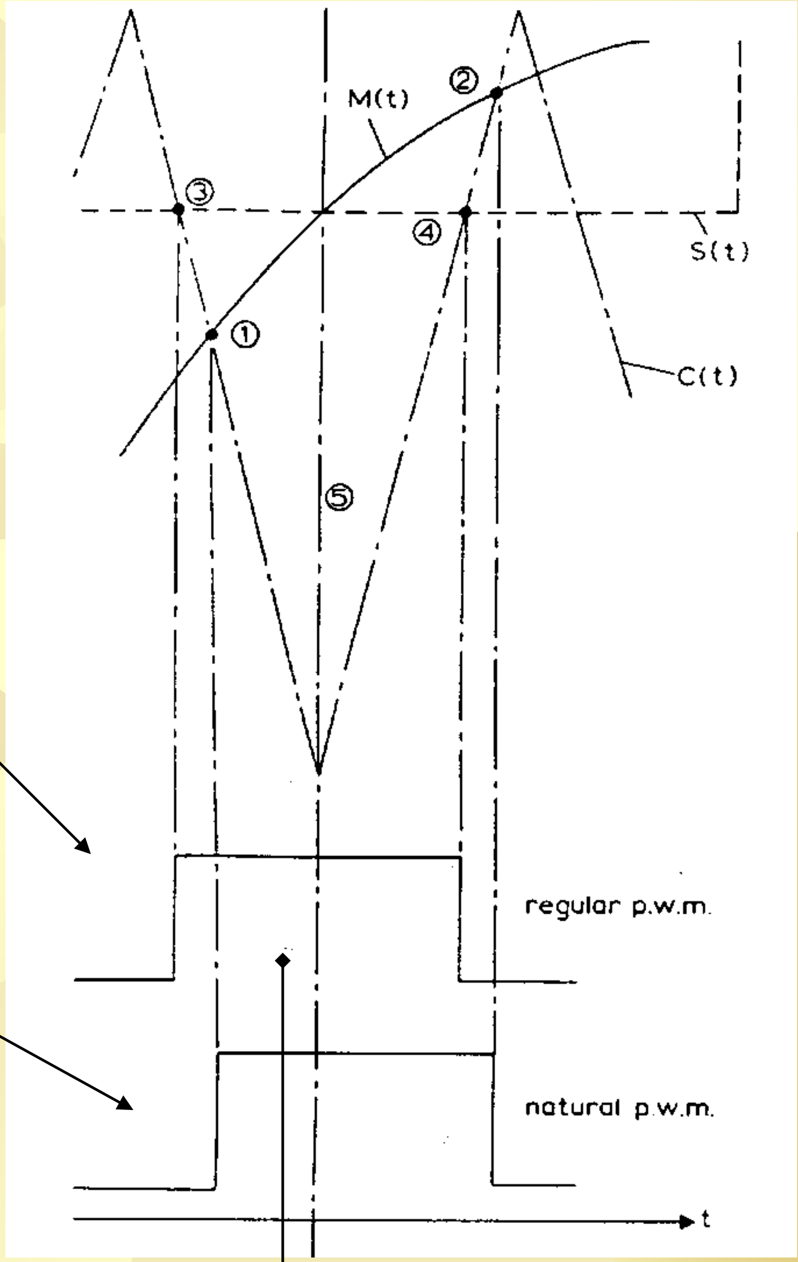
- Pulse width depends on equal distance samples
  - => symmetric
- Distance between pulse centers is constant
- Pulse width is calculated from

$$t_p = \frac{T_c}{2}\left\{1 + m_a \sin \omega_m t_1\right\}$$



Comparison

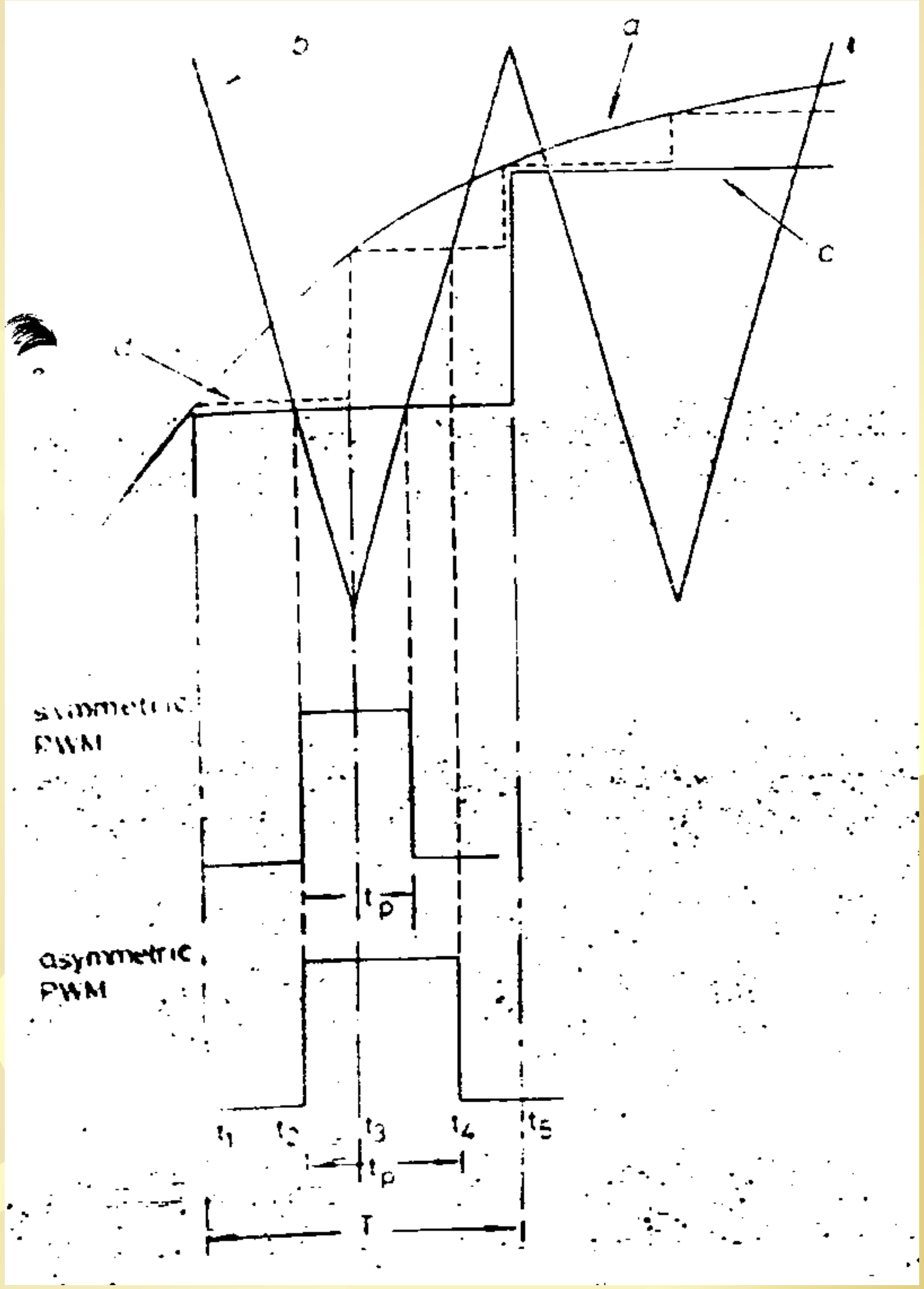
- Symmetric
  - Pulse edges modulated similarly
- Natural
  - Pulses are not symmetric around the center points



Asymmetrical sampling

- Sampling at 2*f_k
- Every pulse edge is modulated separately
- Contains more information than symmetric sampling
  - Harmonics are reduced
- Pulse width is calculated from

$$t_p = \frac{T}{2}\left\{1 + \frac{m_a}{2}\left(\sin\omega_m t_1 + \sin\omega_m t_3\right)\right\}$$



Sampling and harmonics

- Modulating function is replaced by a sampled waveform
  - Harmonics of output voltage are changed when compared to natural sampling
  - Fundamental component is not any more equal to the amplitude of modulating function

Space-vector modulation

The voltage space vector plane with the reference vector $\vec{v}^*$ of line-to-neutral voltages is shown in Figure 7.23(a) and repeated in Figure 7.23(b) in the per-unit format, with the maximum available magnitude of that vector as the base. The modulation index, *m*, constitutes the magnitude of per-unit $\vec{v}^*$. Neglecting the voltage drops in the inverter, the highest available peak value of the output line-to-line voltage equals the dc input voltage, V_i. Thus

$$m = \frac{V^*_{LL,p}}{V_i}$$

where $V^*_{LL,p}$ denotes the reference peak line-to-line voltage. With *m* = 1 the maximum available peak value of the fundamental line-to-line output voltage equals the dc input voltage.

In the steady-state, when the fundamental output voltage and current maintain fixed magnitude and frequency, *m* is constant and $\vec{v}^*$ rotates with a constant speed. However, the space vector PWM technique allows synthesis of an instantaneous voltage vector, which may change in magnitude and speed from one switching cycle to another.

The angular position, β, of the reference vector allows determination of the sector of the complex plane in which the vector is located within the given sampling cycle of the digital modulator. Specifically,

$$S = int\left(\frac{3}{\pi}\beta\right) + 1$$

where β is expressed in radians and *S* is the sector number (I to VI). The in-sector position, α, of $\vec{v}^*$ is then given by

$$\alpha = \beta - \frac{\pi}{3}(S-1)$$

Voltage space vector plane of a three-phase VSI:
(a) in volts, (b) per unit

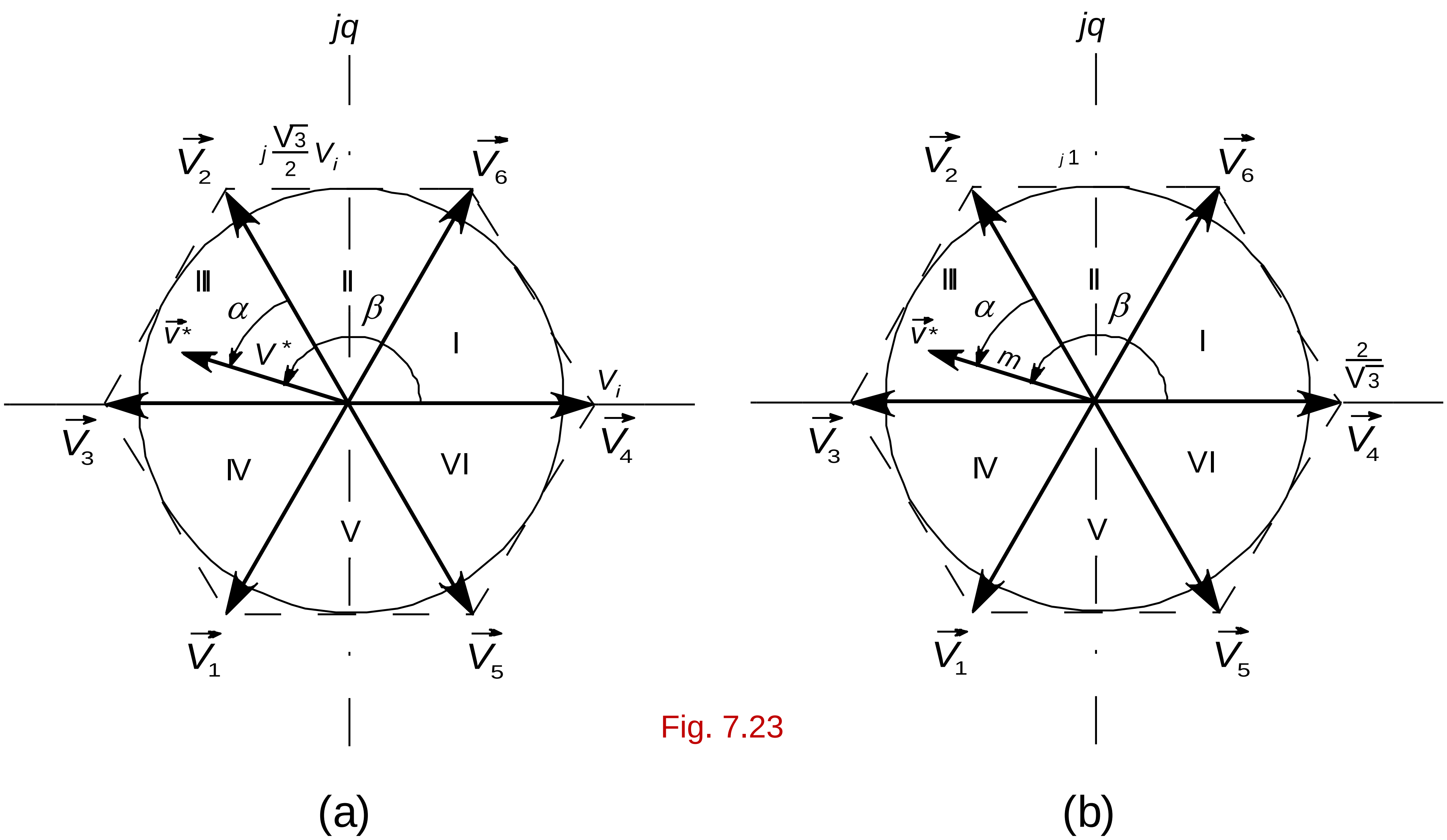


Fig. 7.23

Control of the voltage

- Induction machine is a typical load for VSIs
  - Inductances are smoothening current
- Voltage is caused by a changing flux

$$\underline{u}_s = d\,\underline{\psi}_s / dt$$

- Flux is an integral of voltage

$$\underline{\psi}_s = \int_0^t \underline{u}_s \,\mathrm{d}t + \underline{\psi}_{s0}$$

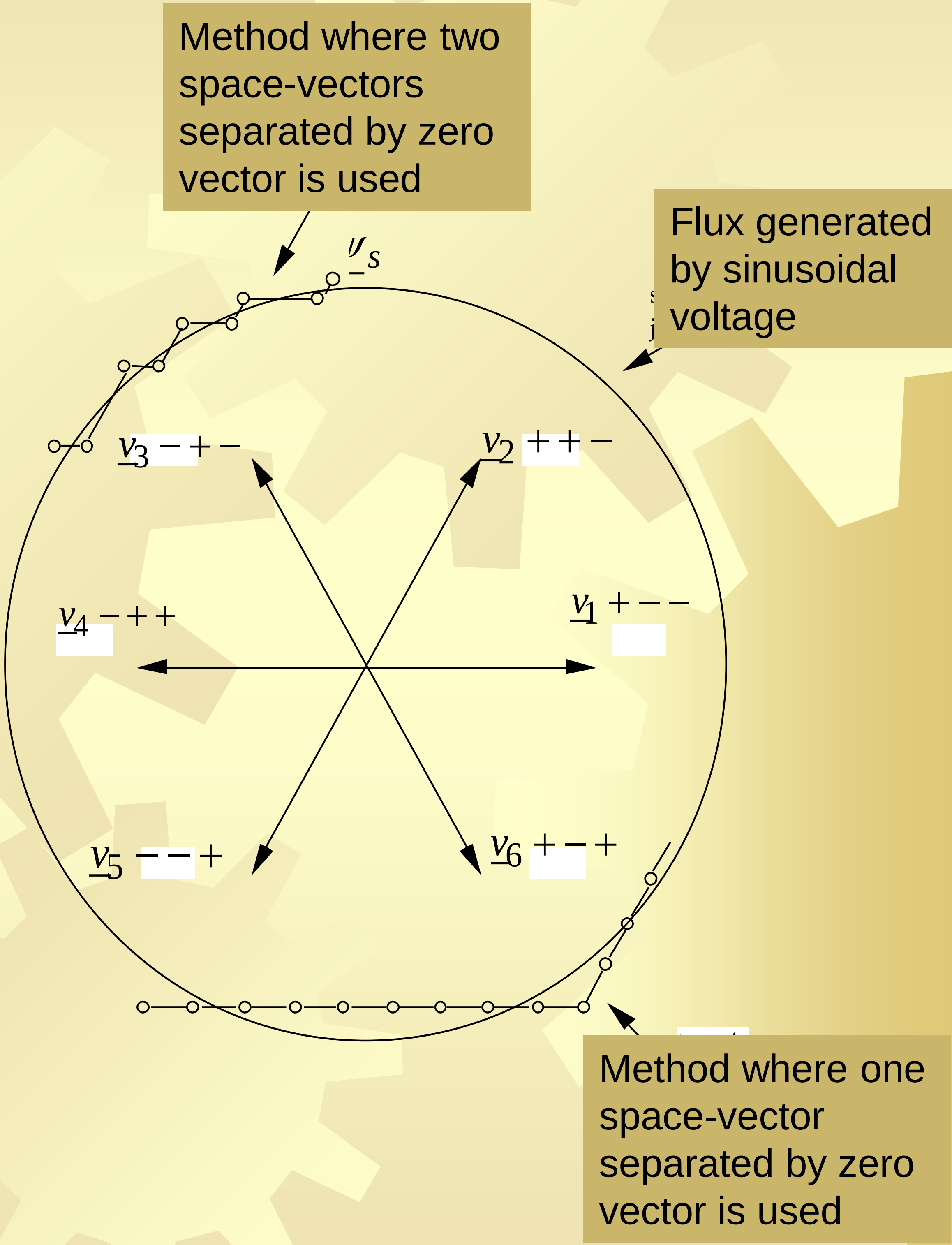
  - Resistances assumed to be small
  - Describes the air gap flux of an induction machine
  - More detailed models needed in accurate control, discussed more in courses on electric drives

Requirements

- Ideal voltage vector causes flux $\underline{\psi}_s = \frac{\hat{u}_s}{\omega_1} e^{j(\omega t - \pi/2)}$
- In VSI
  - Only six non-zero voltage vectors
  - Nevertheless, output voltage integral should be similar to ideal
  - This is true
    - When flux has constant amplitude
    - Rotates smoothly with the wanted angular frequency

Flux vector

- Circle is created by sinusoidal voltage
- VSI
  - Integral stops when zero vector is used
  - Non-zero vectors move flux with constant speed in the direction of the voltage vector



SVM, Space vector modulator

- Every 60° wide sector is divided either to
  - Constant angle slices $\Delta\alpha$
  - Or constant duration time segments ΔT
- Flux change is same with ideal voltage or with VSI

$$\Delta\underline{\psi}_s = \int_0^{\Delta T} \underline{u}_s(\alpha)\,dt = \int_0^{\Delta T} \hat{u}_s \mathrm{e}^{\mathrm{j}\omega t}\,dt$$

The revolving reference voltage vector is synthesized from stationary active (non-zero) vectors, $\vec{V}_X$ and $\vec{V}_Y$, framing the sector in question, and a zero vector, $\vec{V}_0$ or $\vec{V}_7$. Durations, T_X, T_Y, and T_Z, of states generating those vectors are given by:

$$T_X = mT_{sw} \sin(60^{o} - \alpha)$$
$$T_Y = mT_{sw} \sin(\alpha)$$
$$T_Z = T_{sw} - T_X - T_Y.$$

Times T_X, T_Y, and T_Z indicate only how long a given state should last in the given switching cycle, but how the cycle is divided between the employed states must also be specified. The two most commonly used state sequences can be called a *high-quality sequence* and a *high-efficiency sequence*. The high-quality sequence is

$$X - Y - Z_1 - Y - X - Z_2 \ldots$$

where each state in the sequence lasts half of the allotted time. states Z_1 and Z_2, complementarily 0 and 7, are placed in such an order that a transition from one state to another involves switching in one inverter leg only. The number of commutations can further be reduced, at the expense of slightly increased distortion of output currents, when the high-efficiency state sequence

$$X - Y - Z - Y - X \ldots$$

is employed. Now, states X and Y last $T_X/2$ and $T_Y/2$ seconds respectively, and state Z lasts T_Z seconds. Moreover, Z = 0 in the even sectors (II, IV, and VI) and Z = 7 in the odd sectors (I, III, and V).

With this state sequence, the average number of pulses of a switching variable per cycle of output voltage is $2N/3 + 1 \approx 2N/3$. As a result, the switching losses decrease by about 30% in comparison with the high-quality state sequence,

Example high-quality space sequence

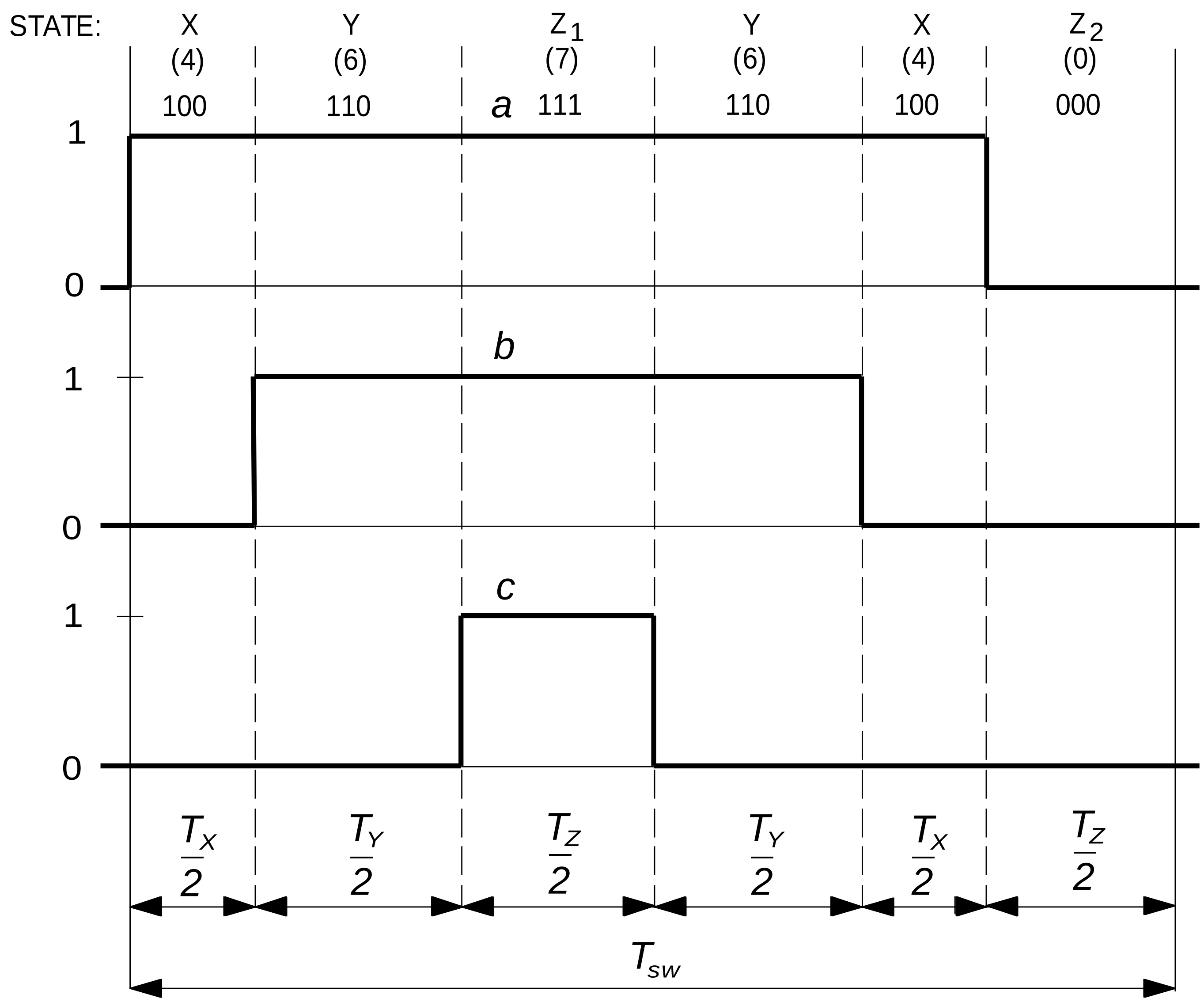


Fig. 7.24

Example high-efficiency space sequence

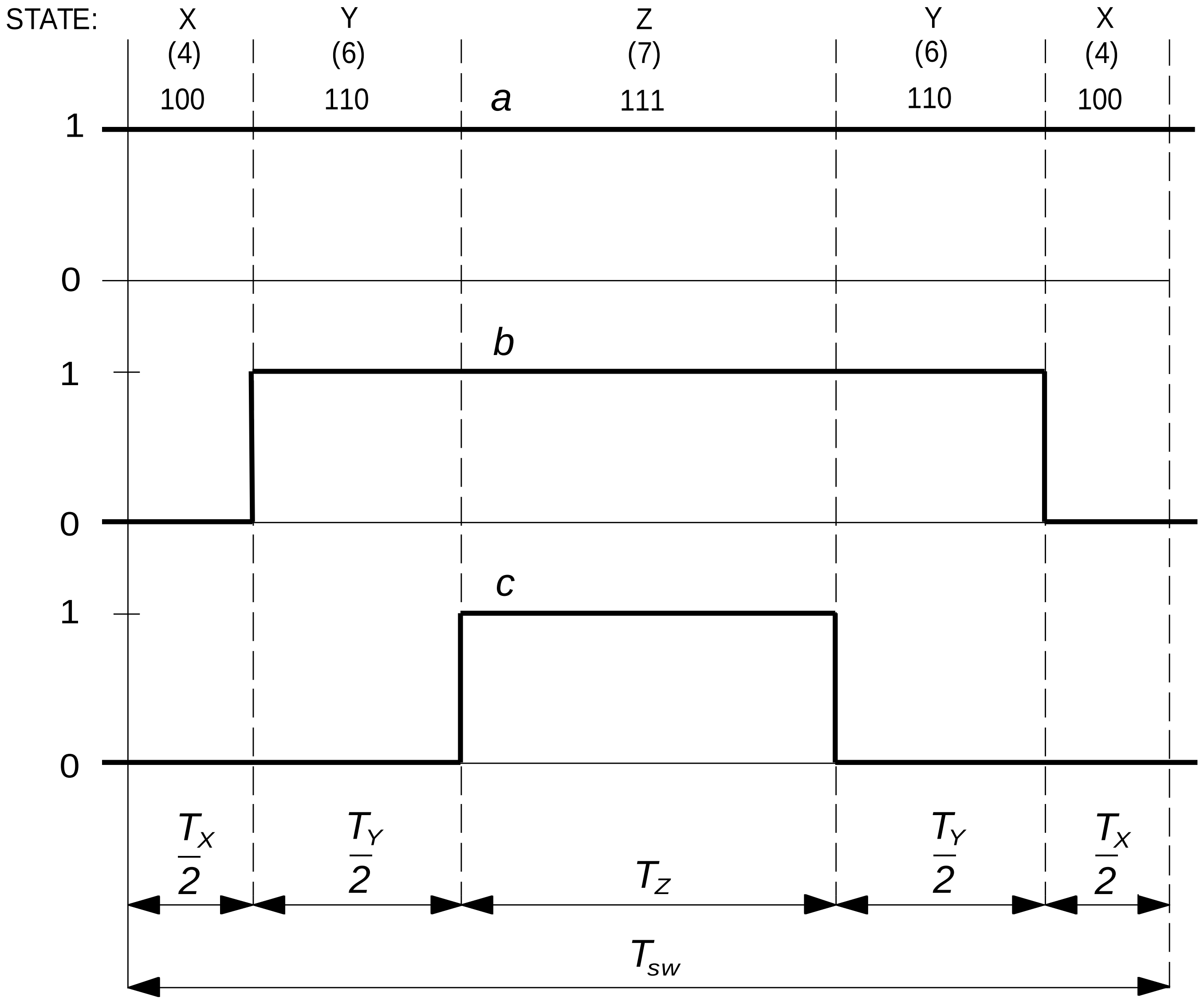


Fig. 7.25

Switching pattern with half- and quarter-wave symmetries

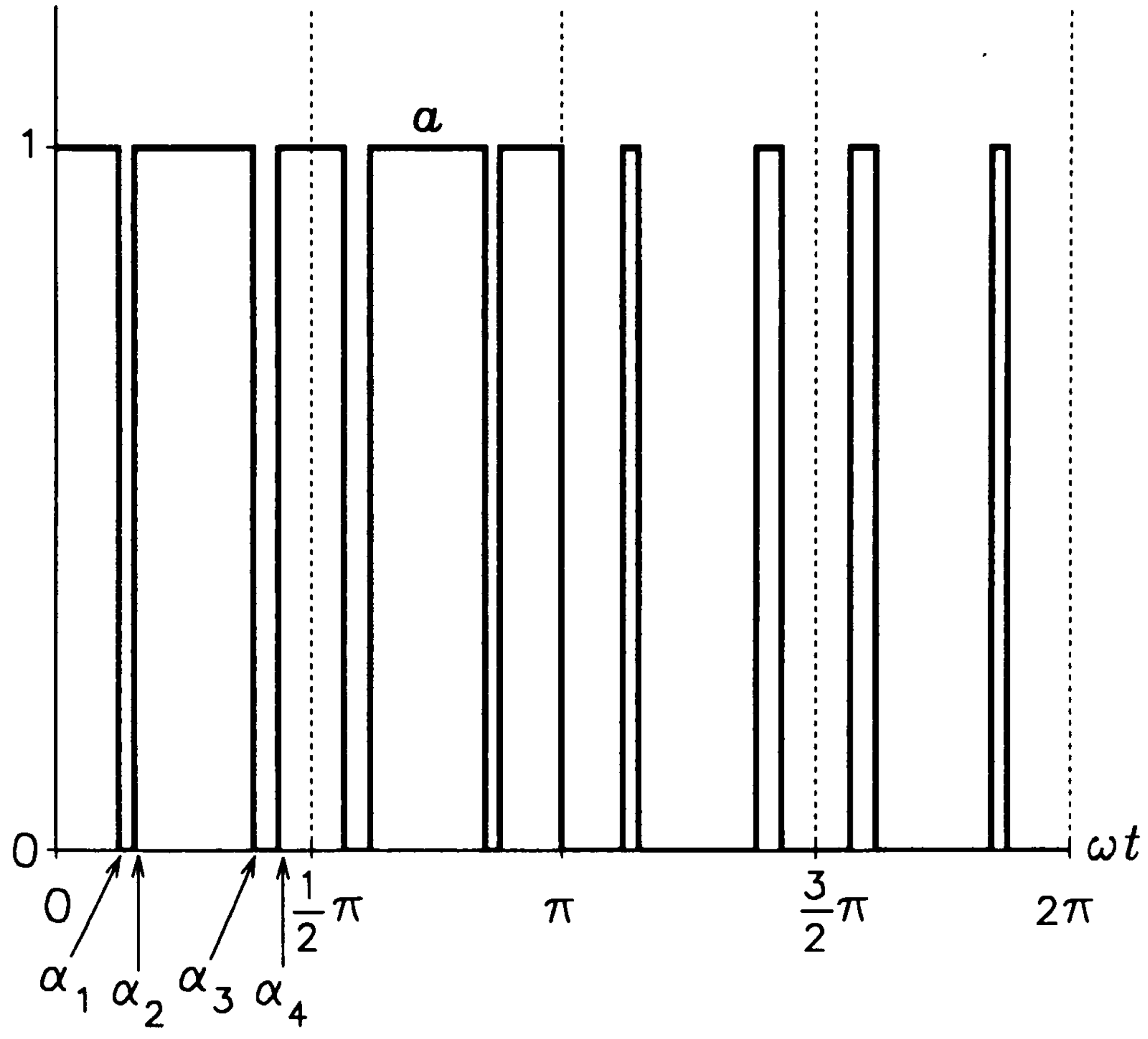


Fig. 7.26

Optimal primary switching angles as functions of the magnitude control ratio ($K = 5$)

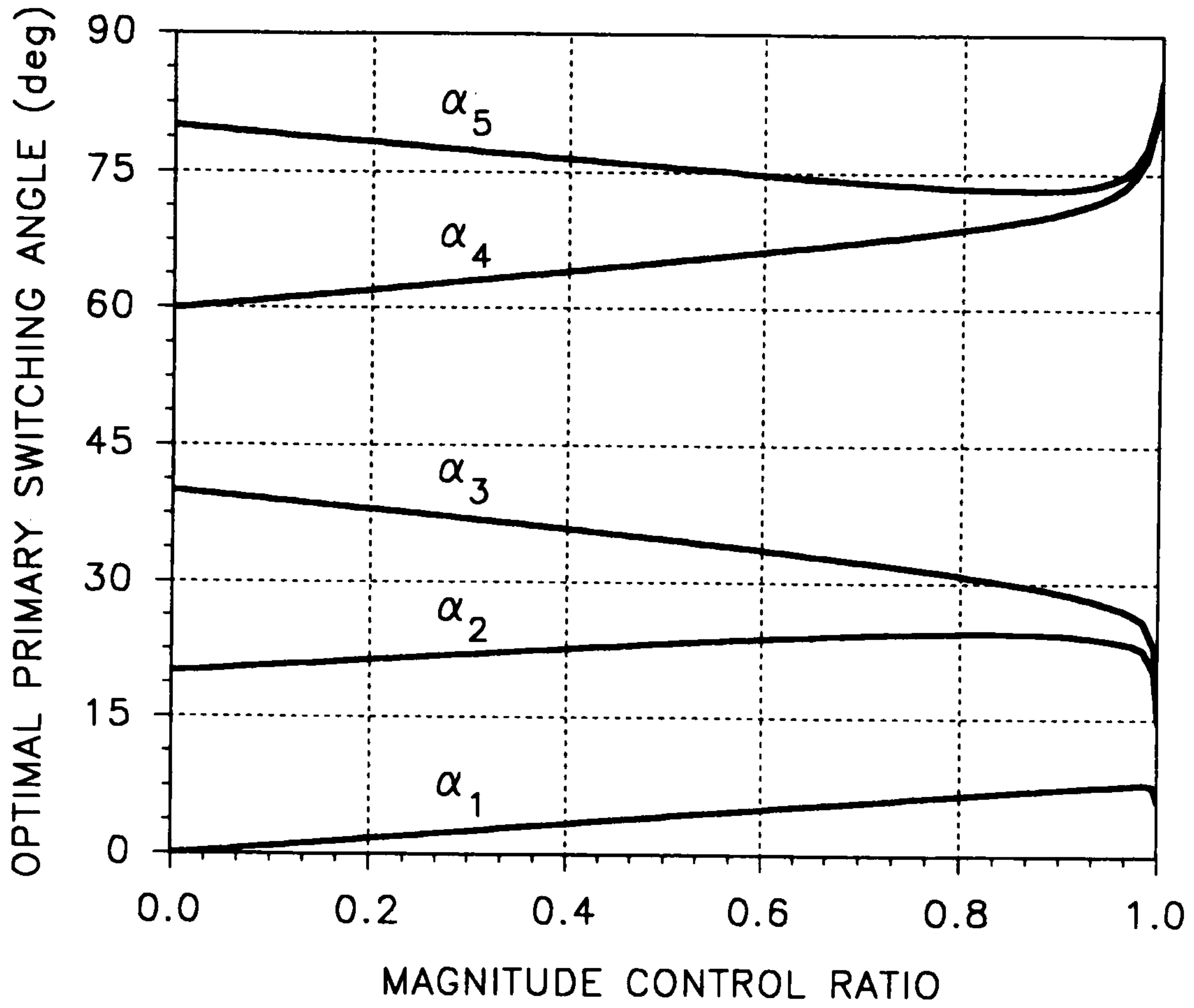


Fig. 7.27

Harmonic spectrum of line-to-neutral voltage with the harmonic-elimination technique ($K = 5$, $M = 1$)

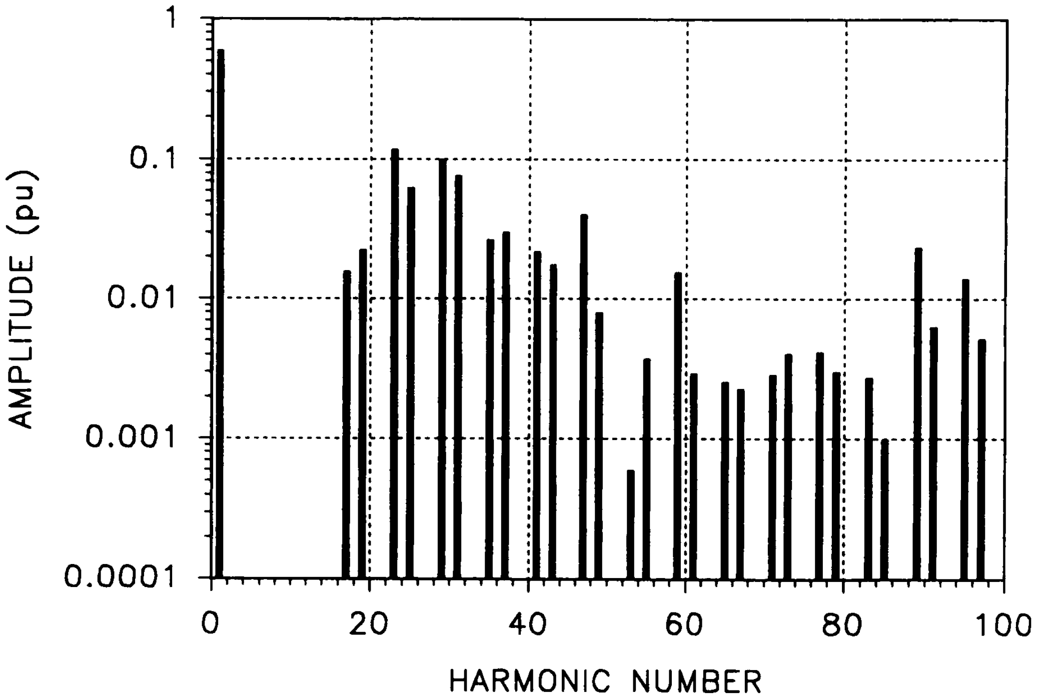


Fig. 7.28

Switching patterns and voltage and current waveforms:
(1) carrier-comparison PWM with sinusoidal reference,
(2) space vector PWM with high-efficiency state sequence,
(3) programmed PWM with harmonic elimination

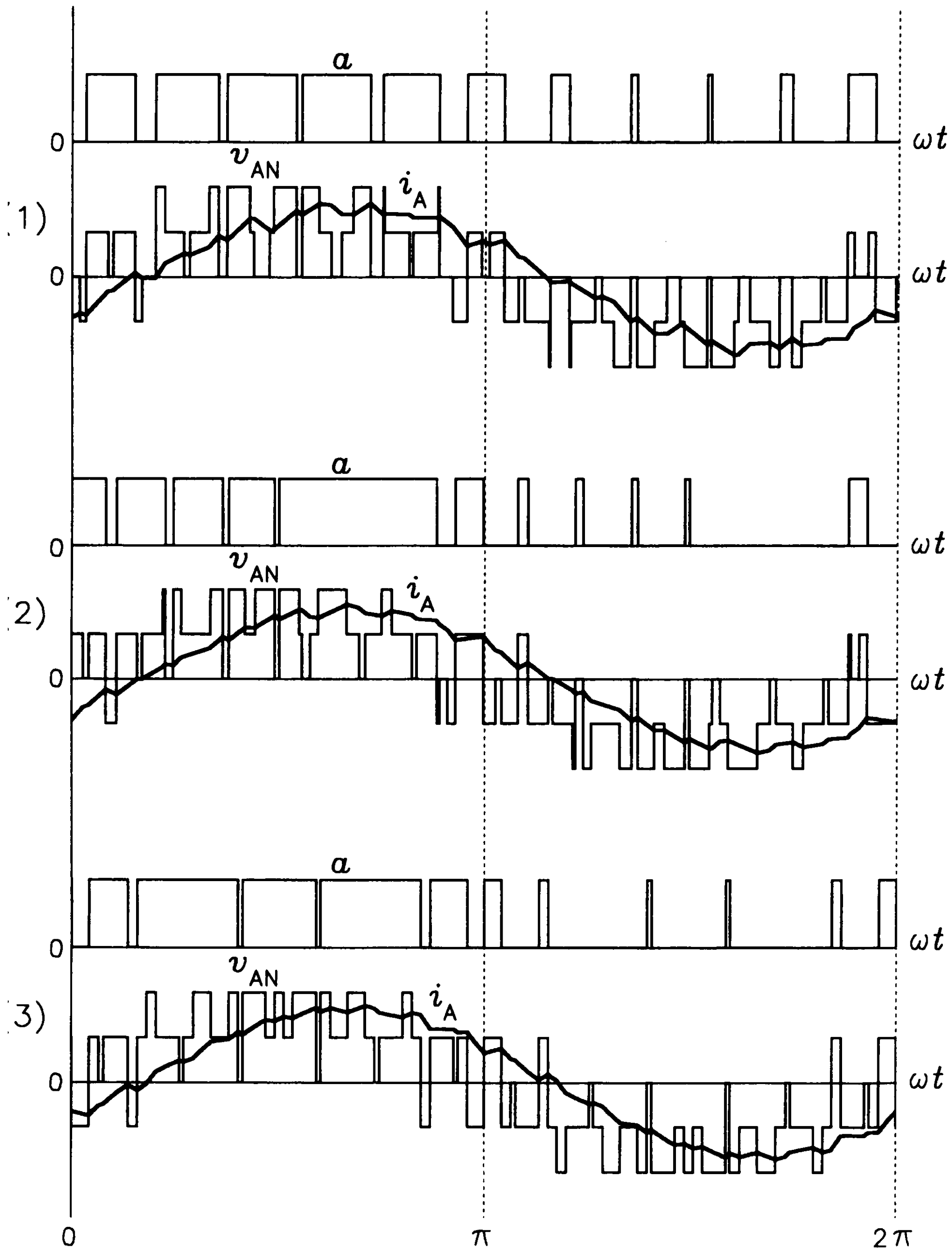


Fig. 7.29

Waveforms of output current in a three-phase VSI: (a) regular PWM, (b) random PWM

Frequency spectra of the line-to-neutral output voltage
in a three-phase VSI: (a) regular PWM, (b) random PWM

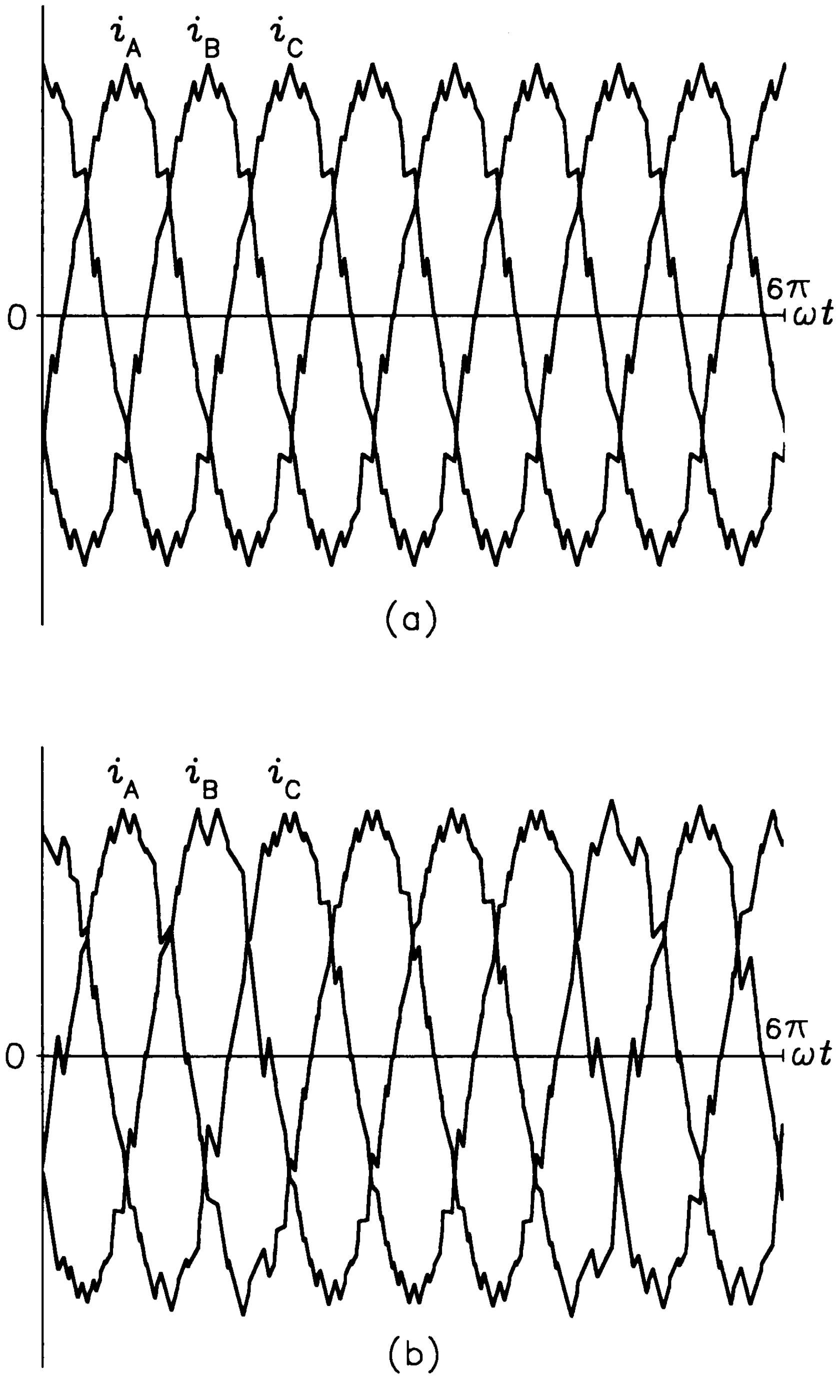


Fig. 7.30

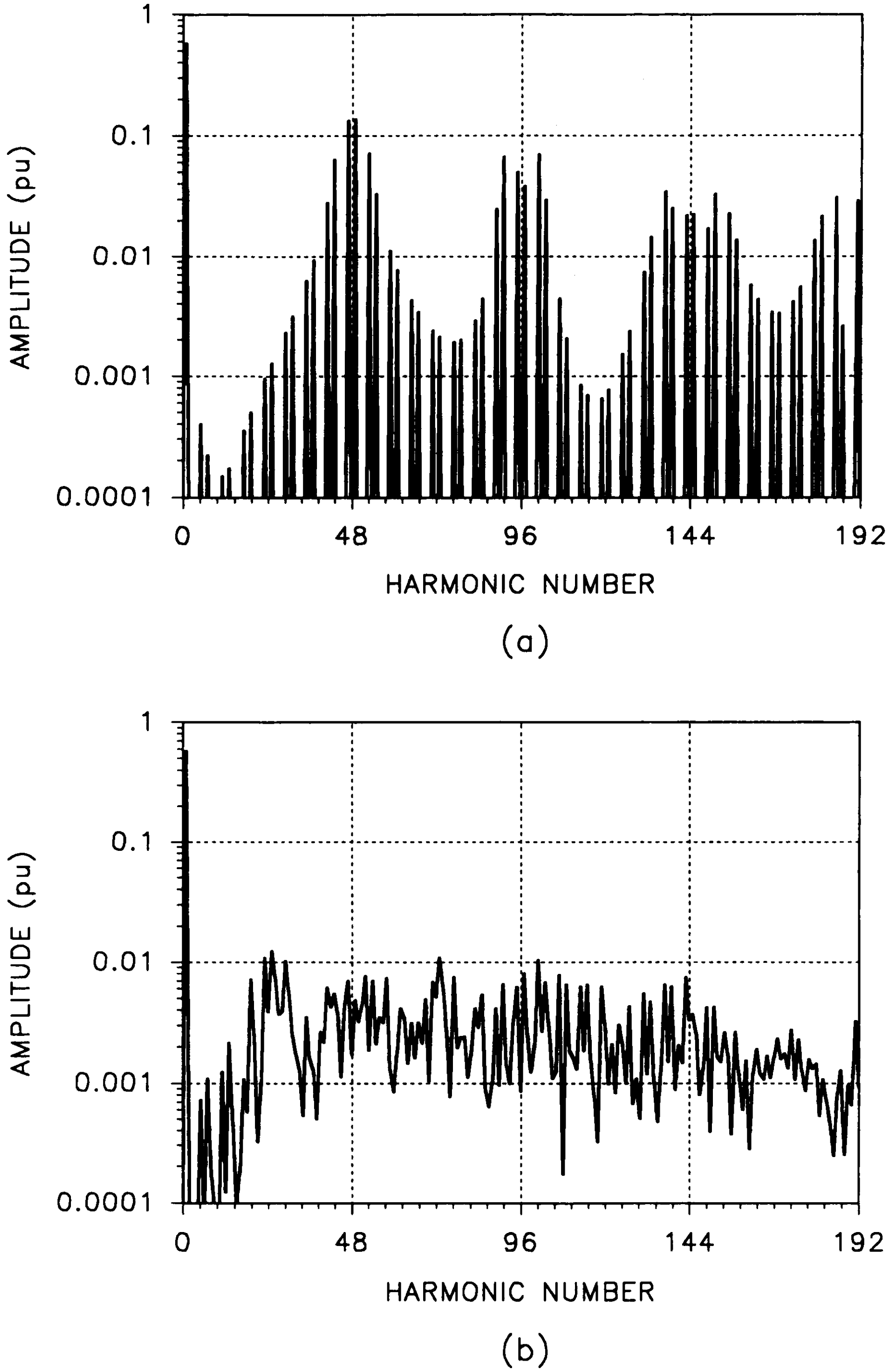


Fig. 7.31

Comparison of random PWM techniques with the regular PWM

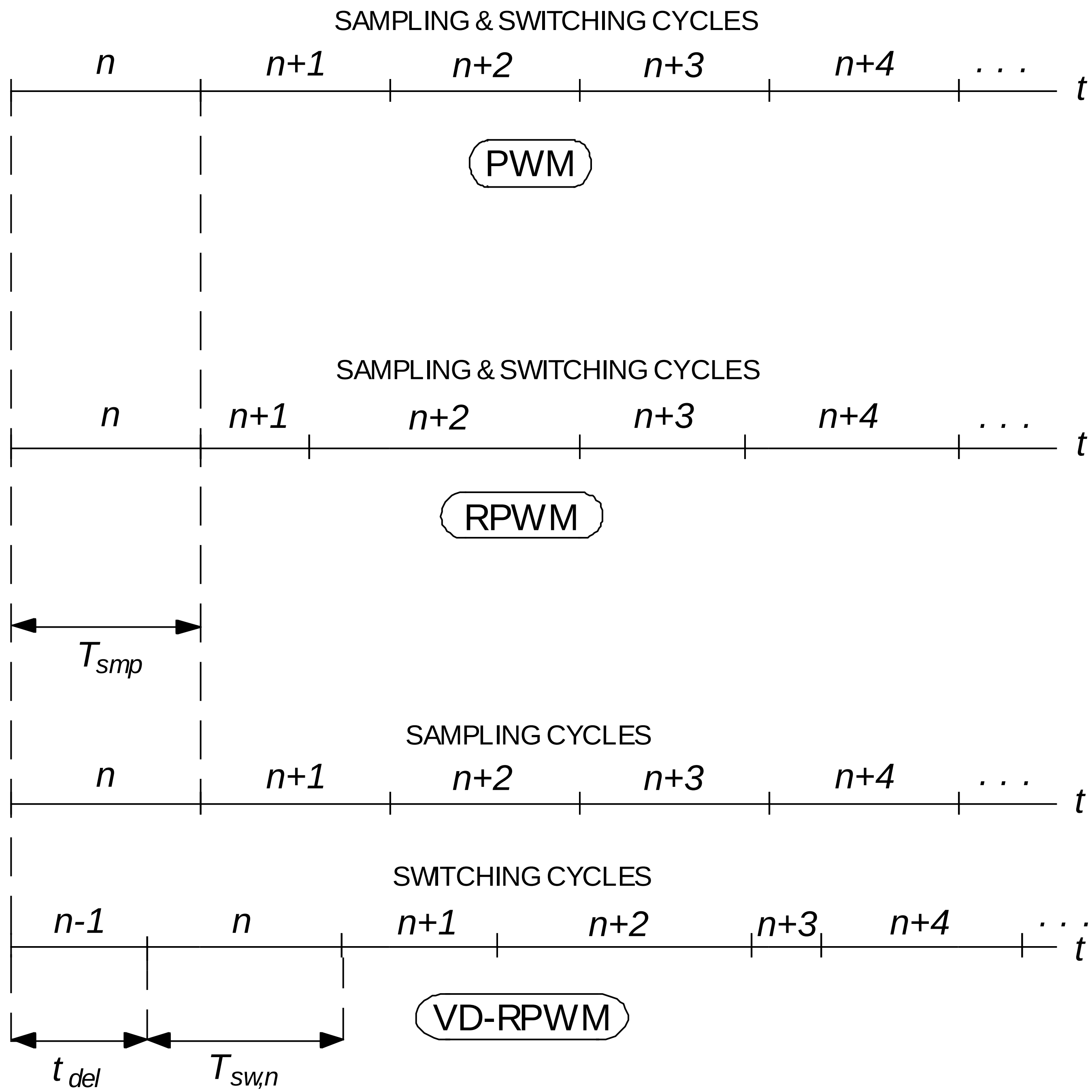


Fig. 7.32

Hysteresis current control scheme

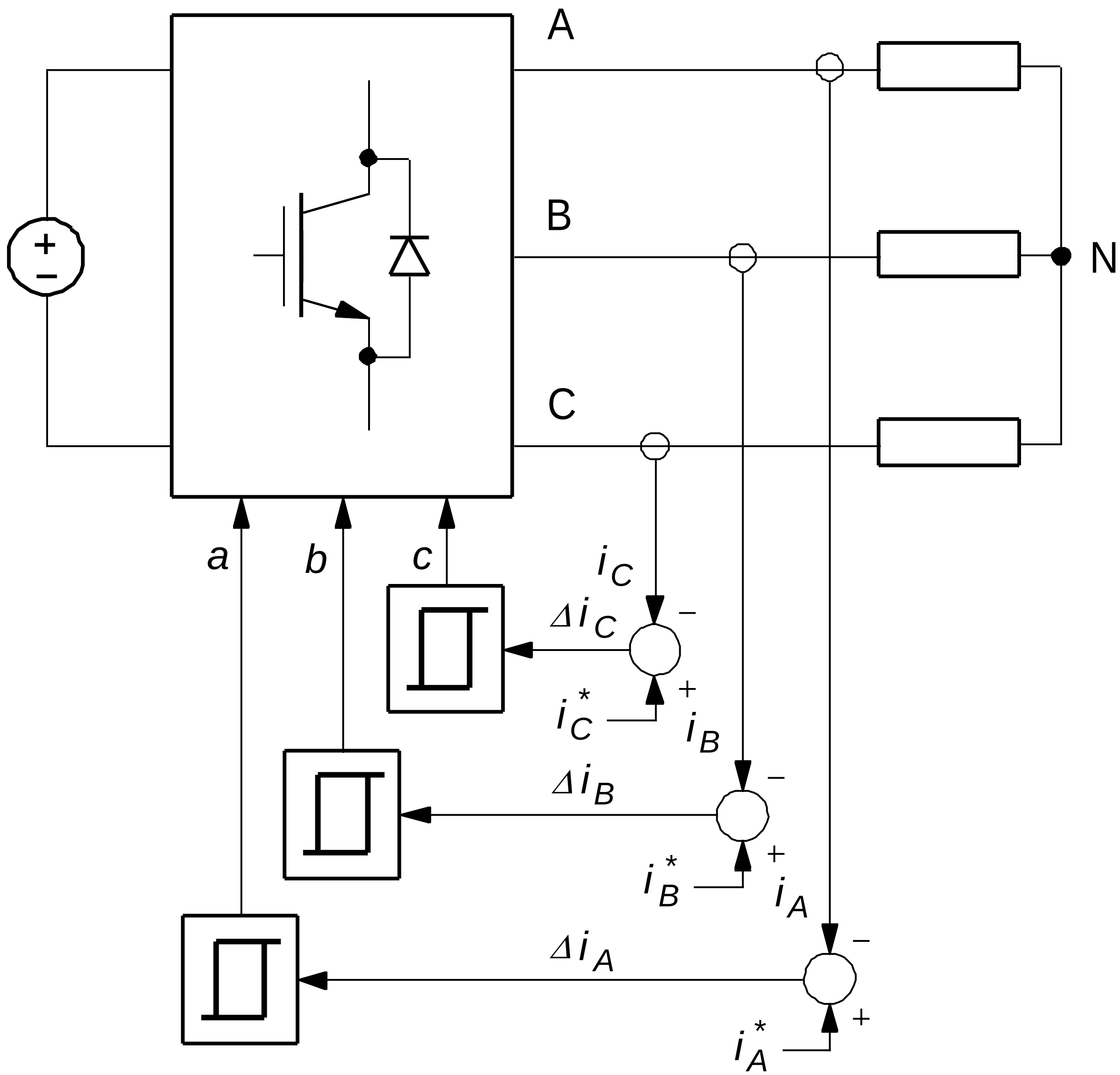


Fig. 7.33

Characteristic of the hysteresis current controller

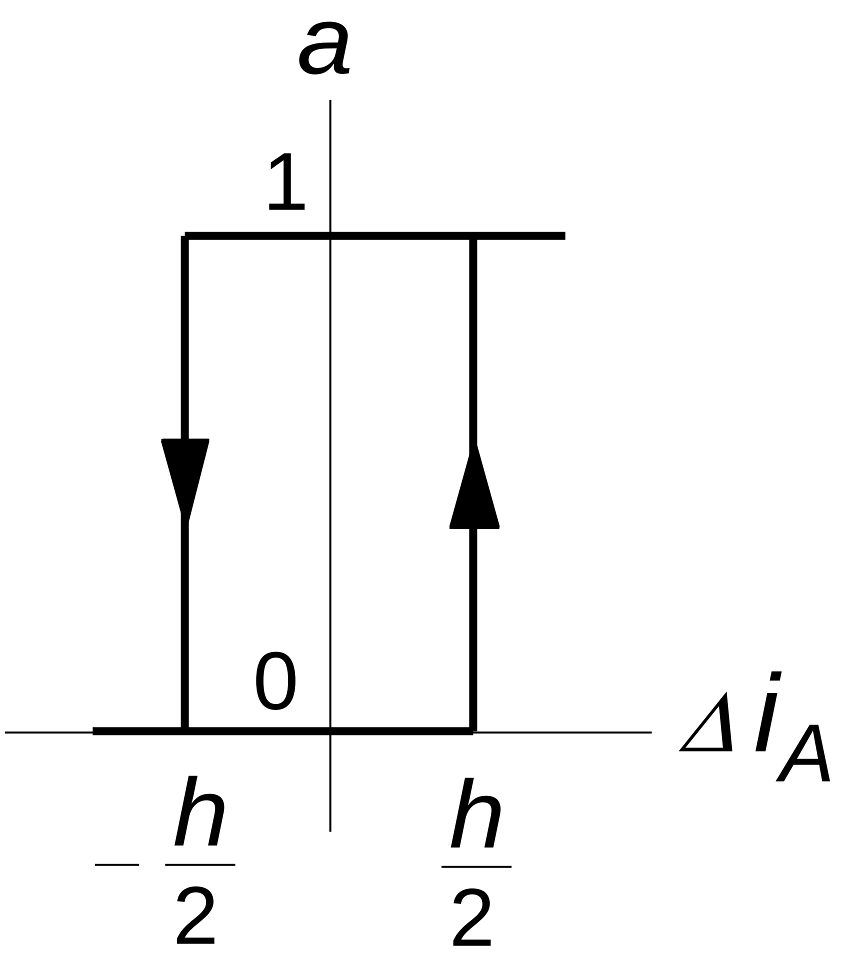


Fig. 7.34

Waveforms of output currents in a VSI
with hysteresis current control:
(a) 20% tolerance, (b) 10% tolerance

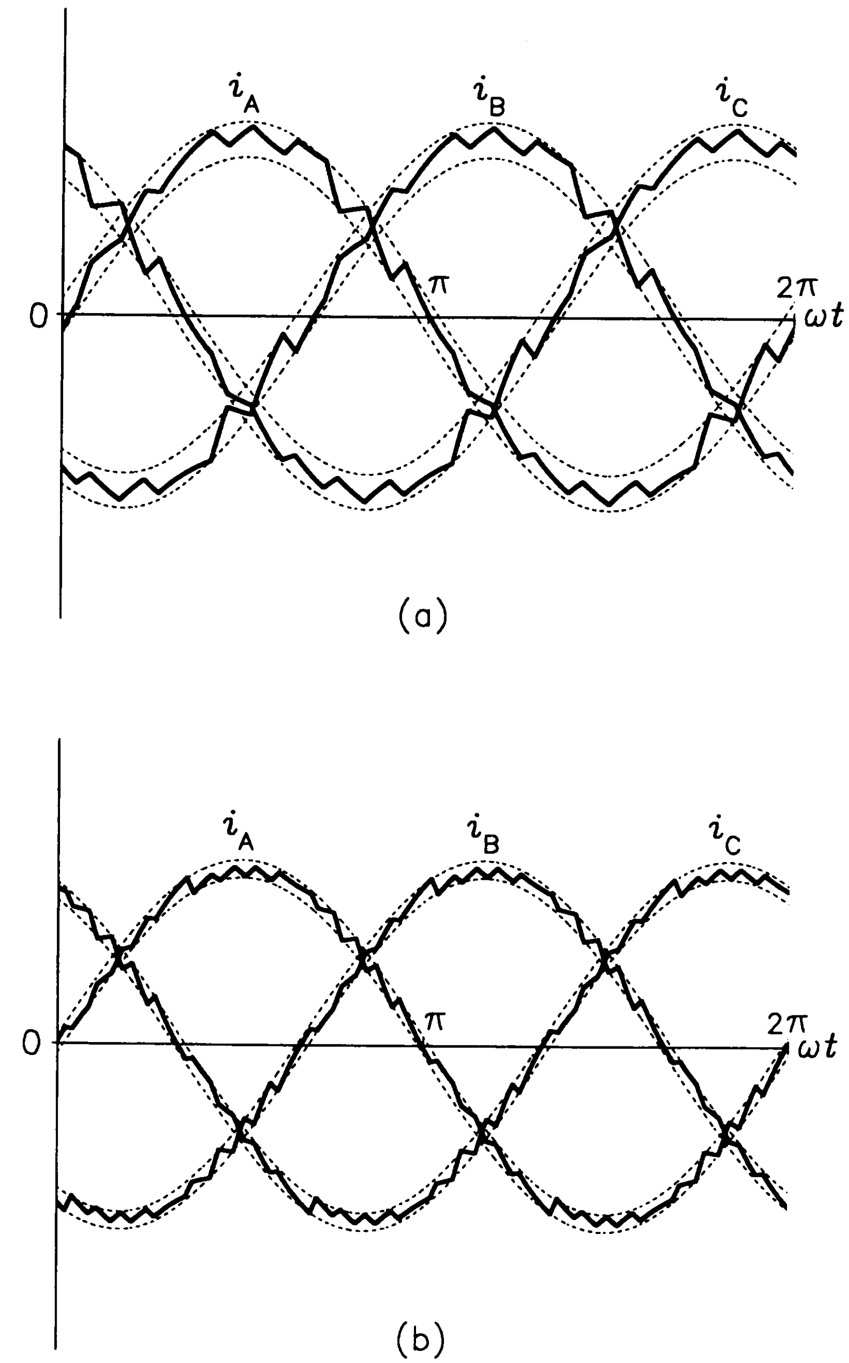


Fig. 7.35

Waveform of output currents in a VSI with hysteresis current control at a rapid change in the magnitude, frequency, and phase of the reference current

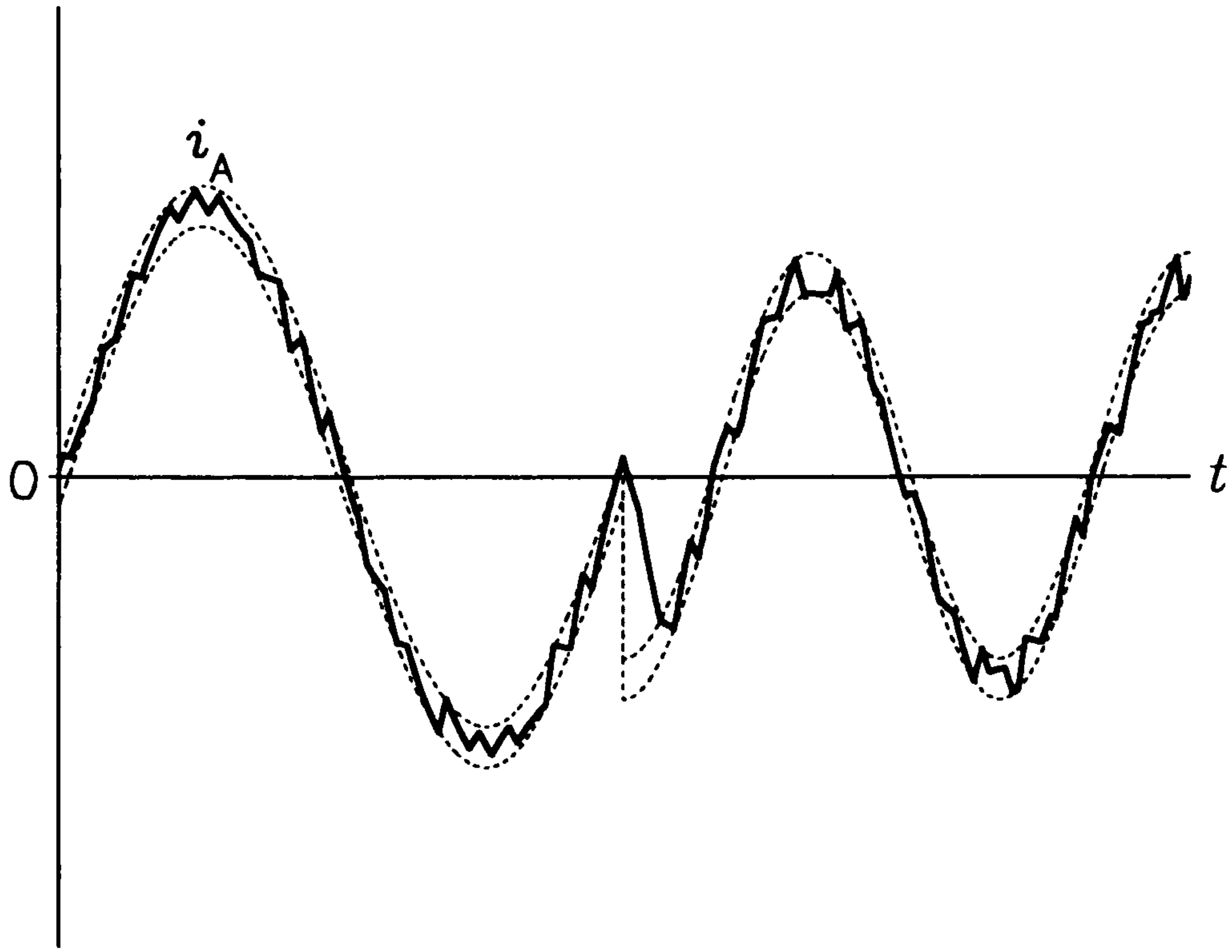


Fig. 7.36

Space vector version of the hysteresis current control scheme

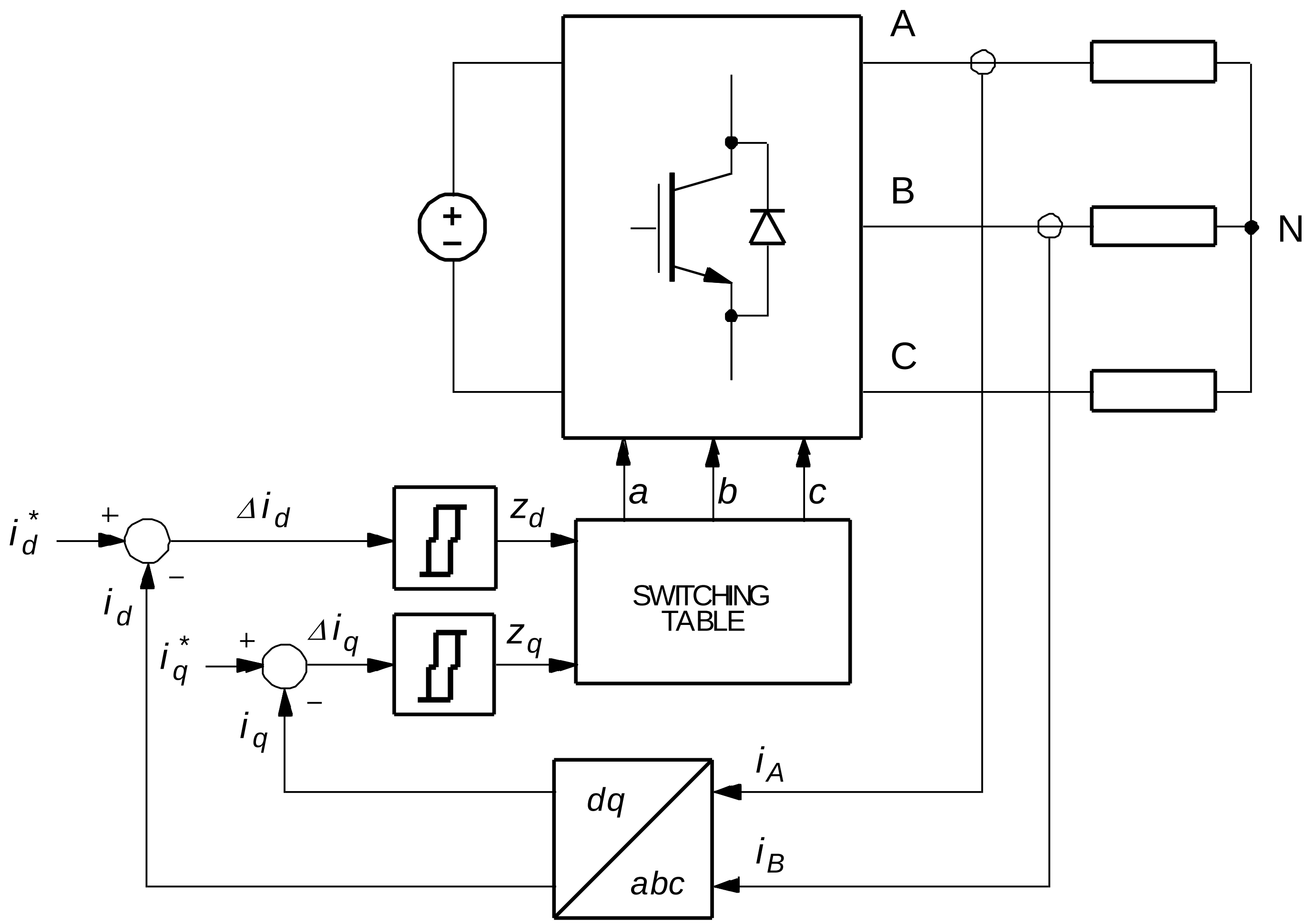


Fig. 7.37

Best control effects are obtained when state 0 or 7 is imposed for (z_d, z_q) = (0, 0), state 1 for (0, 1) and (1, 1), state 2 for (1, -1), state 3 for (1 ,0), state 4 for (-1, 0), state 5 for (-1, 1), and state 6 for (-1, -1) and (0 ,-1).

Characteristic of a current controller
for the space vector version
of the hysteresis current control scheme

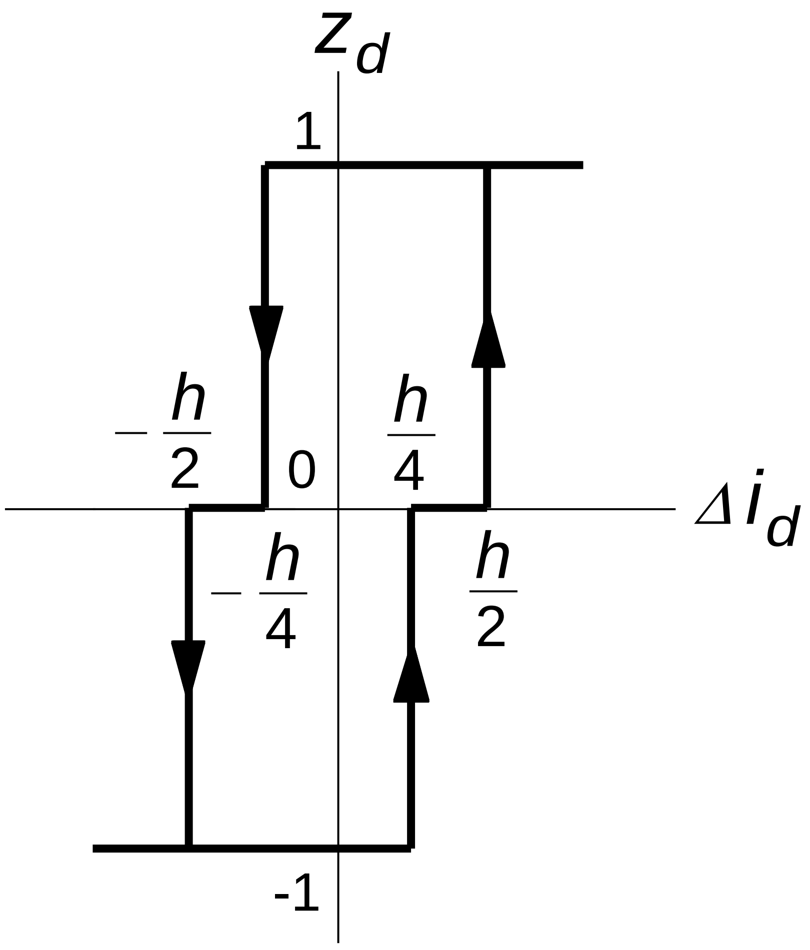


Fig. 7.38

Ramp-comparison scheme for current-controlled VSI

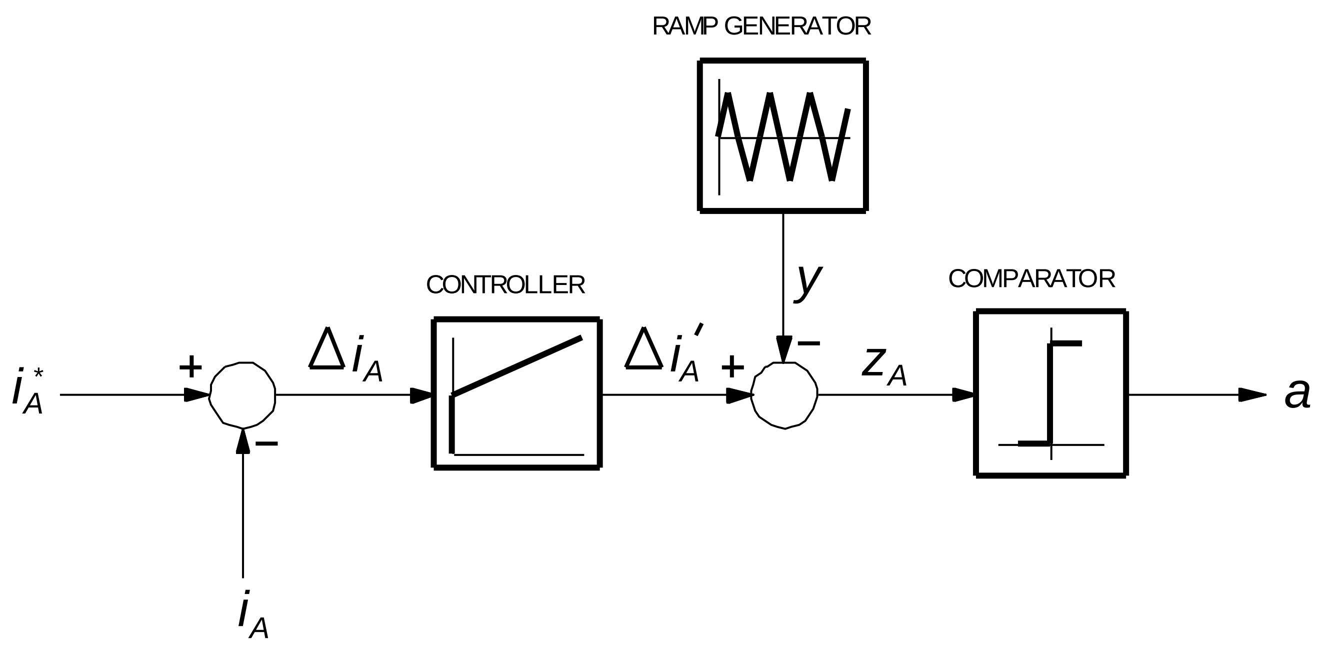


Fig. 7.39

Waveforms of the output current in a VSI with the ramp comparison current control:
(a) $f_r/f_1 = 10$, (b) $f_r/f_1 = 20$

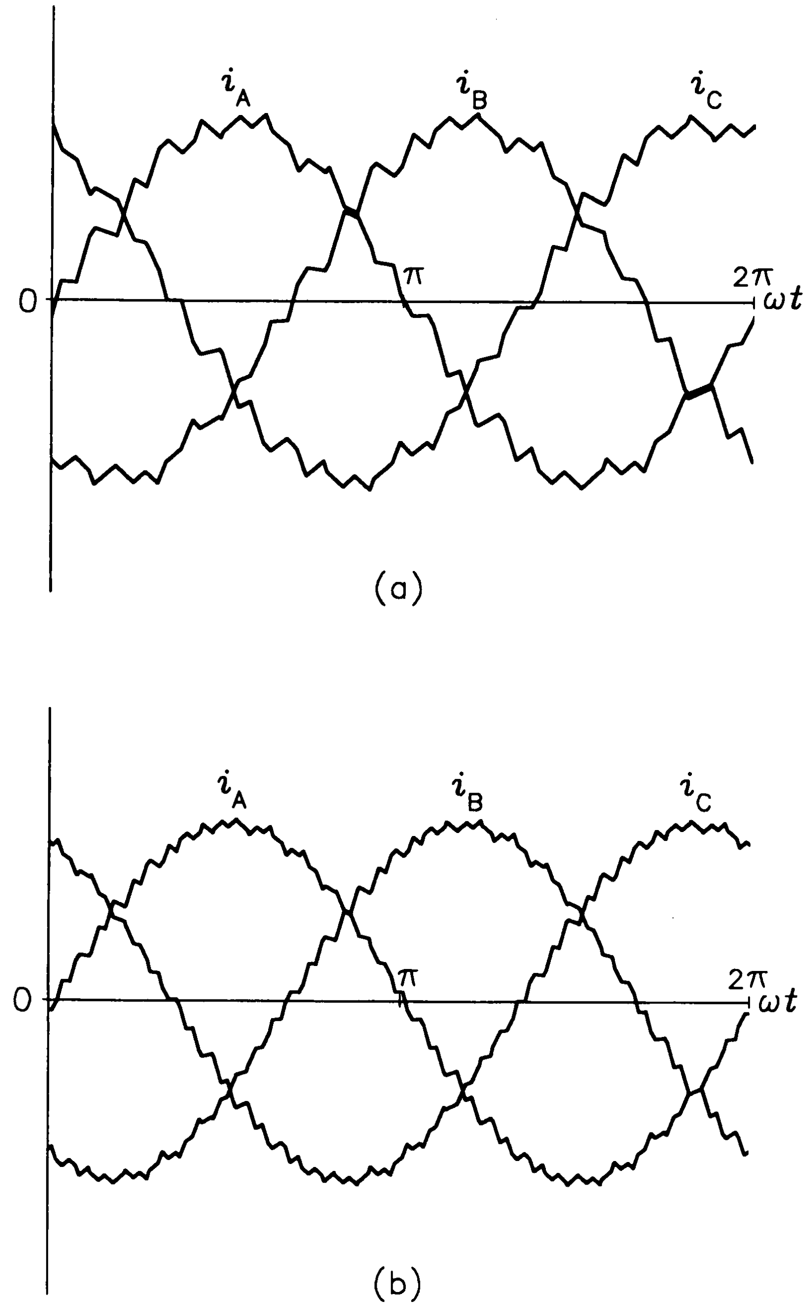


Fig. 7.40

Current-regulated delta modulation scheme
for a current-controlled VSI

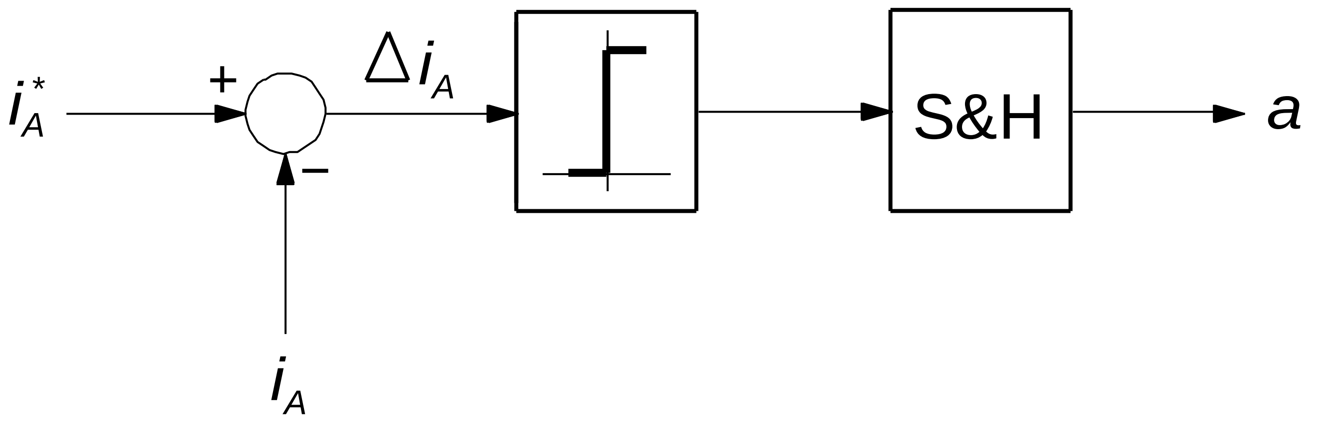


Fig. 7.41

Linear current control scheme for a VSI

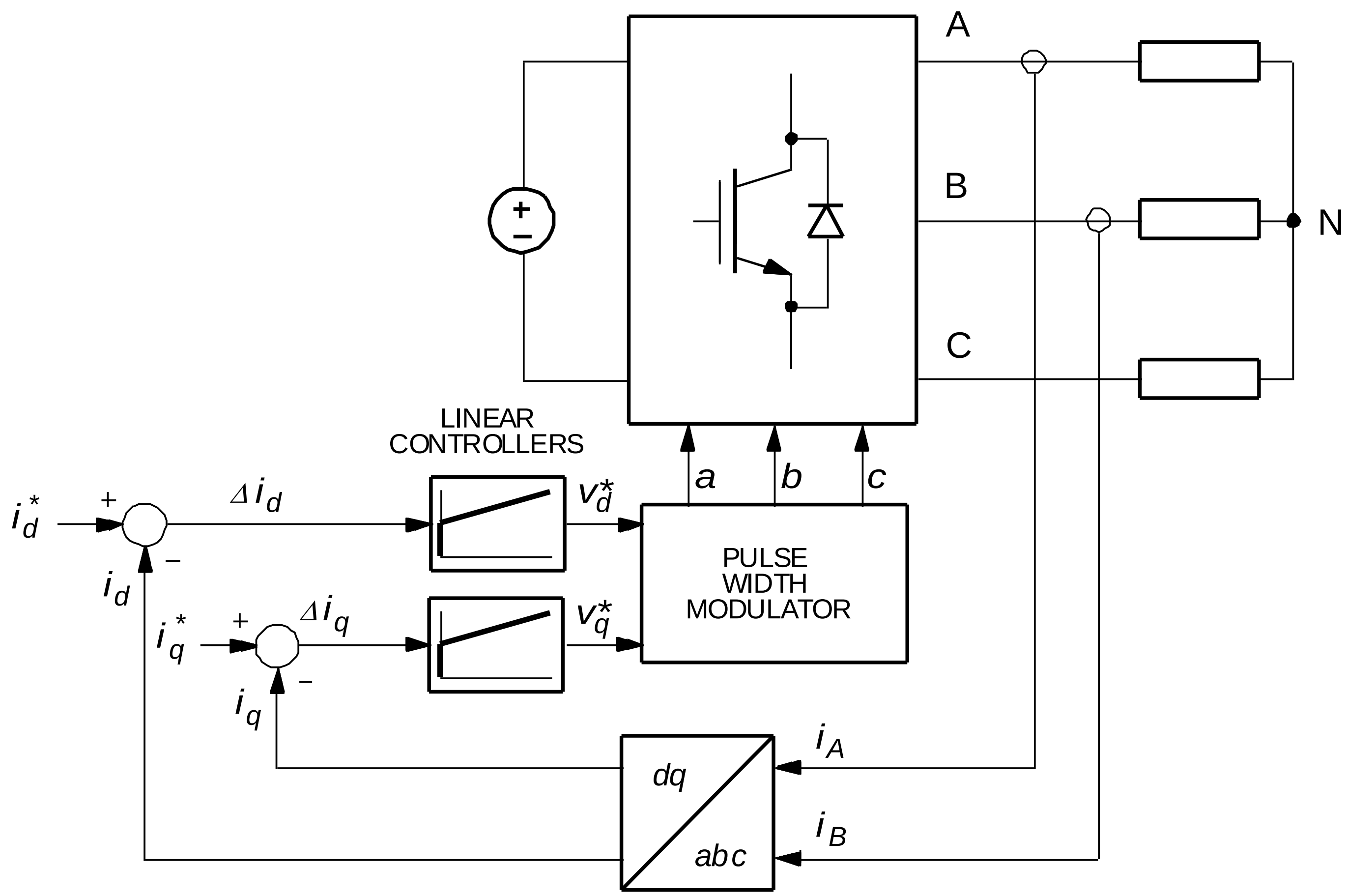
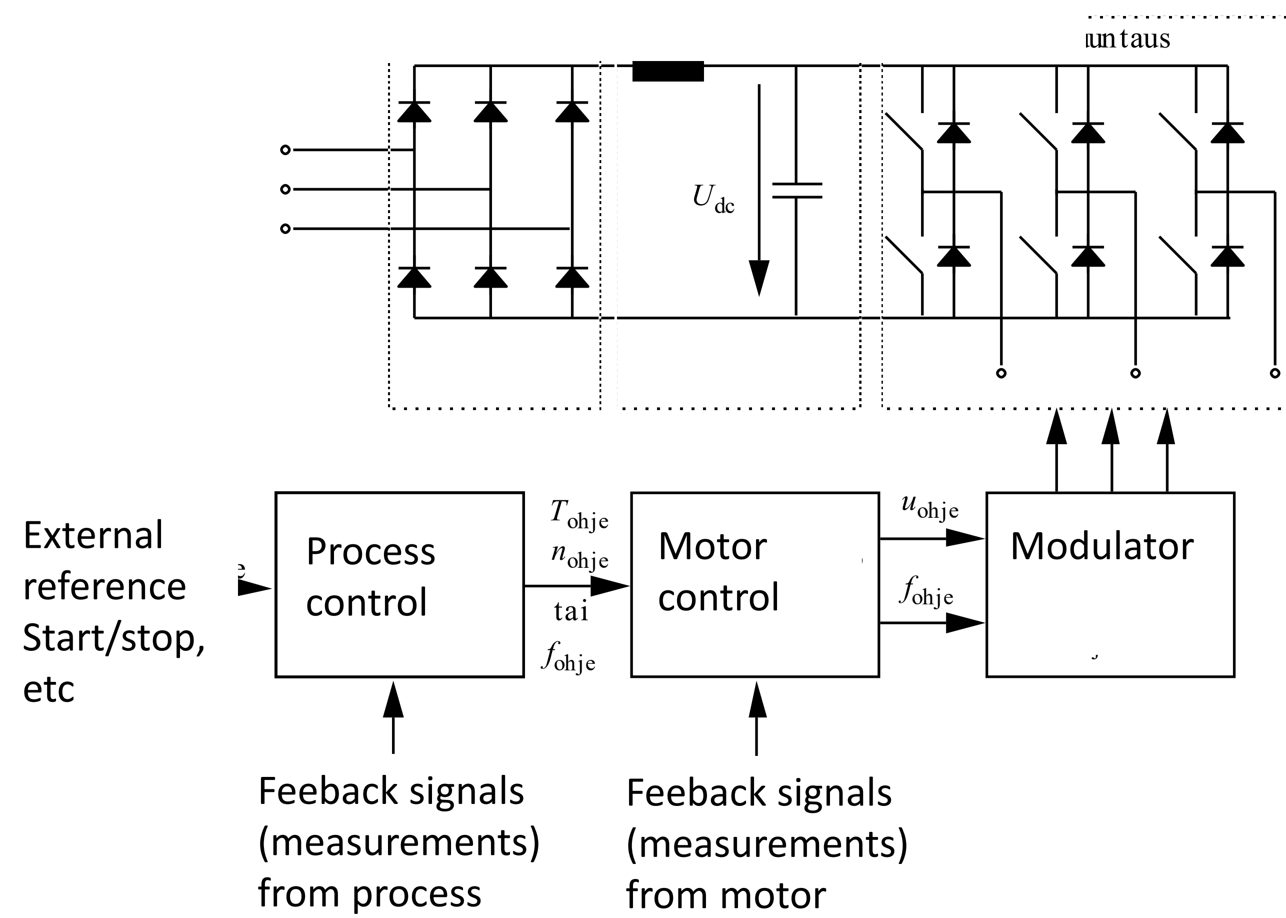


Fig. 7.42

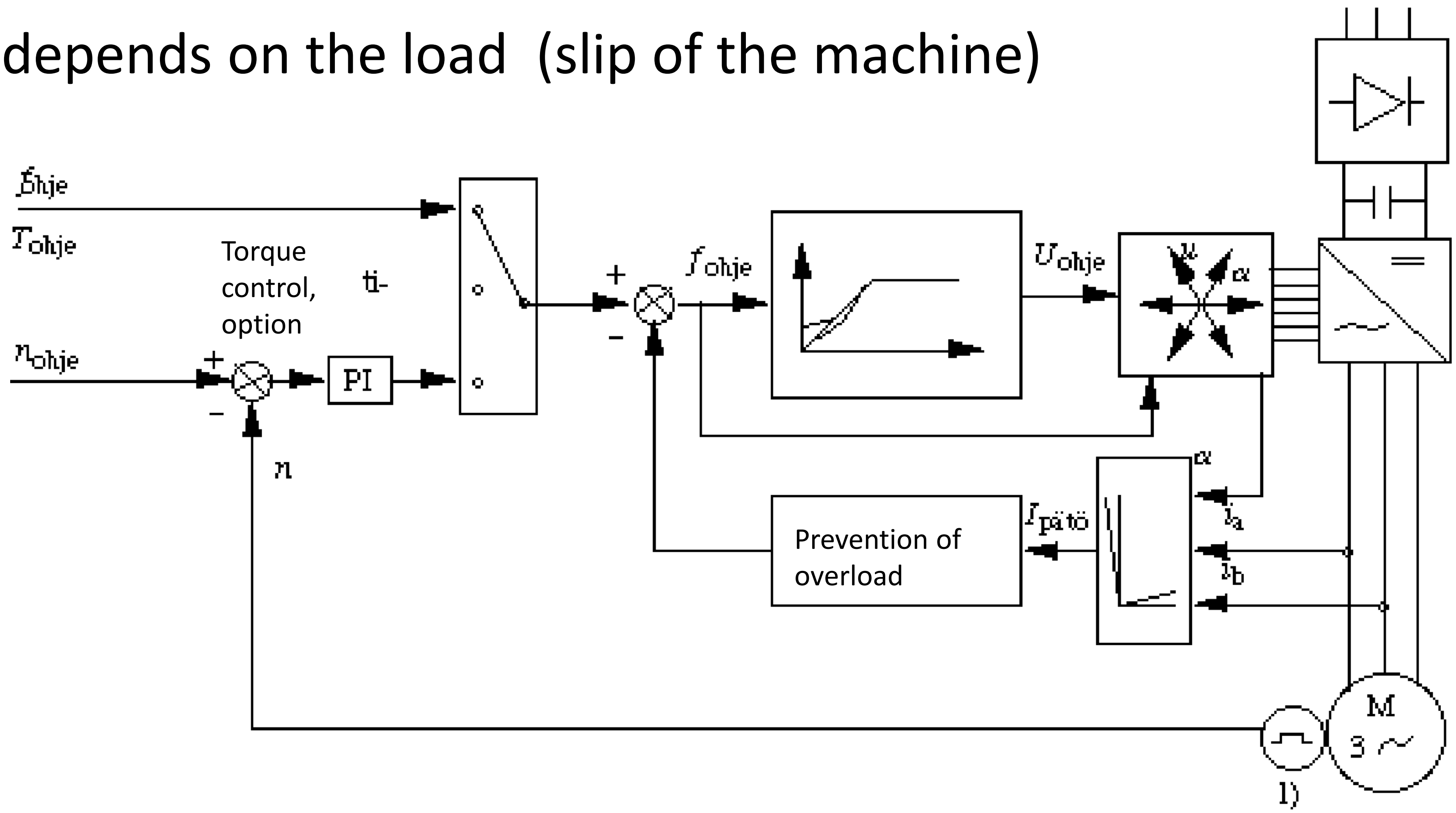
Control methods of VSI supplied induction machines

- Modulator
  - PWM based eg. on comparisons or space vectors
  - Reference comes from outer motor control
    - Scalar control
    - Vector control
- Direct Torque Control
  - Combines both motor control and modulator



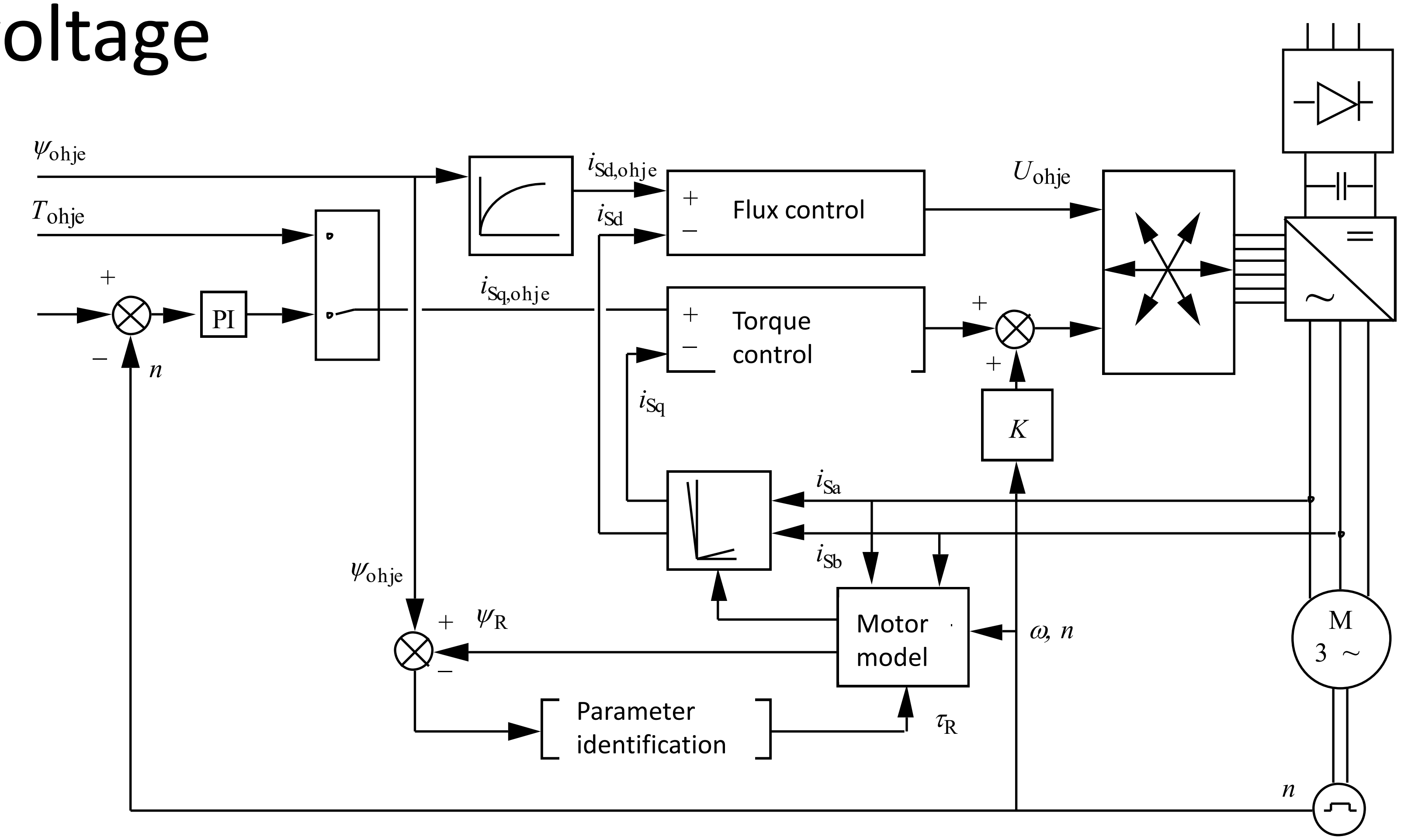
Scalar control

- Motor is controlled by changing supply frequency
- Voltage is increased simultaneus to keep flux constant
  - Modulator gets both voltage amlitude and frequency reference (ohje in the figure below)
- Motor current depends on the load (slip of the machine)



Vector control

- More accurate motor model
- Controls separately flux and torque of the machine
- Modulator receives reference (ohje) to amplitude and frequency of voltage

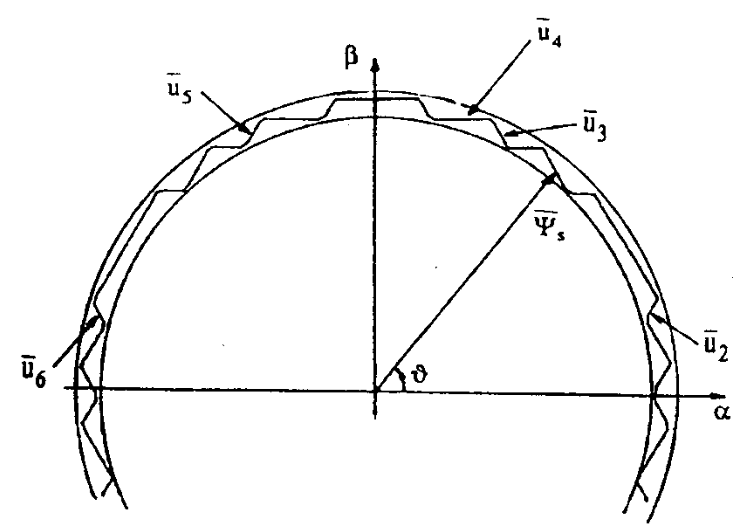


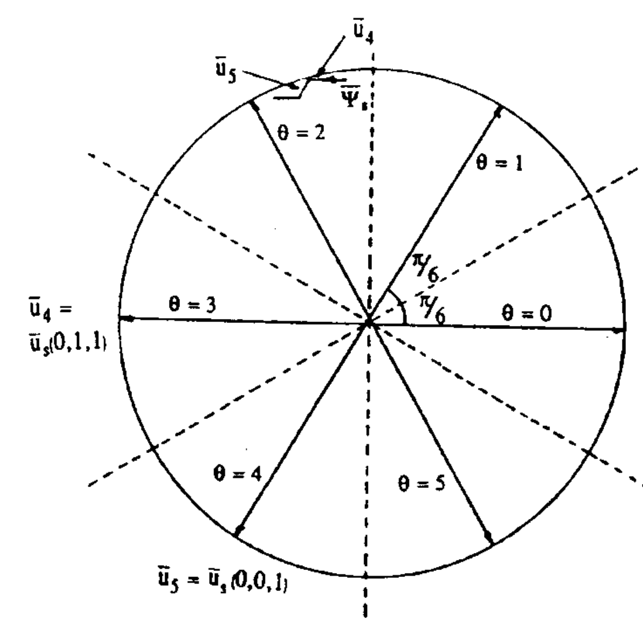
Direct Torque Control, DTC

- Flux vector and torque are controlled simultaneously with hysteresis control
- Suitable voltage vector changing flux and torque in correct way is selected => separate modulator is not needed

Flux control

- Complex plane is divided to six sectors
  - 0-5, dashed lines in the rigth hand figure
- Two voltage vectors are used and they depend on the direction of rotation
  - One vector to increase
  - One vector to decrease flux



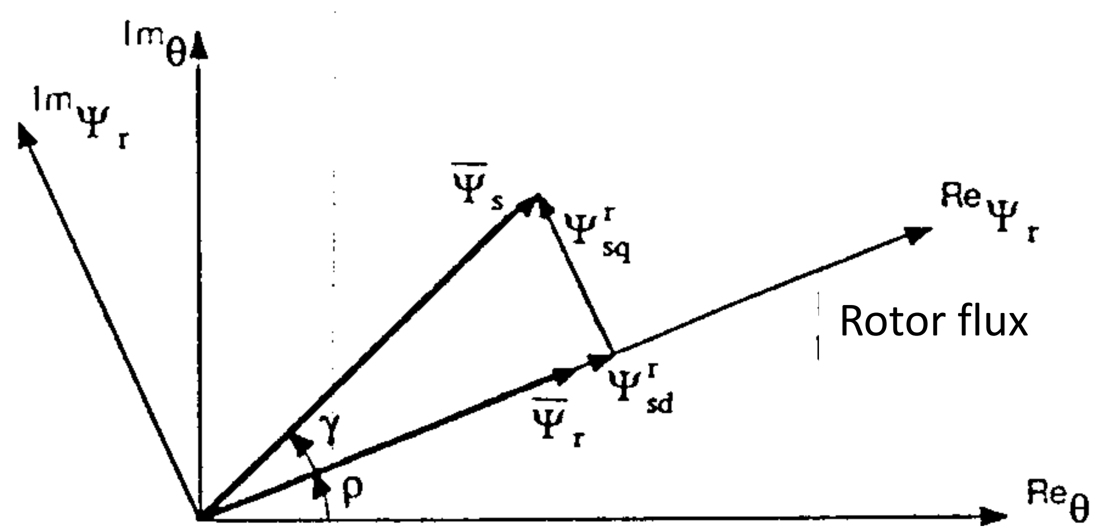


Torque control

- Induction machine torque can be expressed as

$$T = -\frac{3}{2} p \, \vec{\psi}_S \times \vec{i}_S = \frac{3}{2} p \frac{1-\sigma}{\sigma L_m} \vec{\psi}_R \times \vec{\psi}_S = \frac{3}{2} p \frac{1-\sigma}{\sigma L_m} \psi_R \psi_S \sin \gamma$$

- If angle between rotor flux
  - Increases torque increases
  - Decreases torque decreases

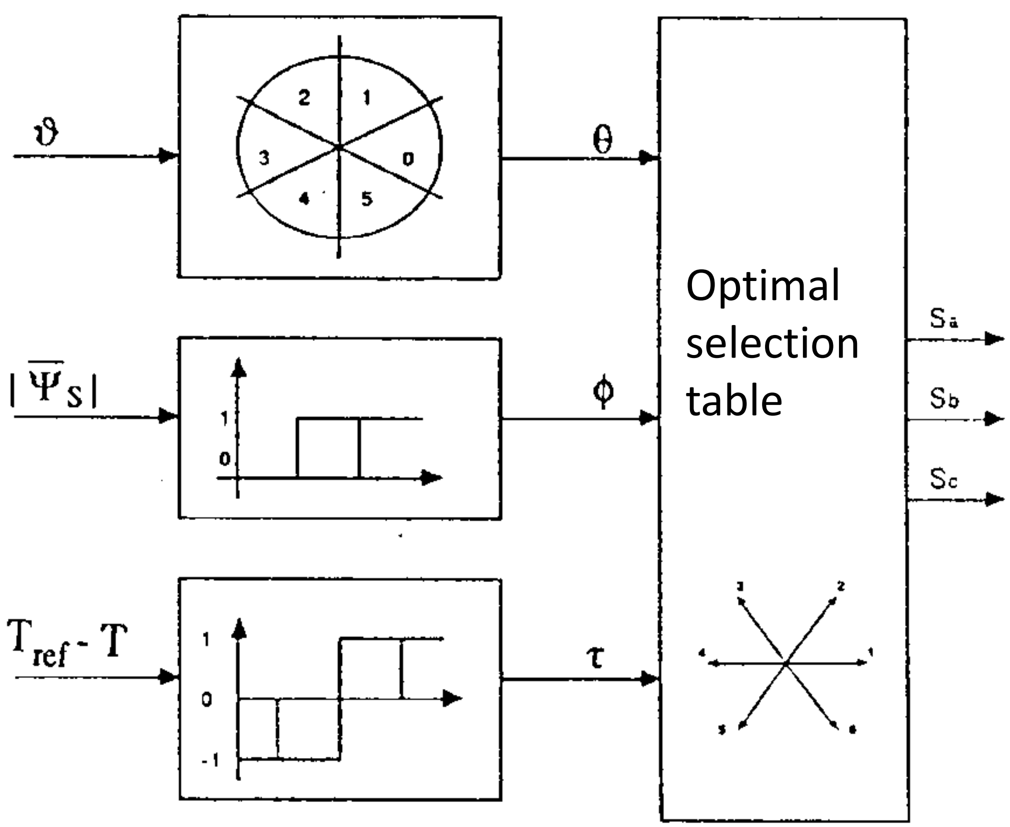


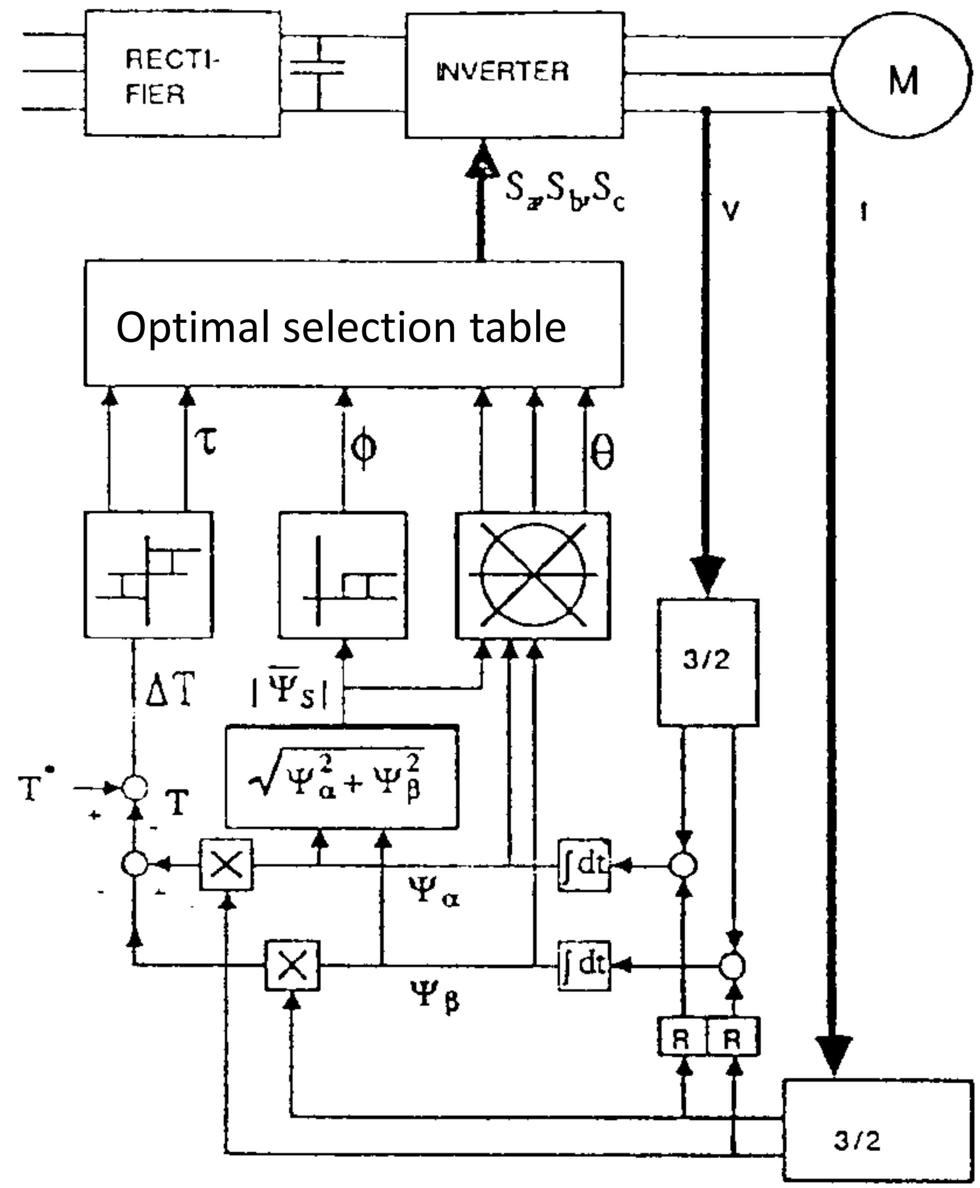
Optimal selection table

- Space vector is selected based on the
  - sector θ of flux
  - torque
    - τ = –1, too large
    - τ = 1, too small
    - τ = 0, ok, use zero vector
  - Flux
    - ϕ = 0, decrease flux
  - ϕ = 1, increase flux

Optimal selection table, space vector is selecteettävä jänniteosoitin eri sektoreissa θ määräytyy momentti- ja ja vuobittien τ,ϕ perusteella.						
sektori θ	0	1	2	3	4	5
τ = –1, ϕ = 0	5	6	1	2	3	4
τ = –1, ϕ = 1	6	1	2	3	4	5
τ = 1, ϕ = 0	3	4	5	6	1	2
τ = 1, ϕ = 1	2	3	4	5	6	1
τ = 0	0	0	0	0	0	0

Block diagram of DTC





CSI, Current Source Inverter

Current-source inverter supplied from a controlled rectifier

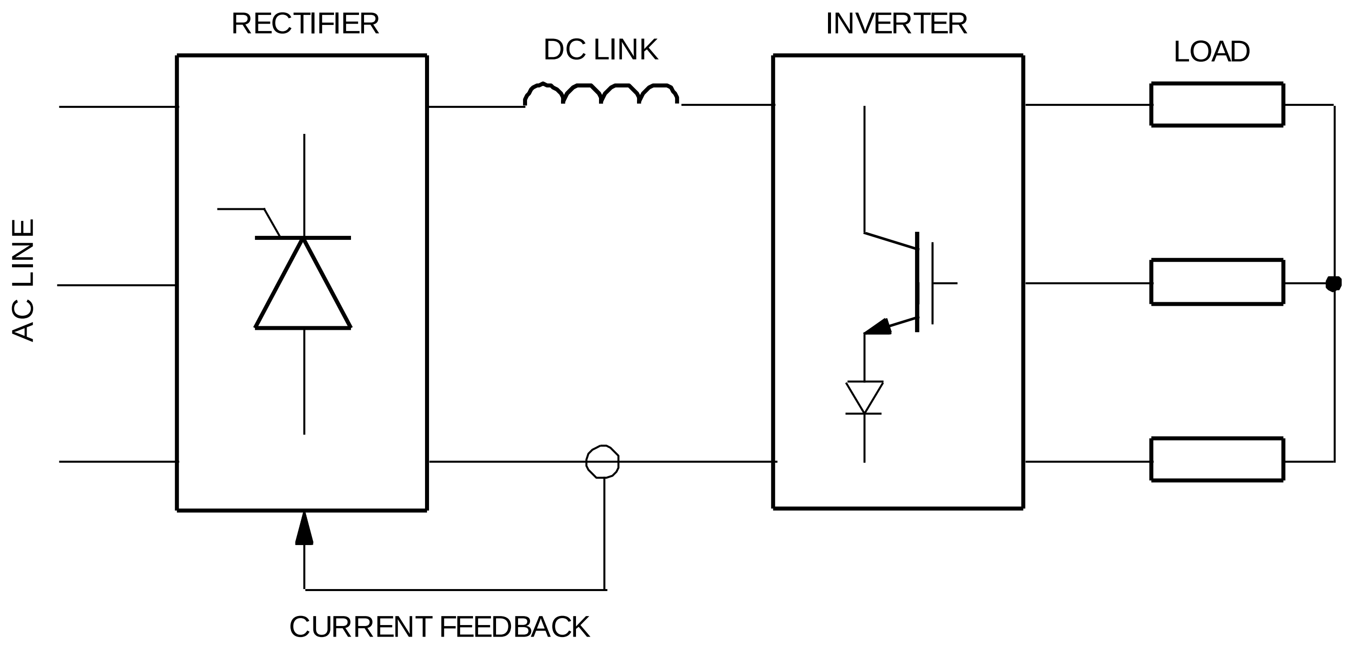


Fig. 7.43

Three-phase current-source inverter

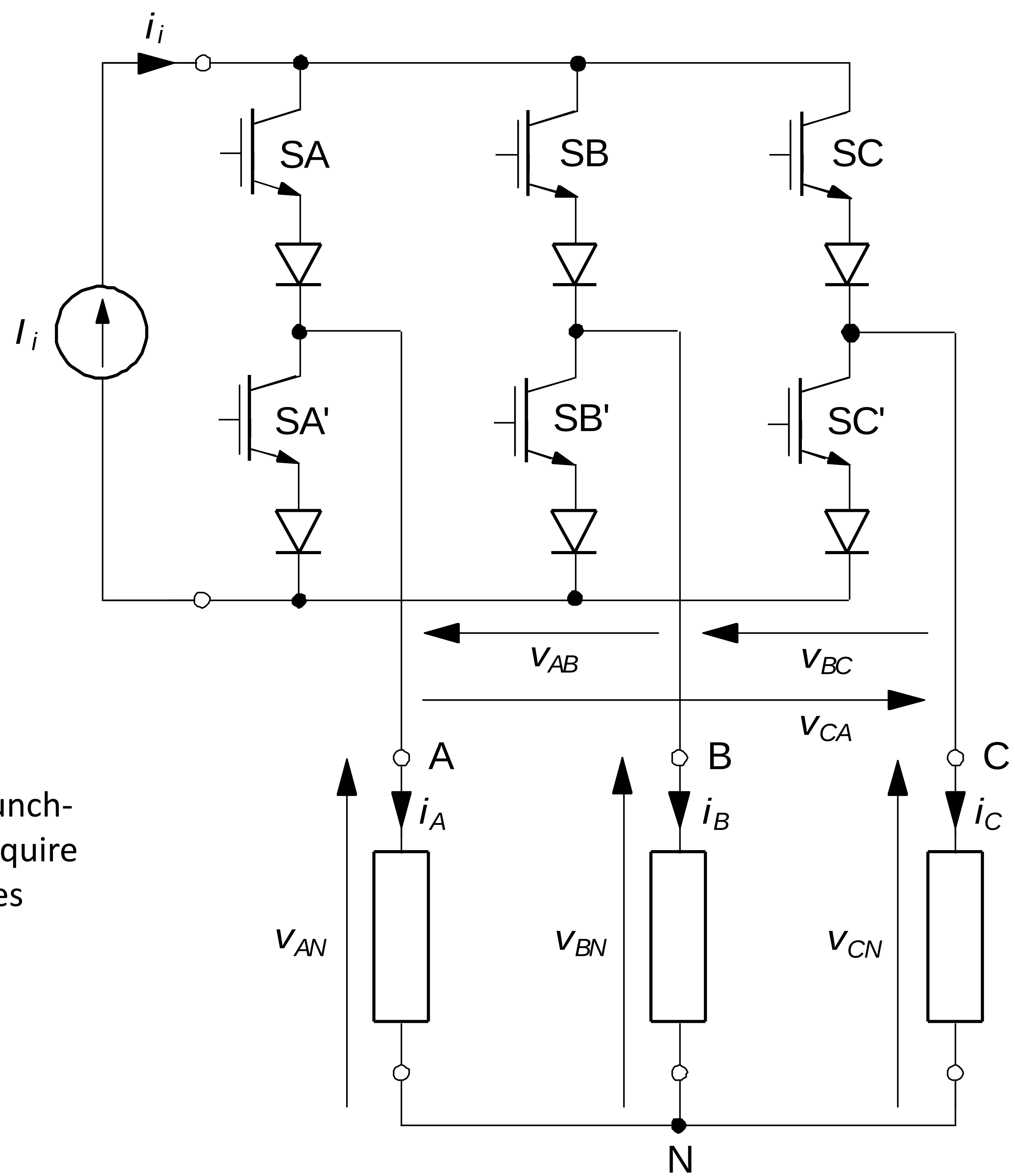


Fig. 7.44

Note:
Symmetrical power semiconductors like punch-through IGBTs don't require series connected diodes

Switching variables in a three-phase CSI in the square-wave mode

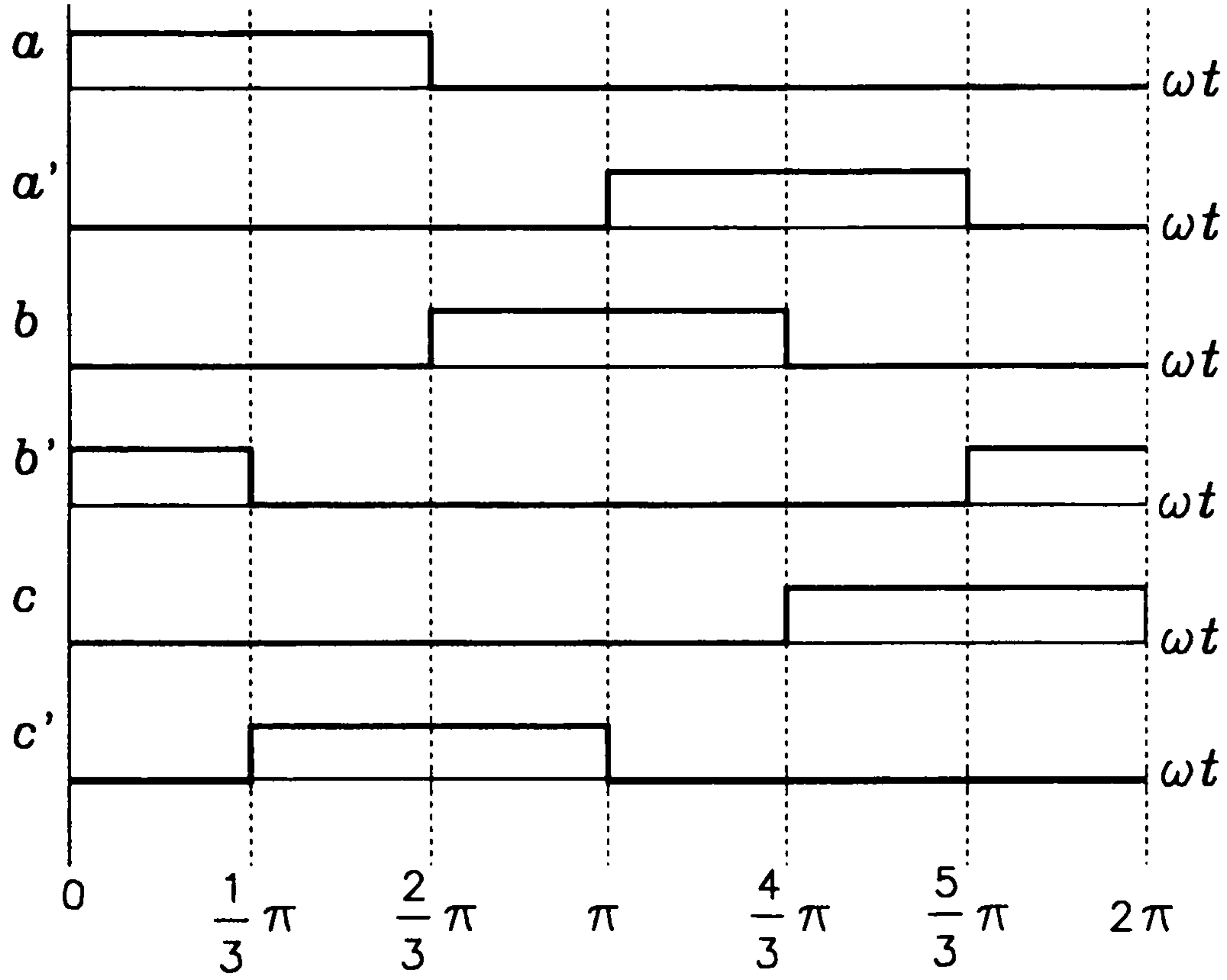


Fig. 7.45

Idealized waveforms of output currents in a three-phase CSI in the square-wave mode

Wye-connected load

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} a-d' \\ b-b' \\ c-c' \end{bmatrix} I_i$$

Delta-connected load

$$\begin{bmatrix} i_{AB} \\ i_{BC} \\ i_{CA} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}$$

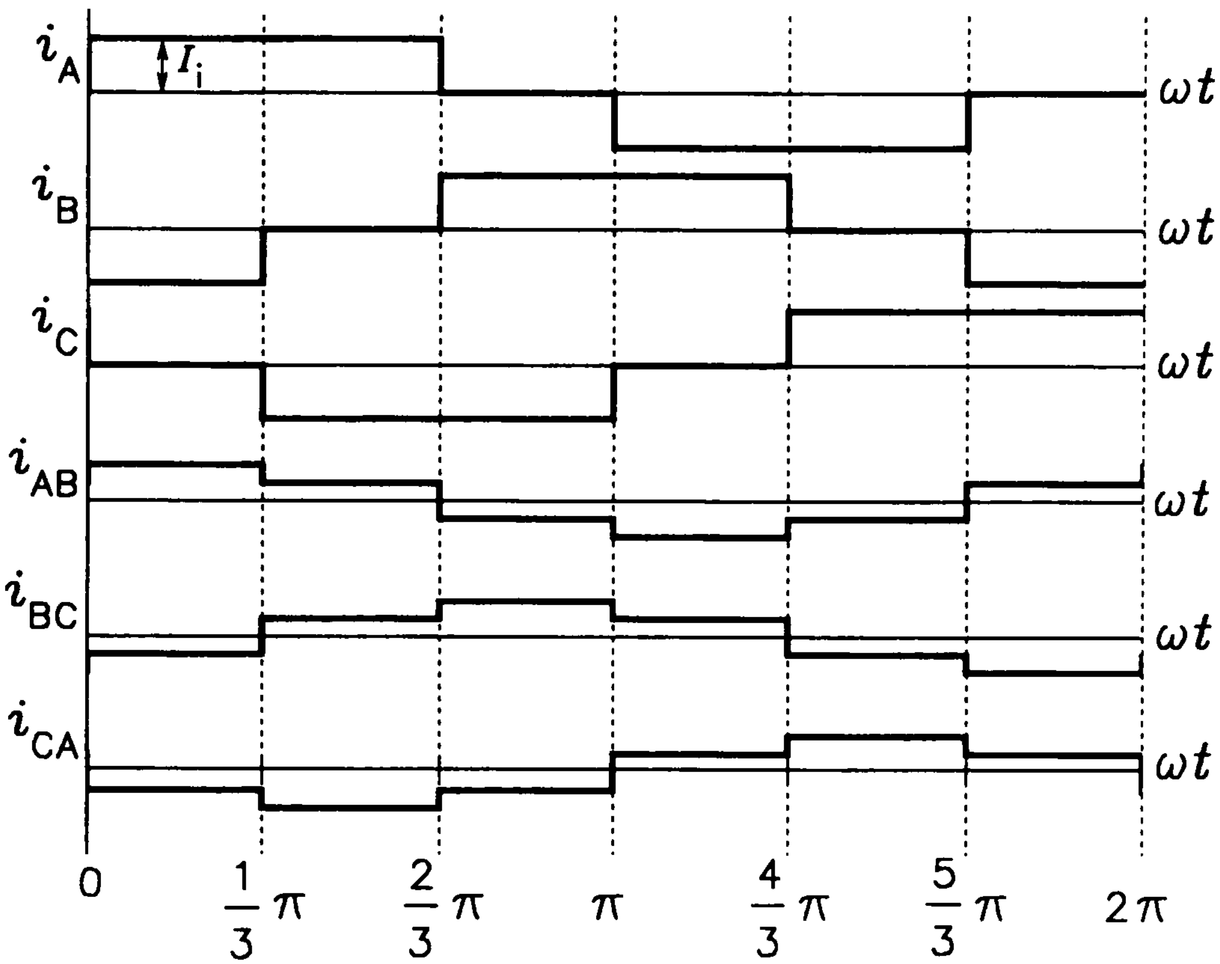


Fig. 7.46

Waveforms of output voltage and current in a three-phase CSI in the square-wave mode:

- Load current is given by the CSI
- In an inductive load current cannot change instantaneously => induced voltage spike in voltage waveforms

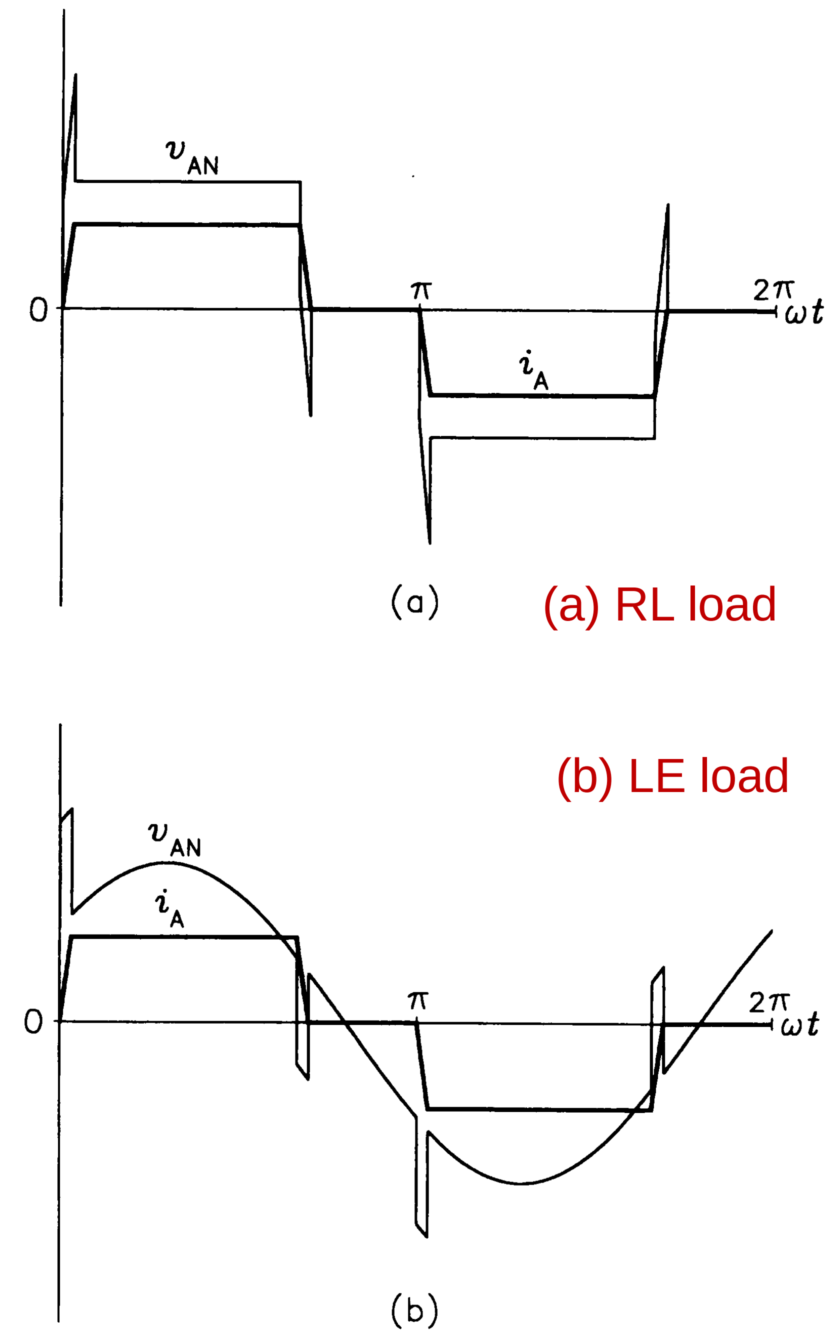


Fig. 7.47

Three-phase PWM current-source inverter

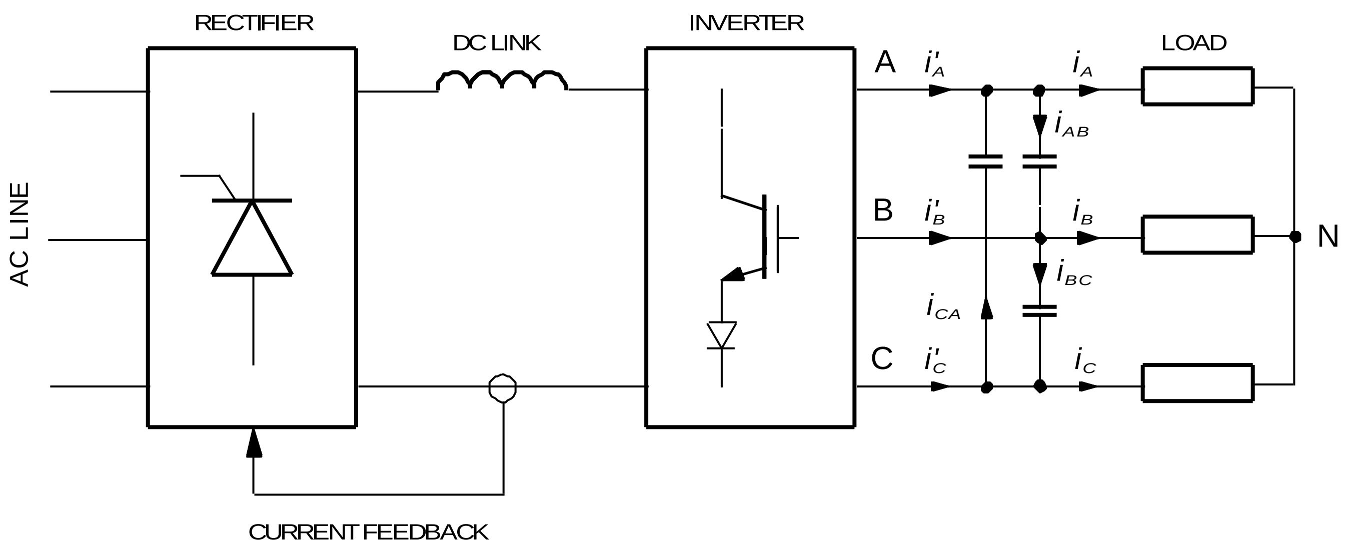


Fig. 7.48

Carrier-comparison method for the PWM CSI

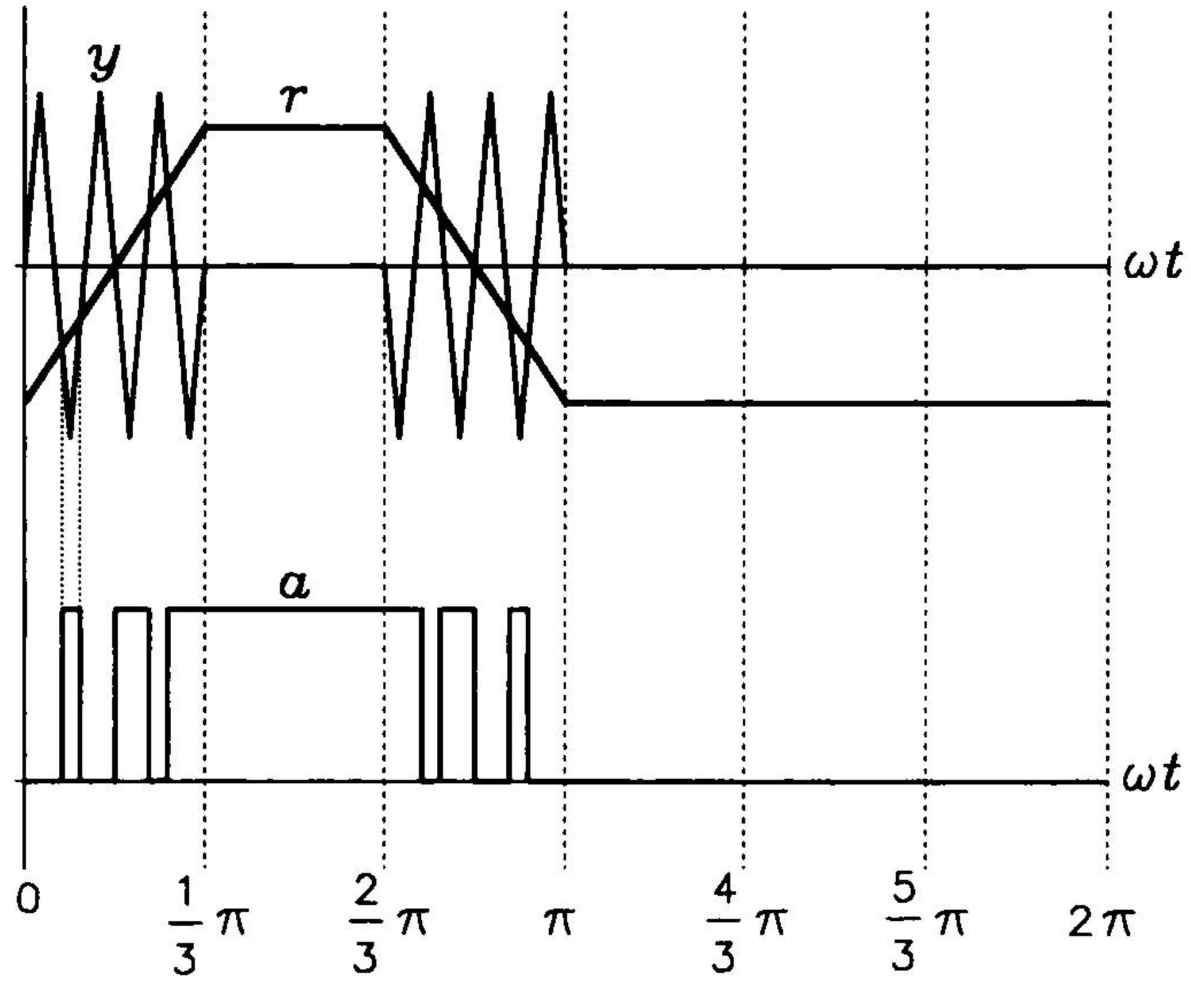


Fig. 7.49

Optimal switching pattern for the PWM CSI with two primary switching angles

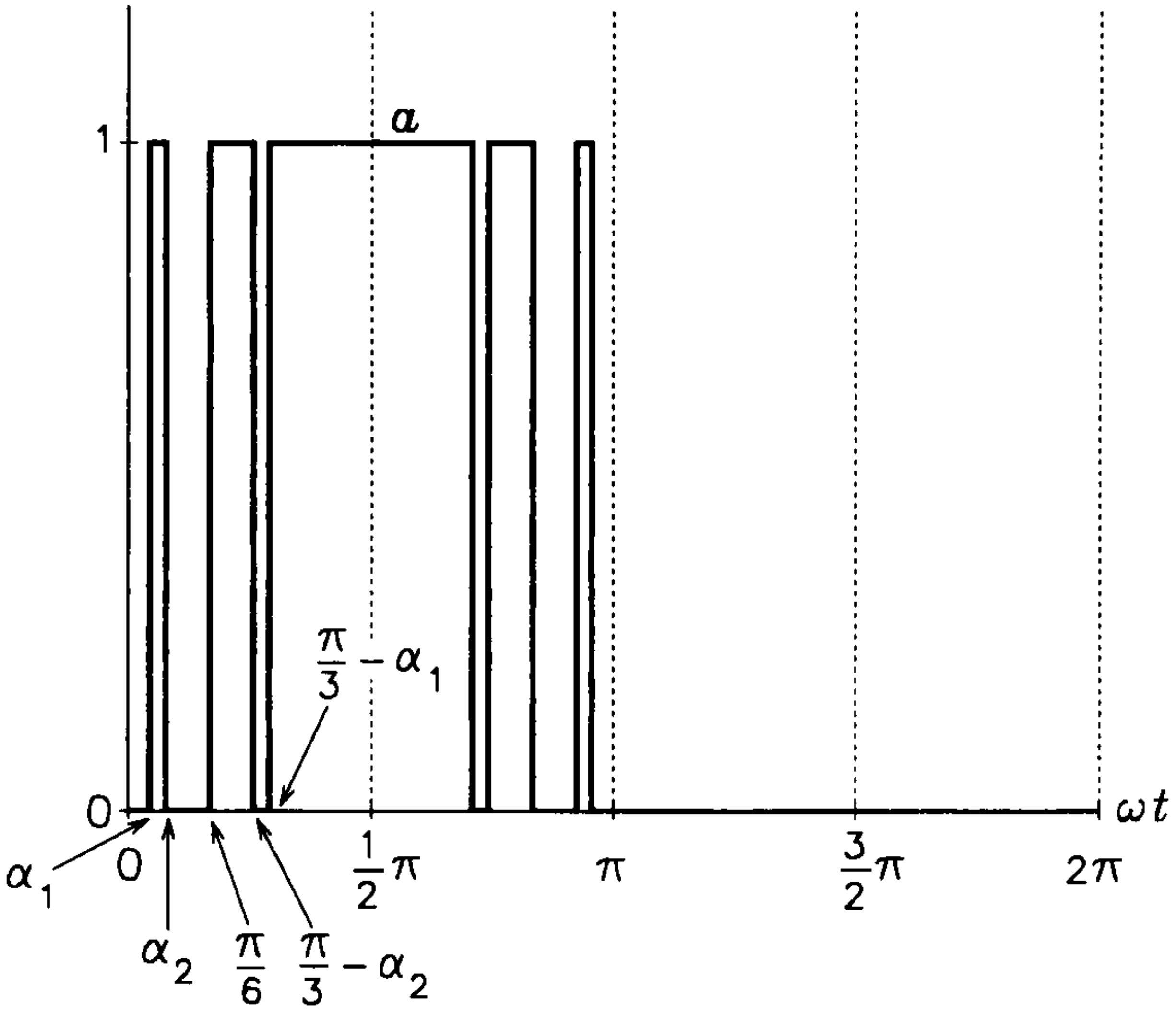


Fig. 7.50

Waveforms of the output current, capacitor current, and output voltage in a three-phase PWM CSI (wye-connected RL load, P = 9)

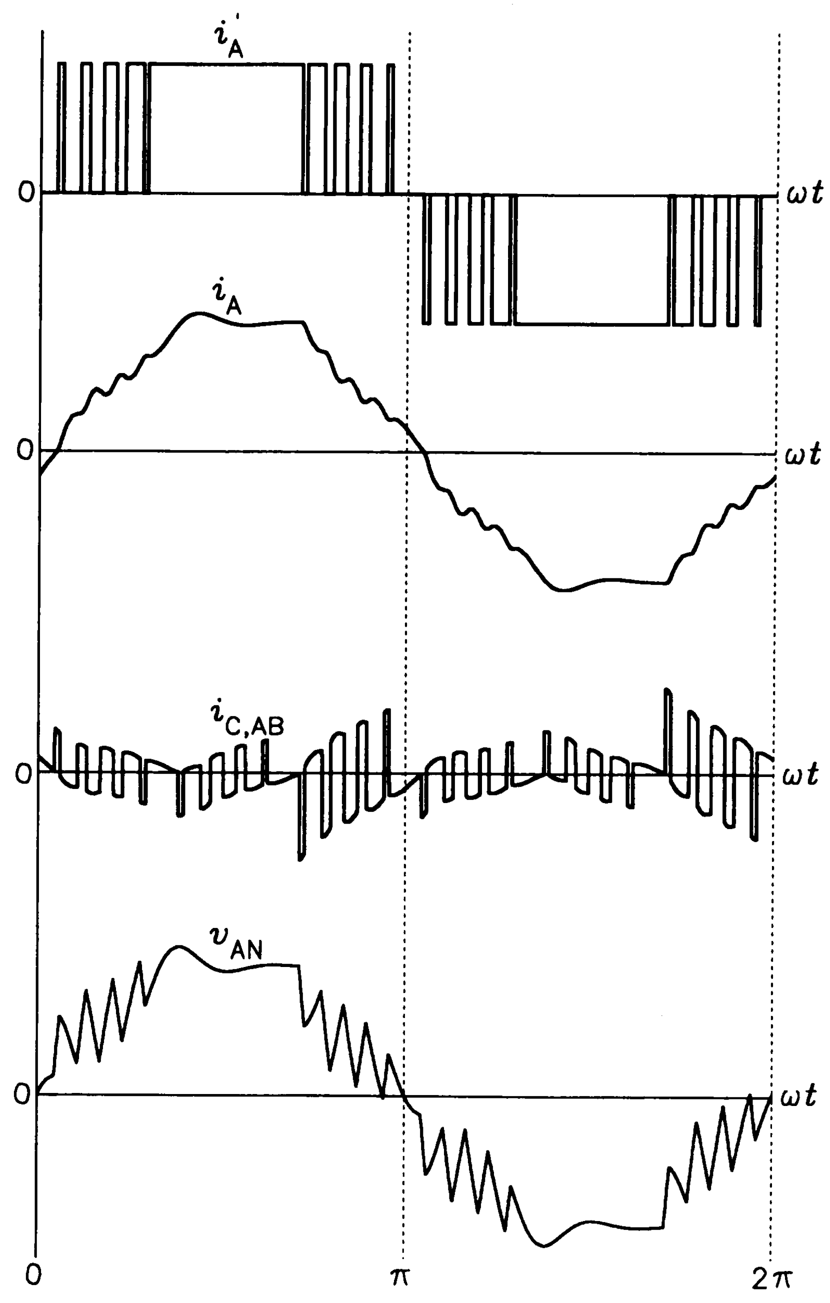


Fig. 7.51

Generic five-level inverter

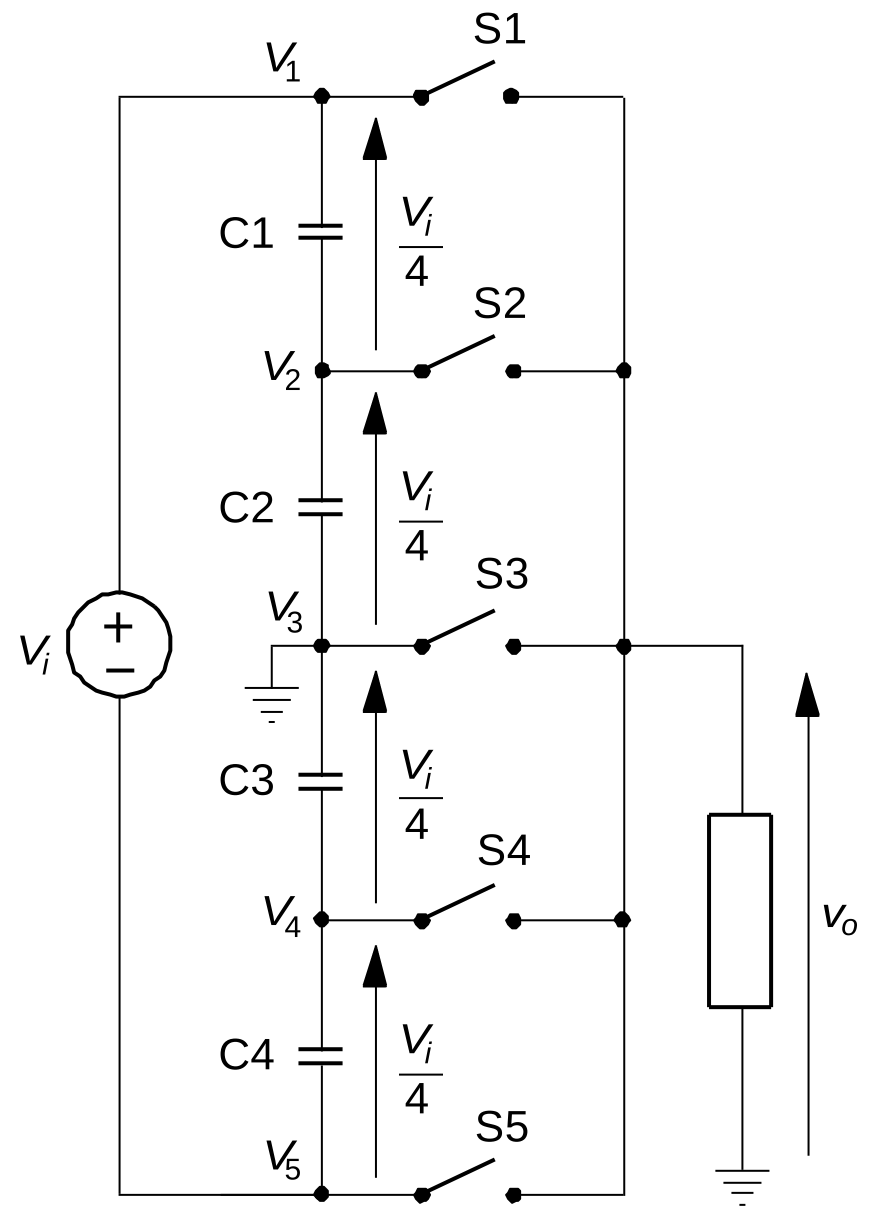


Fig. 7.52

Half-bridge voltage-source inverter

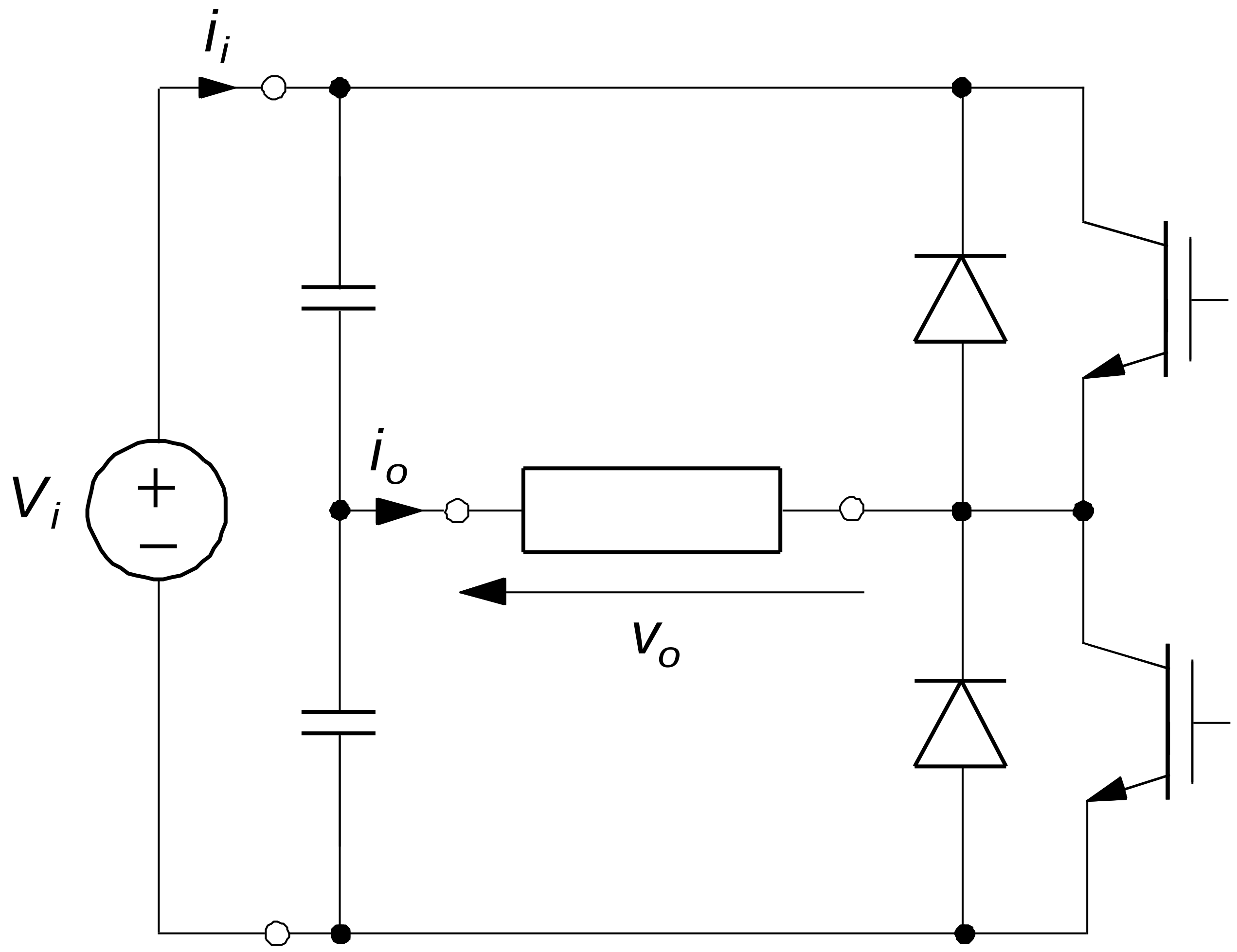


Fig. 7.53

Three-level neutral-clamped inverter

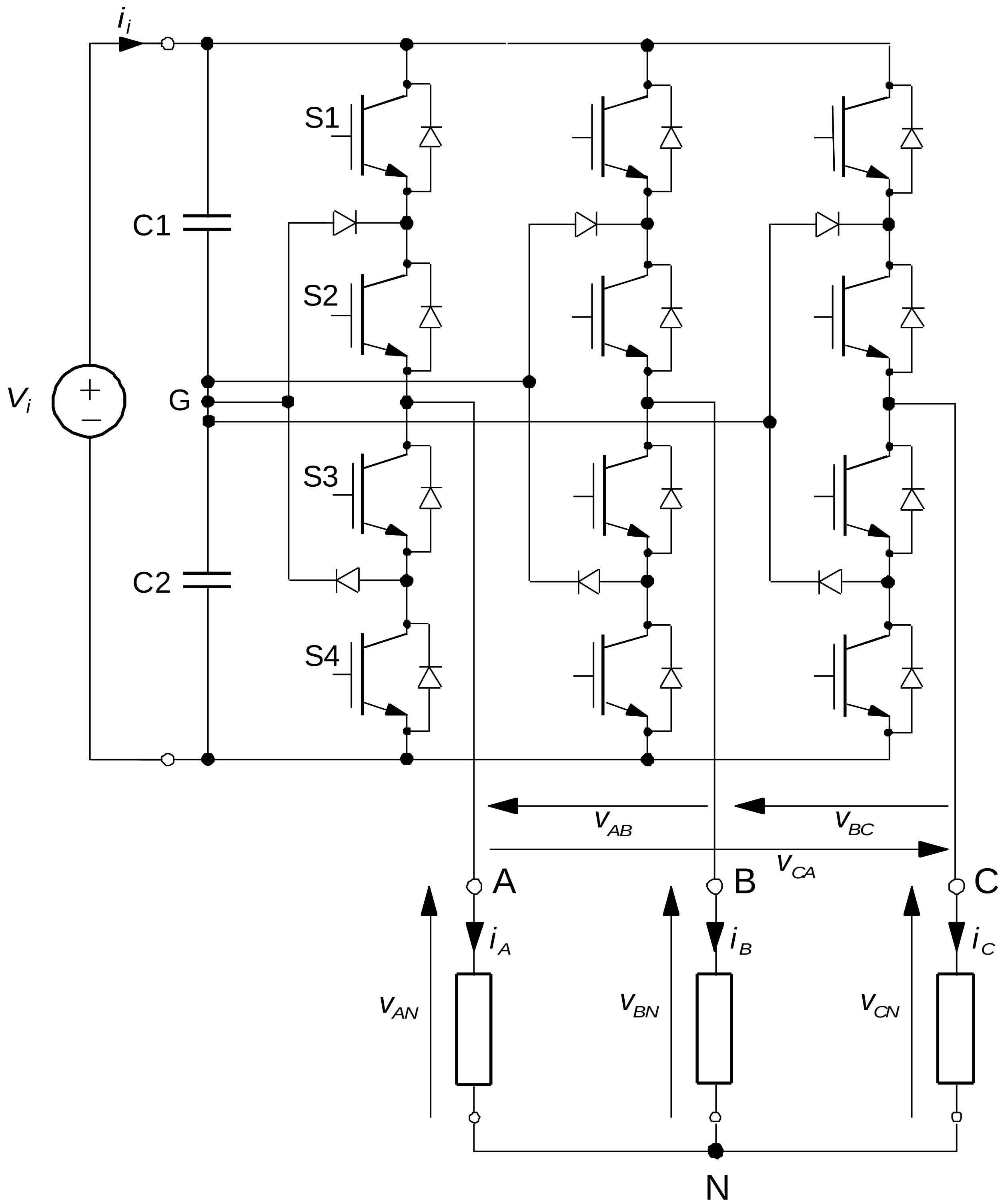


Fig. 7.54

In practice only three states of a leg are used, which makes for the total of twenty-seven states of the three-level inverter. The limiting condition is that two and only two adjacent switches must be ON at any time. A ternary switching variable can thus be assigned to each inverter phase and, for phase A, defined as

$$a = \begin{cases} 0 & if\ S3\ \&\ S4\ are\ ON \\ 1 & if\ S2\ \&\ S3\ are\ ON \\ 2 & if\ S1\ \&\ S2\ are\ ON \end{cases}$$

Switching variables *b* and *c* for the other two phases are defined analogously. It is easy to see that the potential a given output terminal of the inverter with respect to the "ground" (inverter's neutral), G, can be expressed in terms of the associated switching variable and input voltage. For instance, the voltage, v_A, at terminal A is

$$v_A = \frac{a-1}{2} V_i.$$

Consequently, the output line-to-line voltages are given by

$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = \frac{V_i}{2} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

and the line-to-neutral voltages by

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{V_i}{6} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Listing all possible values of the line-to-line and line-to-neutral voltages it can be seen that they can assume five and nine values, respectively. Generally, these numbers in an *l*-level inverter are 2*l* – 1 and 4*l* – 3.

Voltage space vectors
of a three-level neutral-clamped inverter

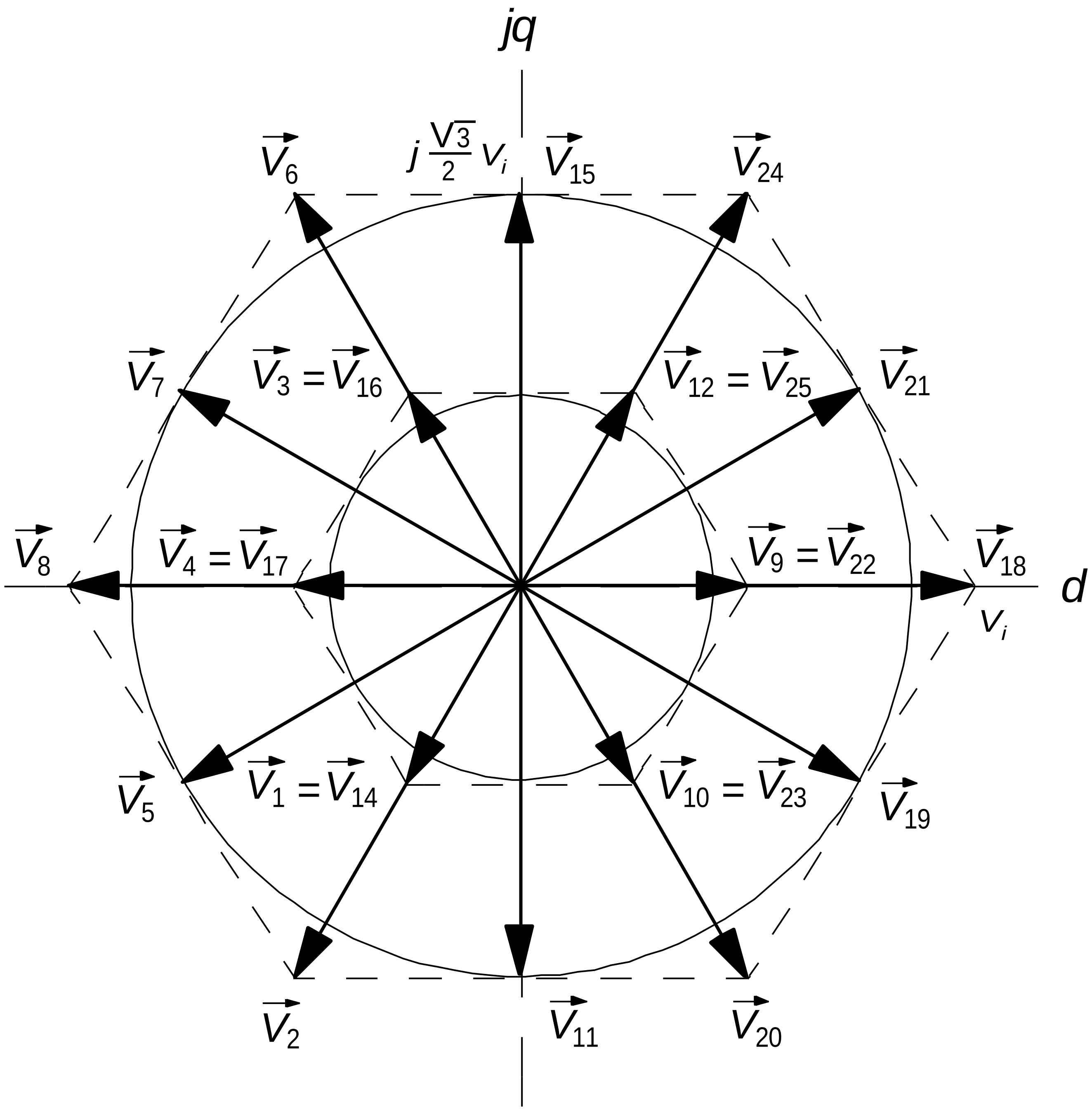


Fig. 7.55

States, switching variables, and waveforms of output voltages in a three-level neutral-clamped inverter in the square-wave mode

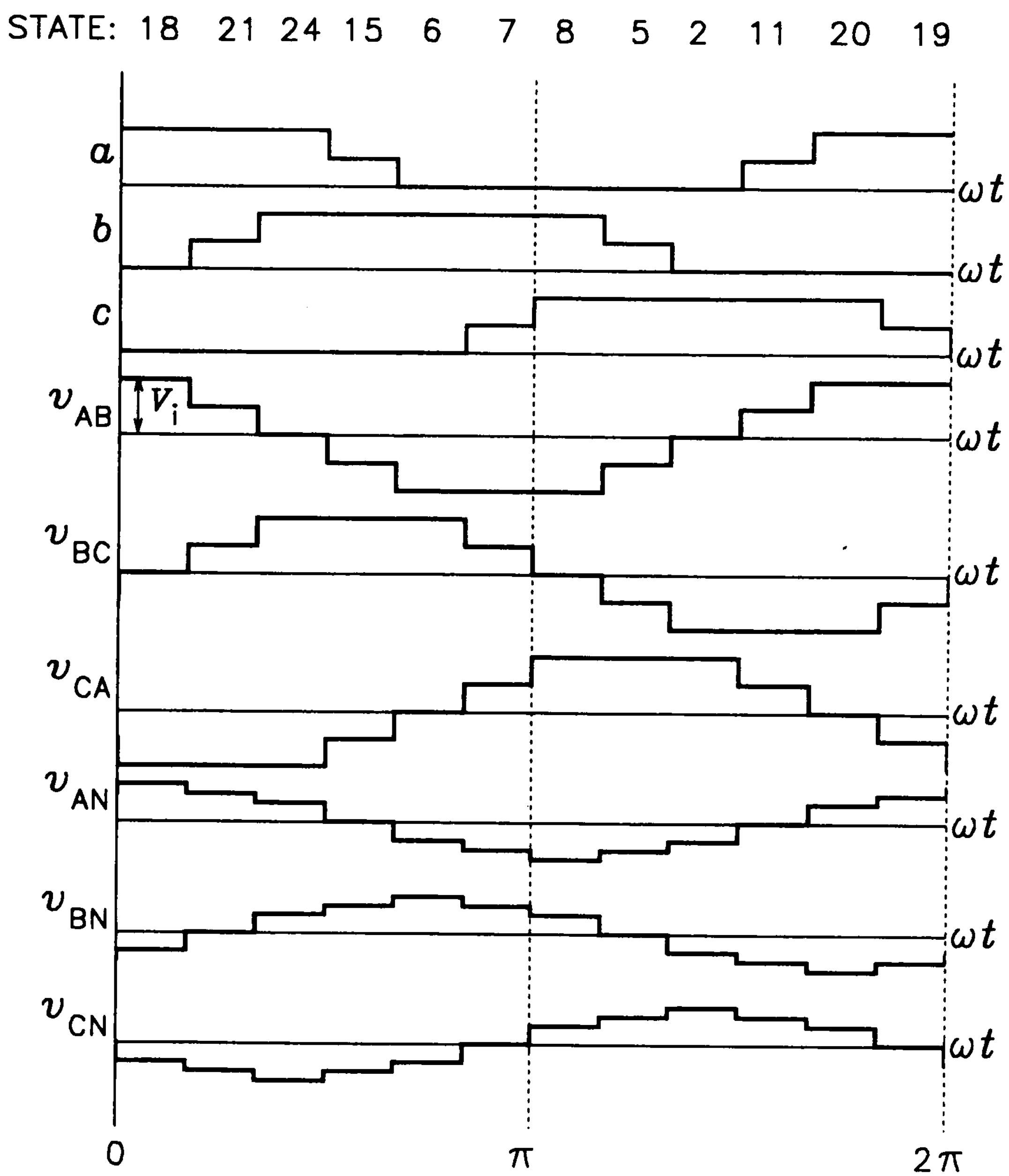


Fig. 7.56

Waveforms of output voltage and current in a three-level neutral-clamped inverter in the square-wave mode

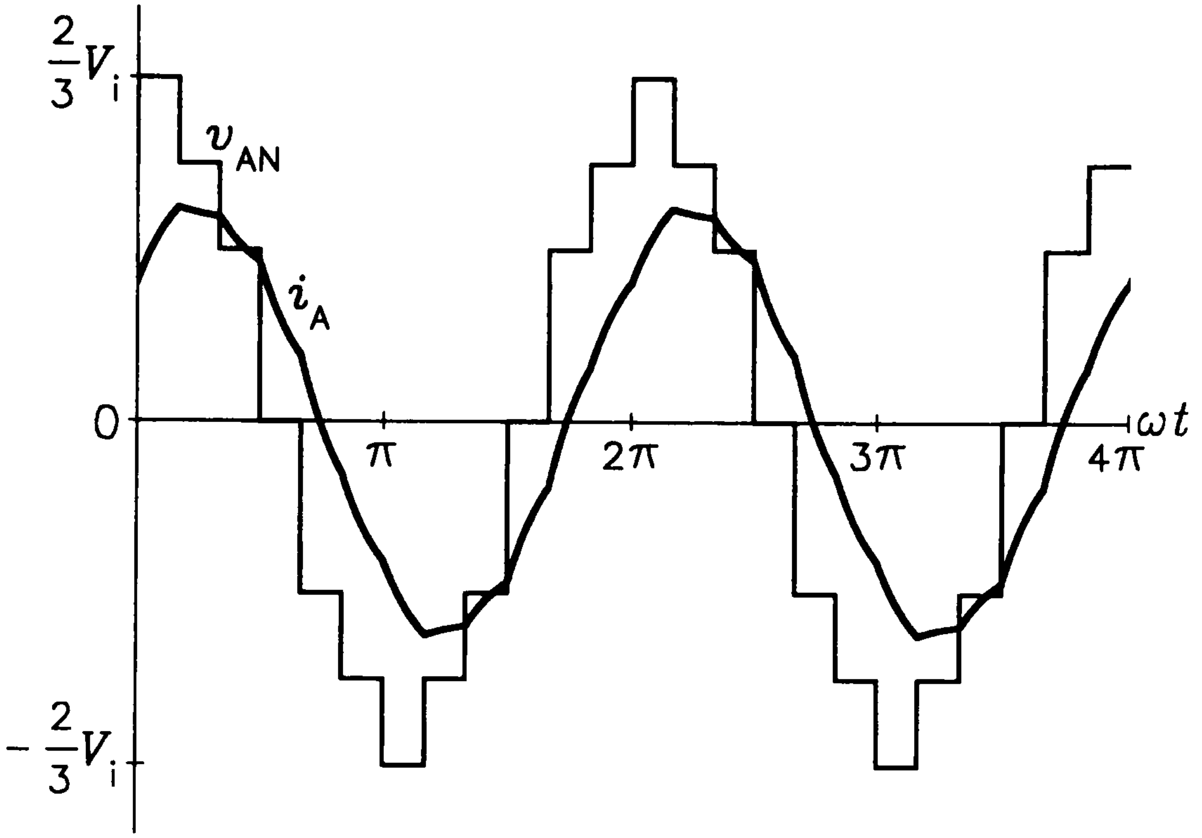
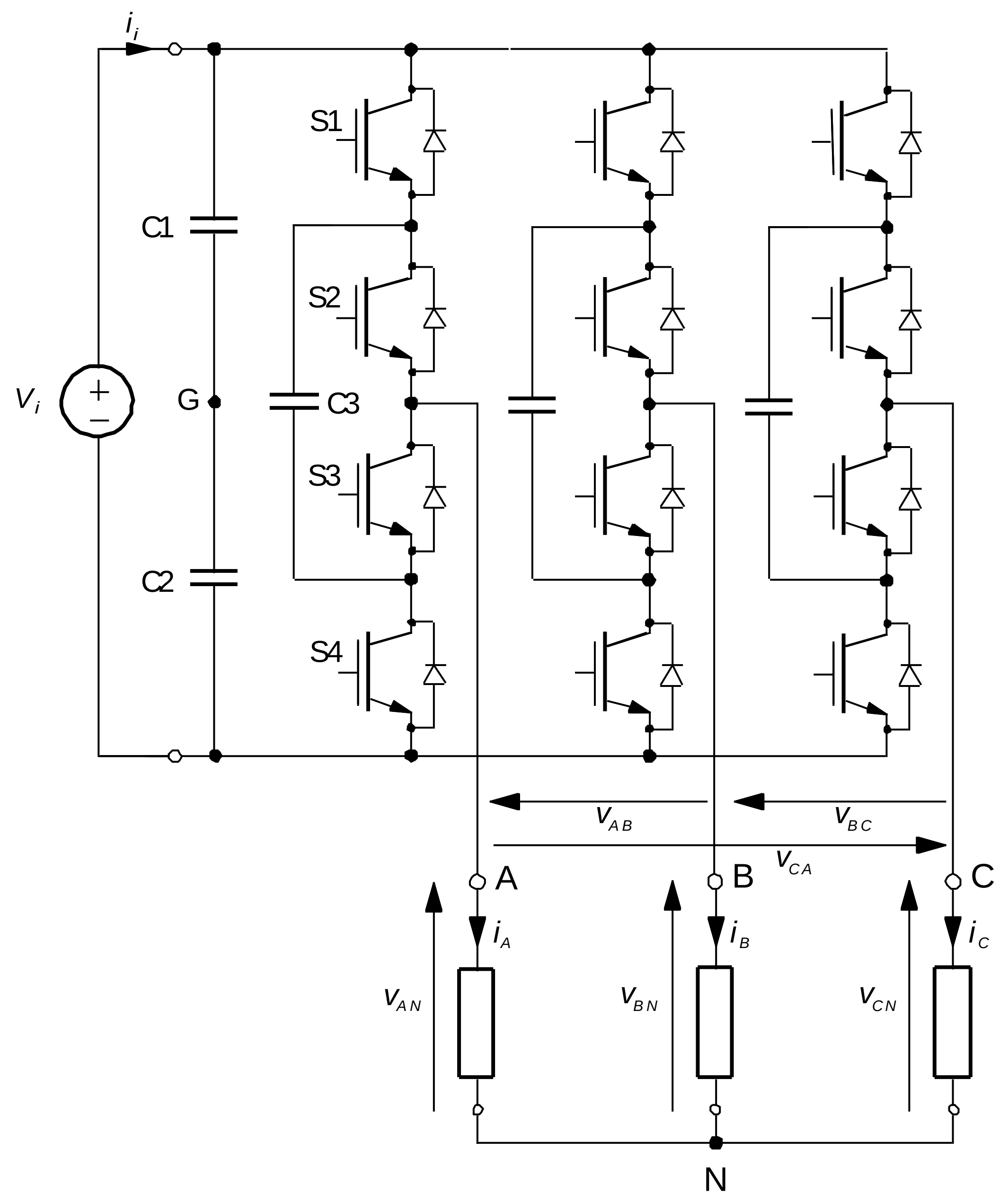


Fig. 7.57

Three-level flying-capacitor inverter



$$a = \begin{cases} 0 & if\ S3\&S4\ are\ ON \\ 1 & if\ S1\&S3\ or\ S2\&S4\ are\ ON \\ 2 & if\ S1\&S2\ are\ ON \end{cases}$$

Fig. 7.58

Cascaded H-bridge inverter:
(a) block diagram, (b) constituent bridge

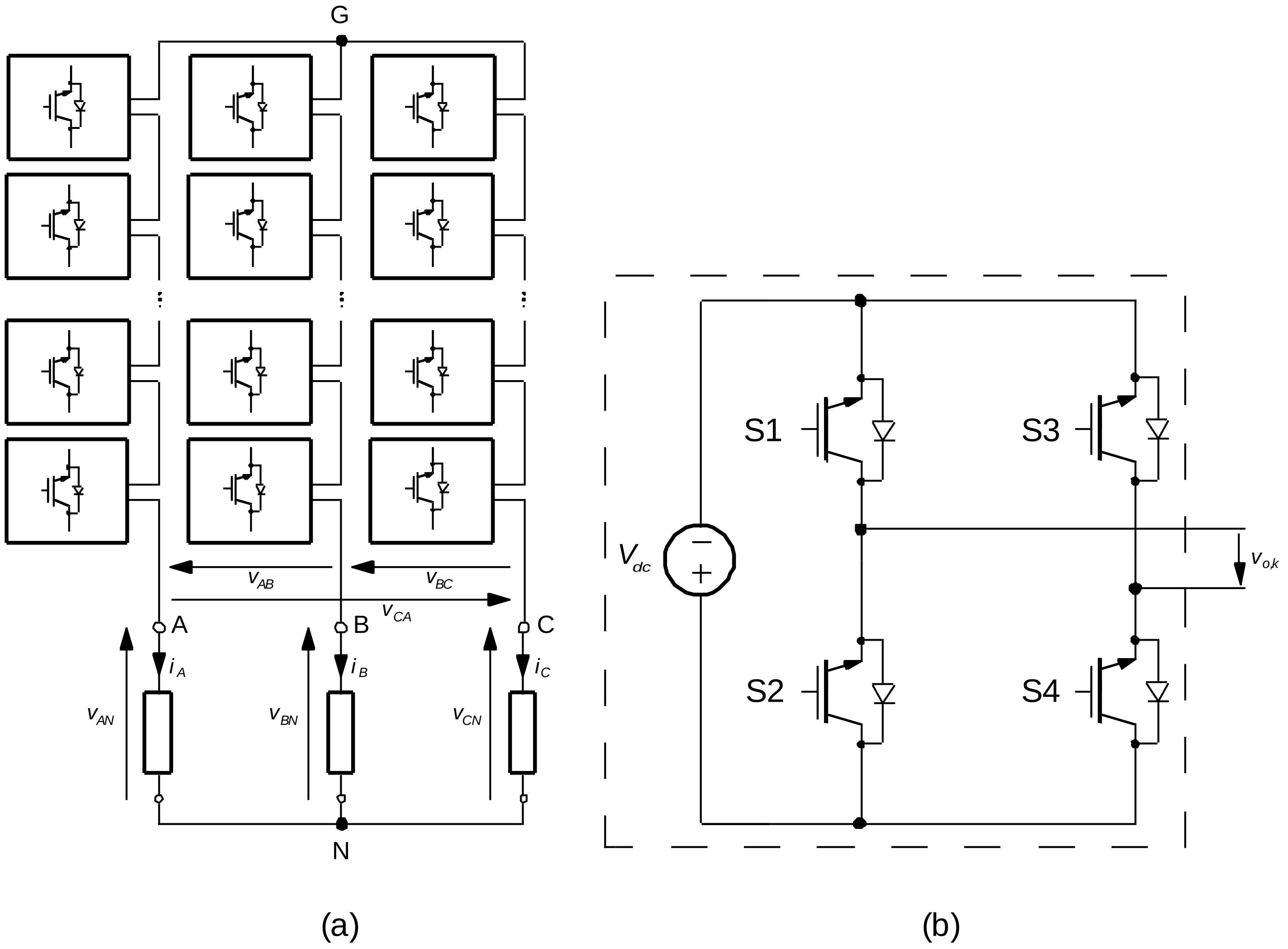


Fig. 7.59

An H-bridge can generate three voltage levels between its output terminals, namely $-V_{dc}$, 0, and V_{dc}. The number N of H-bridges in an l-level inverter is $(l - 1)/2$, thus N bridges form an inverter with $2N + 1$ levels. In the constituent H-bridge two and only switches can be ON at any time. For the k-th bridge in a leg of a three-phase inverter, the ternary switching variable, a_k, is defined as follows:

$$a_k = \begin{cases} 0 & if\ S2\&S3\ are\ \mathrm{ON} \\ 1 & if\ S1\&S3\ or\ S2\&S4\ are\ ON \\ 2 & if\ S1\&S4\ are\ ON. \end{cases}$$

The output voltage, $v_{o,k}$, of the bridge is then given by

$$v_{o,k} = (a_k - 1)V_{dc}.$$

The voltage of terminal A, v_A, with respect to the inverter's neutral, G, is a sum of output voltages of all the bridges. Consequently, a switching variable of phase A of the inverter can be defined as

$$a = \sum_{k=1}^{N} a_k$$

where, depending on control of the individual bridges, a can assume any integer value from the 0 to $2N$ range. Then,

$$v_A = (a - N)V_{dc}.$$

Switching variables *b* and *c* are defined analogously, yielding the following equations for the line-to-line and line-to-neutral output voltages of the inverter:

$$\begin{bmatrix} v_{AB} \\ v_{BC} \\ v_{CA} \end{bmatrix} = V_{dc} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

and

$$\begin{bmatrix} v_{AN} \\ v_{BN} \\ v_{CN} \end{bmatrix} = \frac{V_{dc}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Approximation of a sinewave by a stepped waveform in the H-bridge cascaded inverter

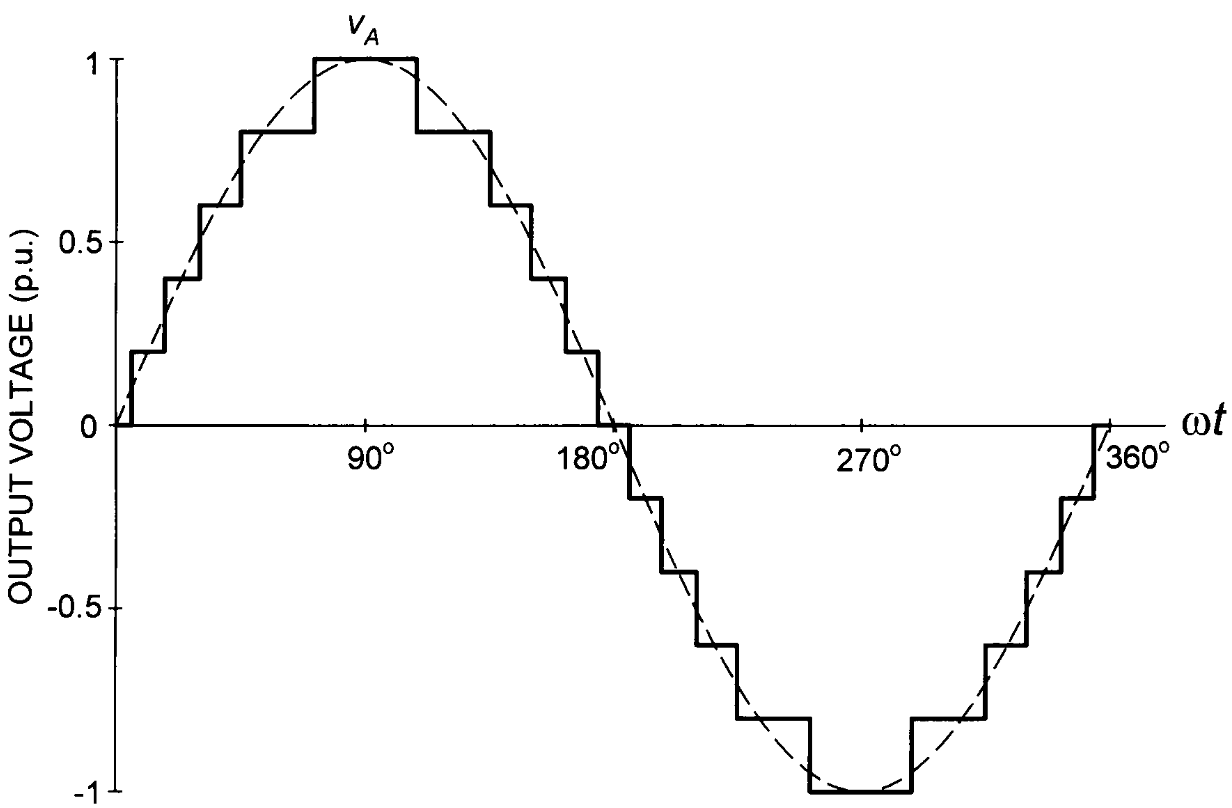


Fig. 7.60

Endpoints of line-to-neutral vectors
of two-bridge cascaded inverter

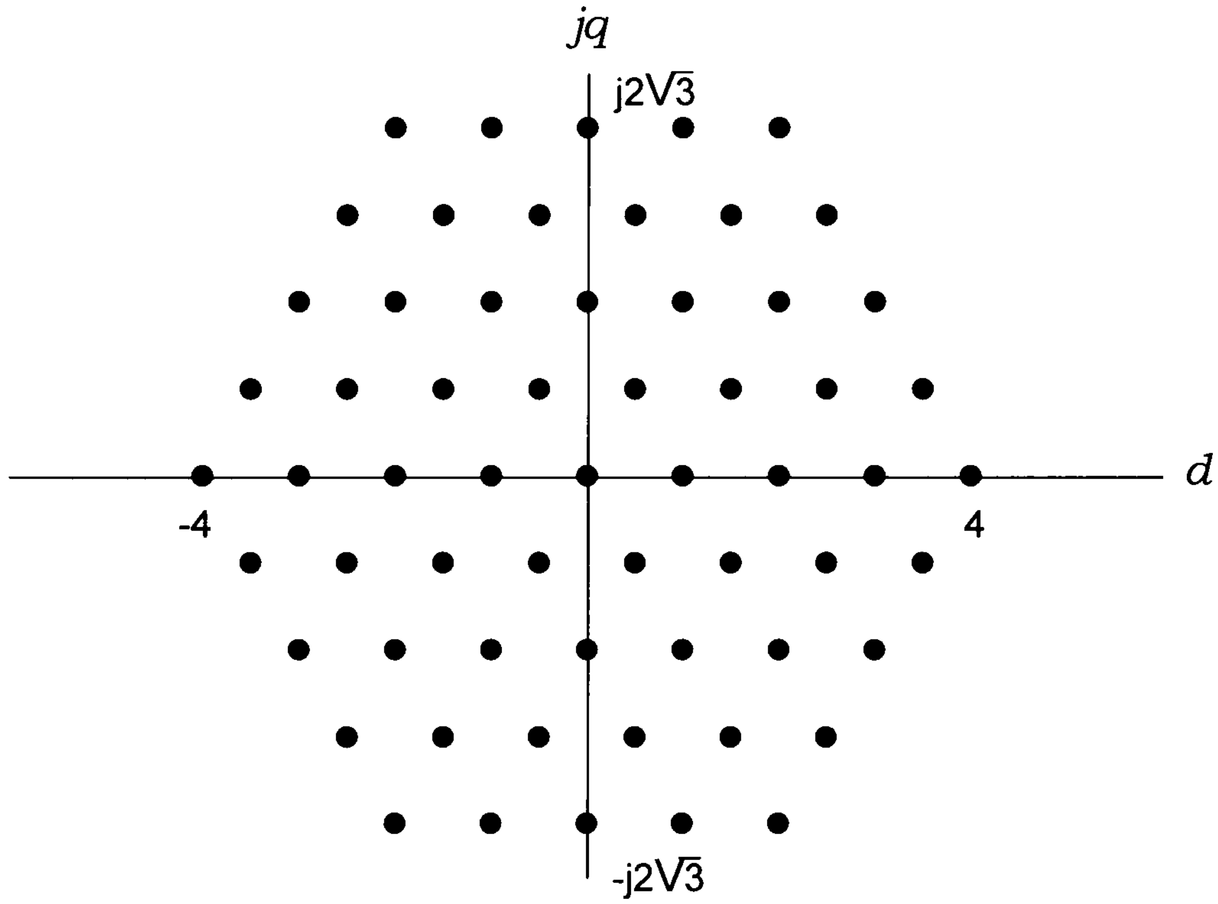
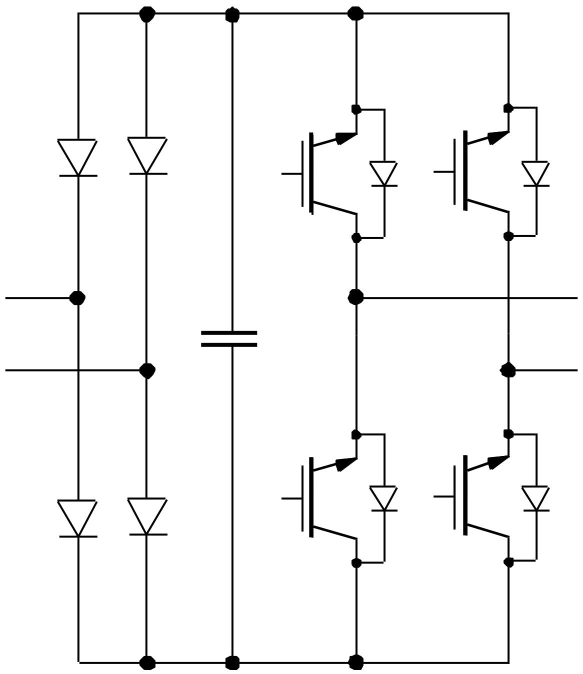
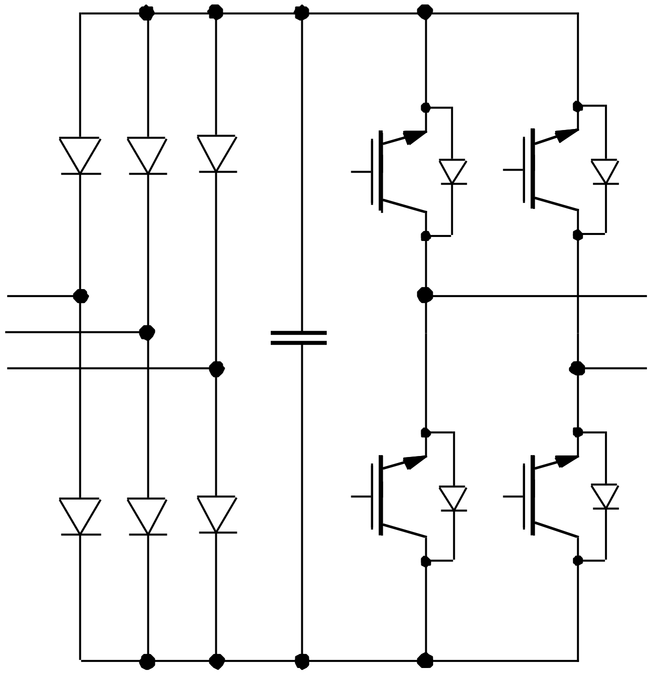


Fig. 7.61

Cells with diode rectifiers of two-bridge cascaded inverter:
(a) single-phase, (b) three-phase



(a)



(b)

Fig. 7.62

Cells with PWM rectifiers for ac-supplied cascaded inverter:
(a) single-phase, (b) three-phase

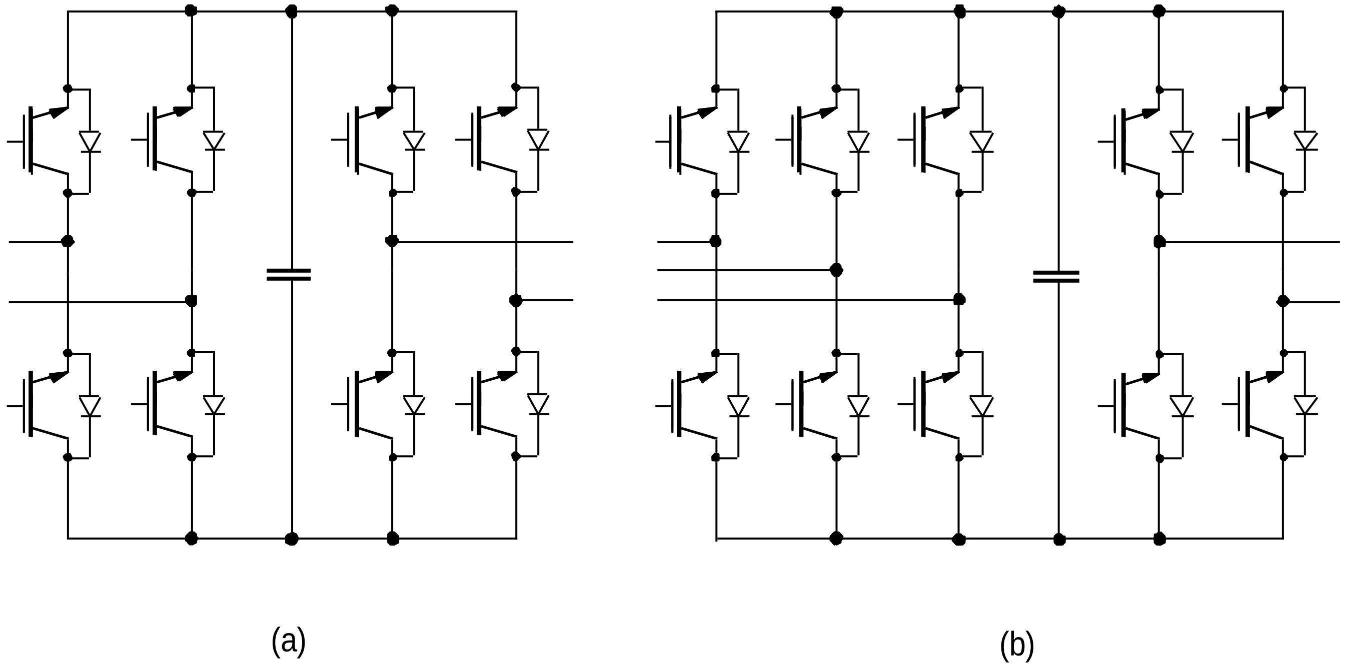


Fig. 7.63

Note: additional reading, not required in the exam

Switched network for illustration of the operating principle of a resonant dc link

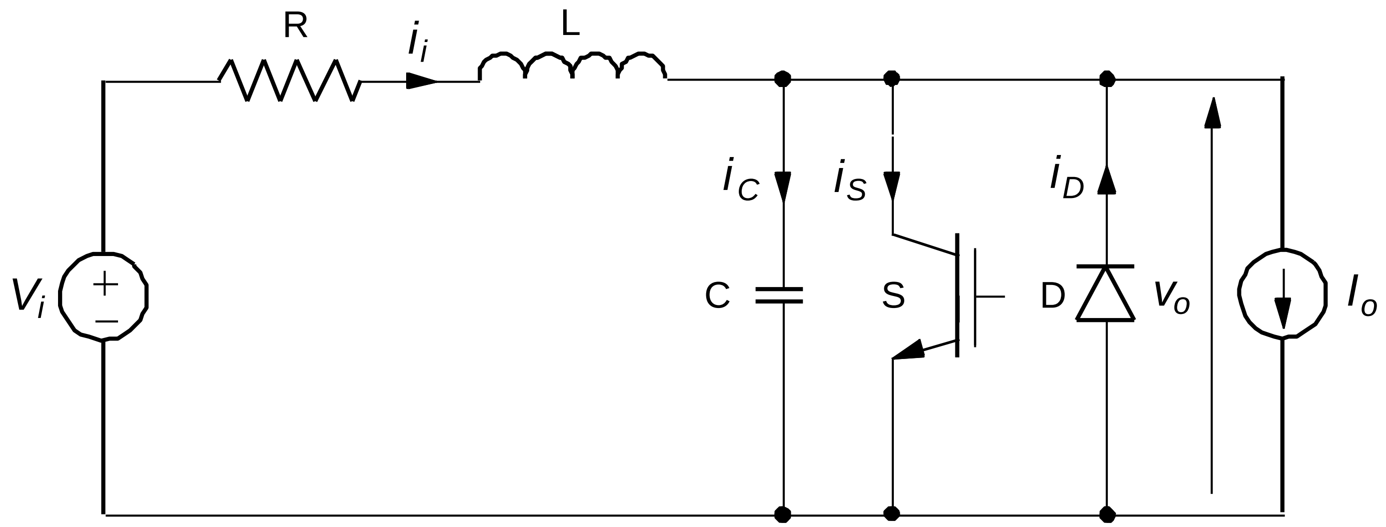


Fig. 7.64

Waveforms of voltage and current in the resonant dc link

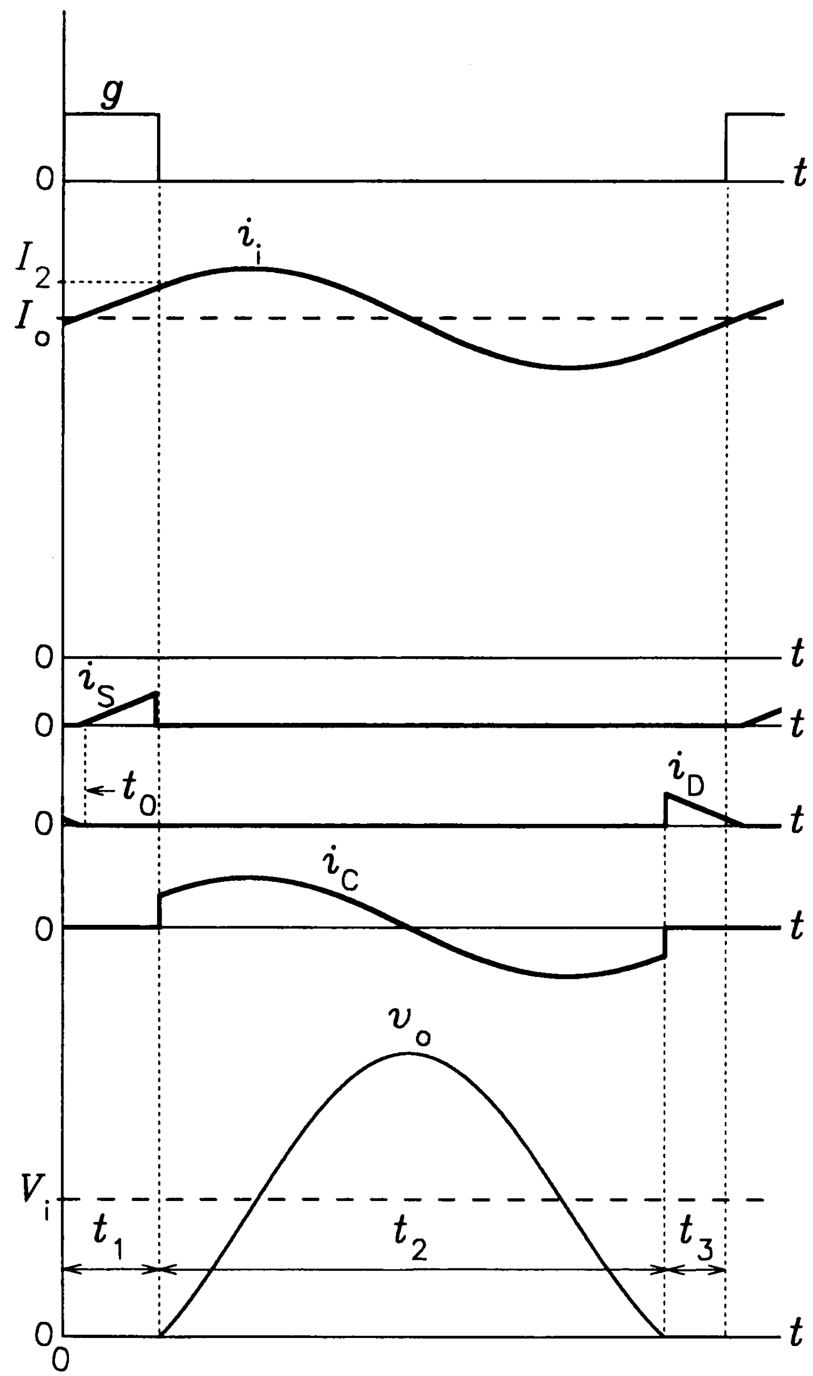


Fig. 7.65

Three-phase resonant dc link inverter with an active clamp

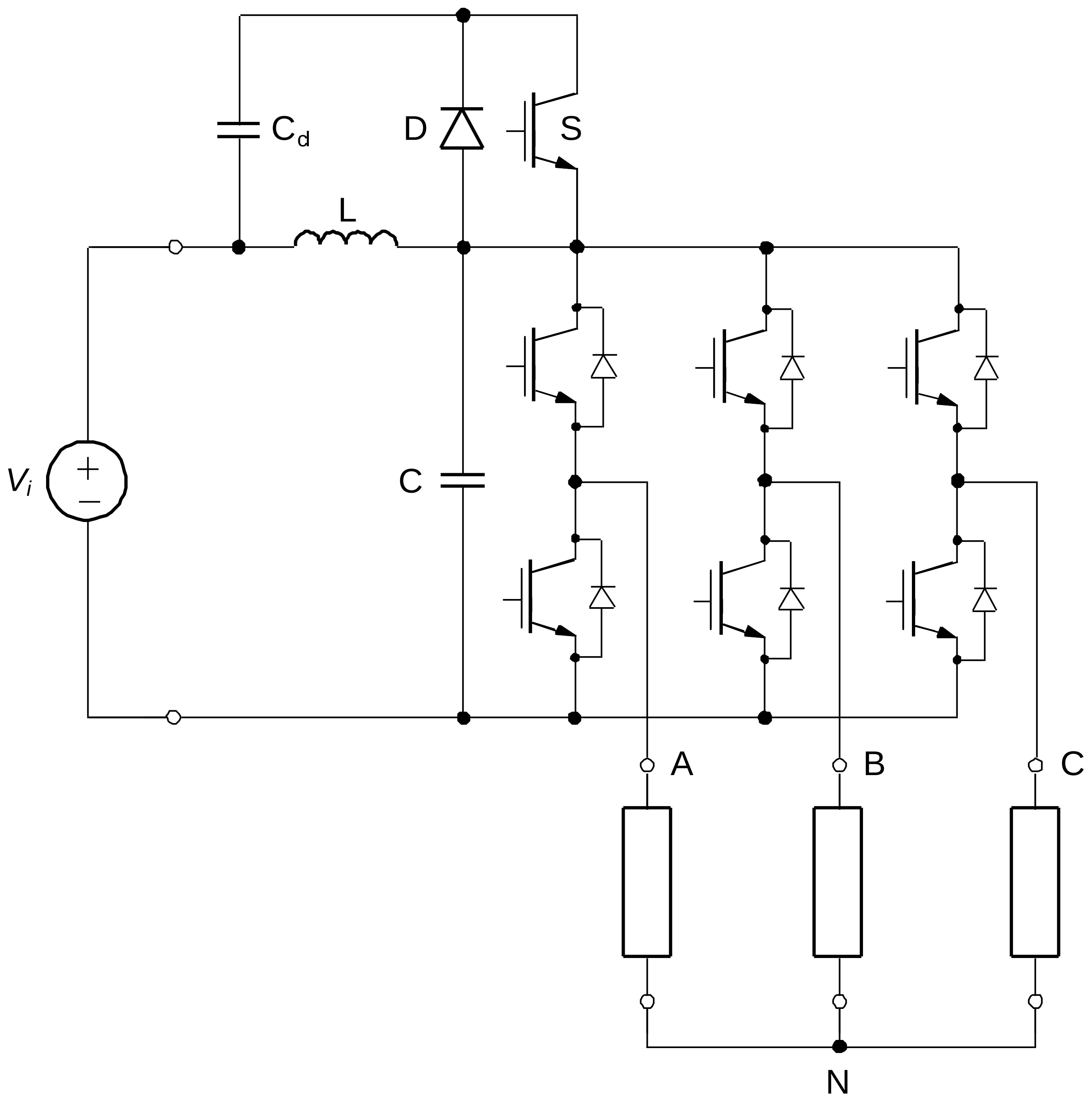


Fig. 7.66

Waveforms of line-to-line output voltages in a resonant dc-link inverter

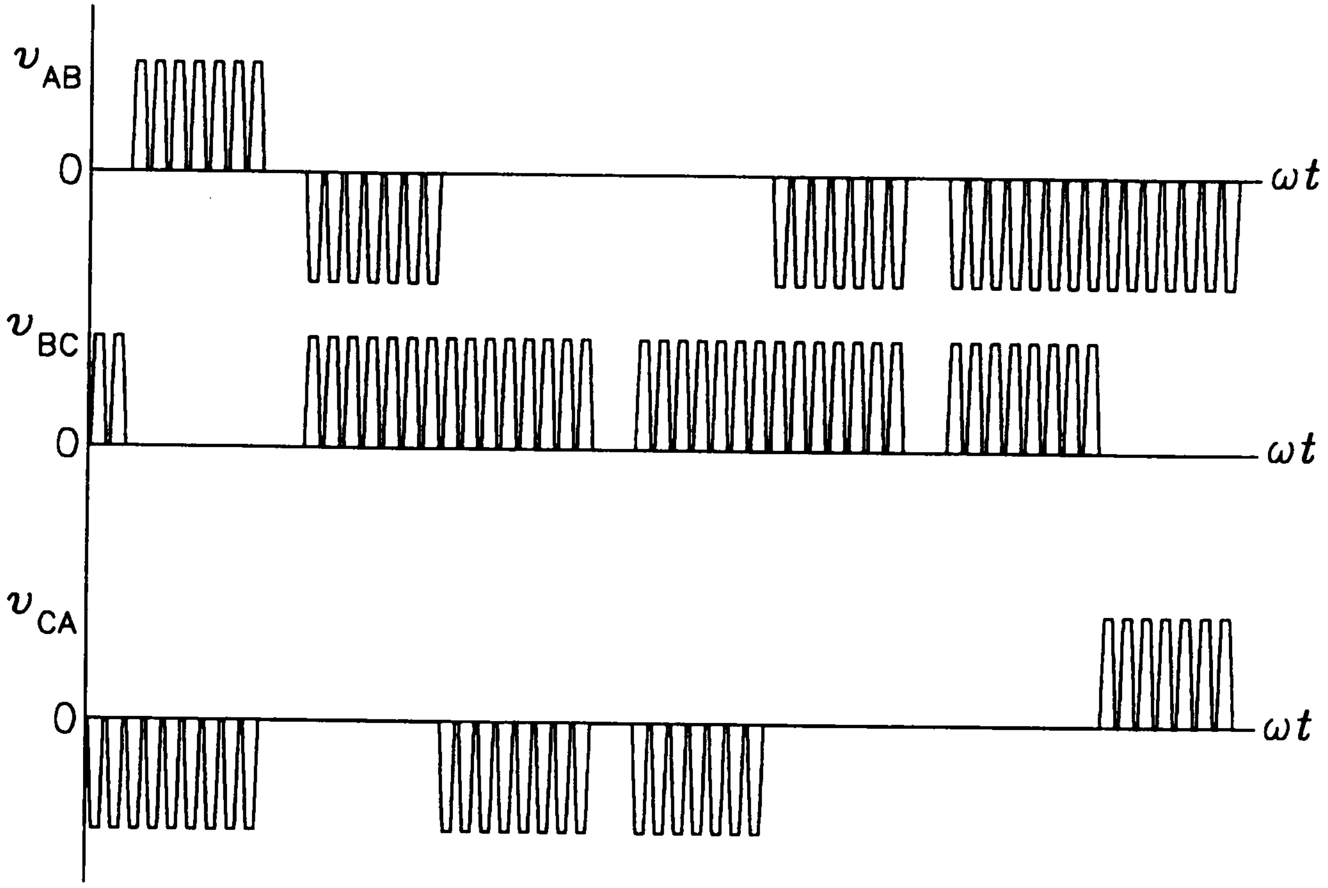
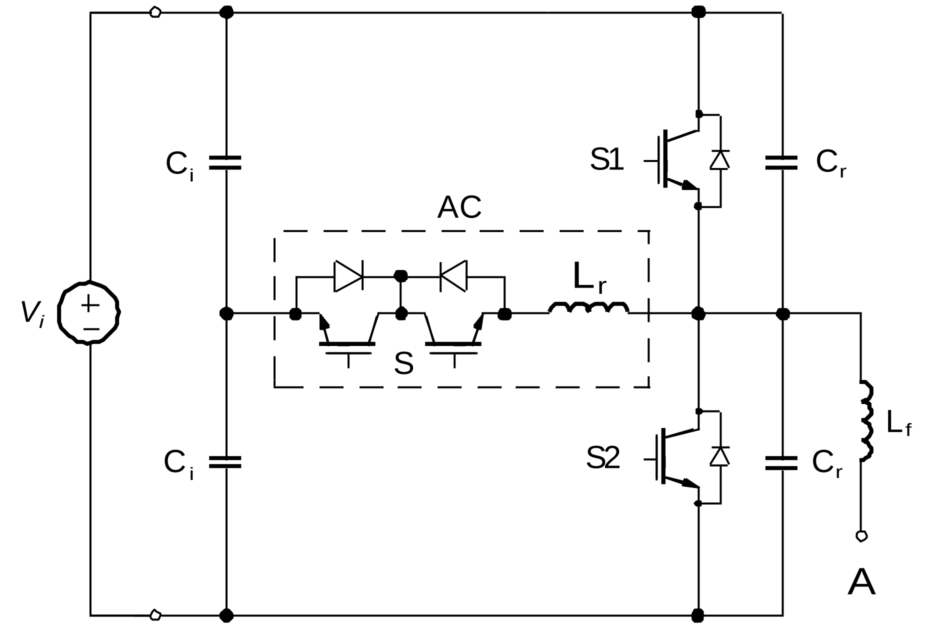
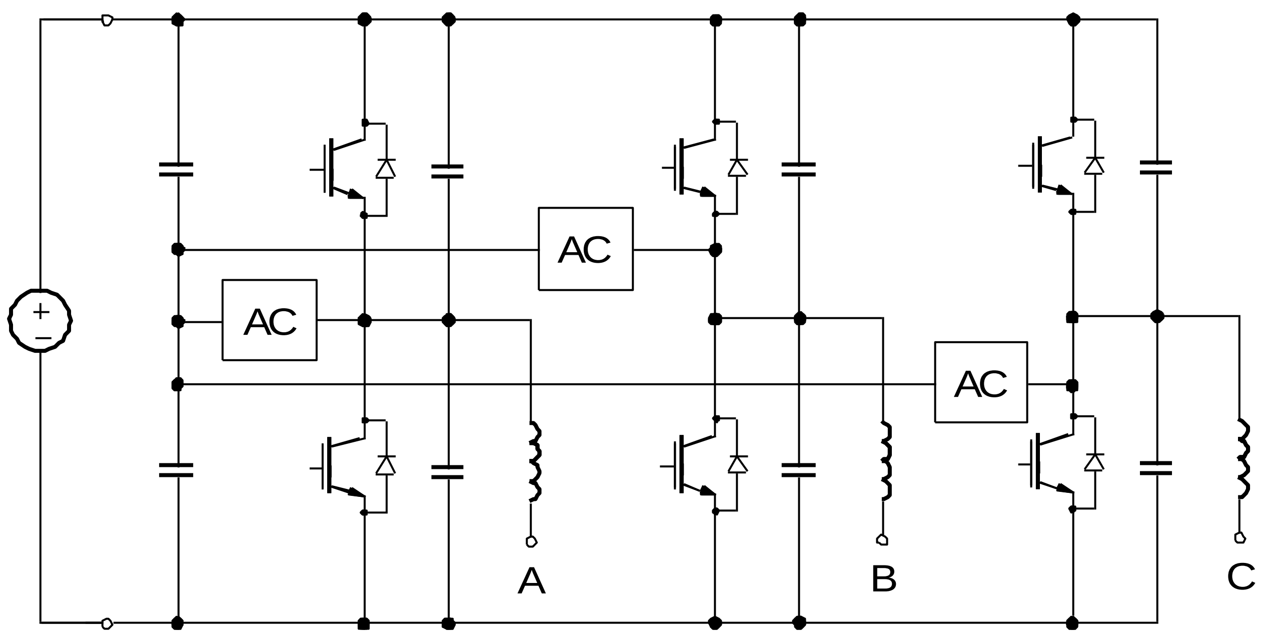


Fig. 7.67

Auxiliary resonant commutated pole inverter:
(a) one phase with the auxiliary circuit, (b) the entire inverter



(a)



(b)

Fig. 7.68

Idealized line-to-neutral voltage and line current waveforms in a VSI in the square-wave mode

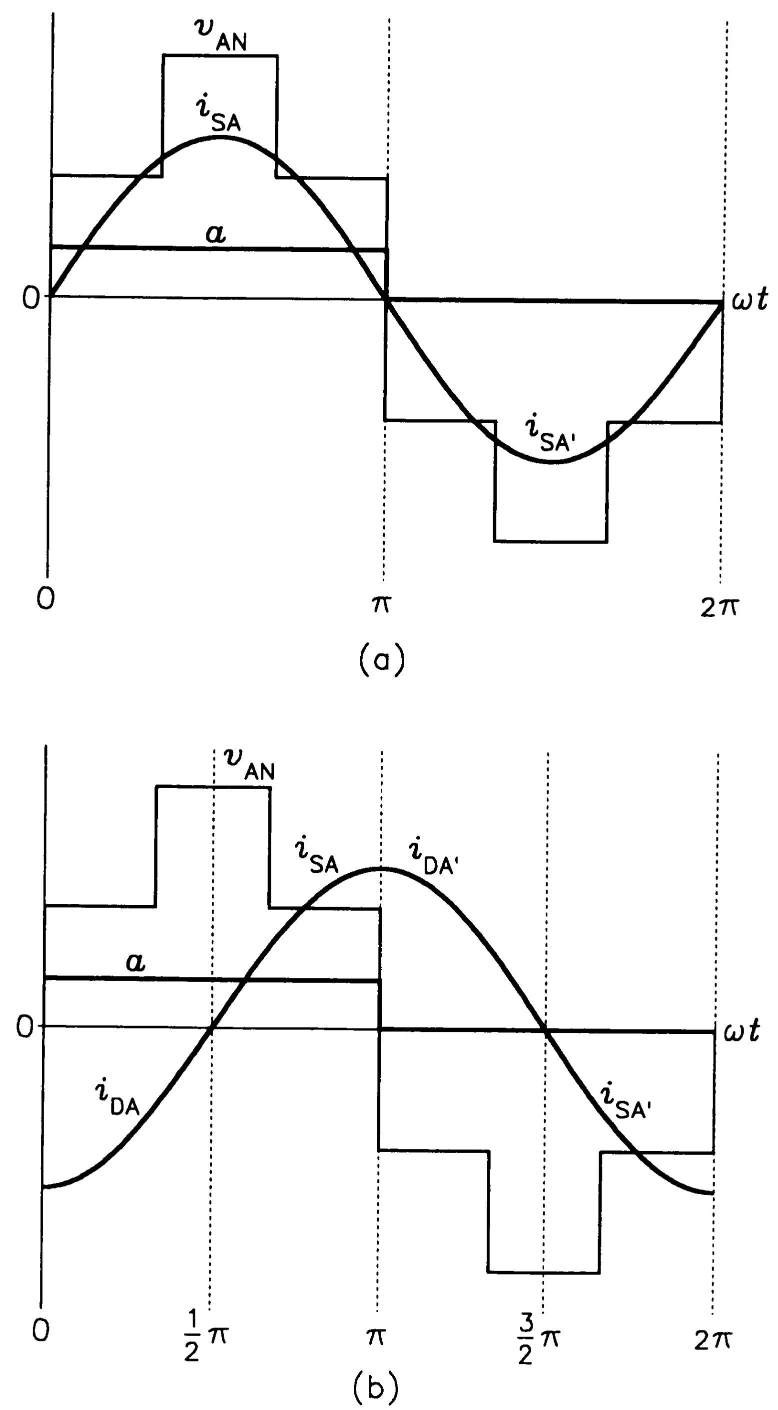


Fig. 7.69

Block diagram of a photovoltaic utility interface

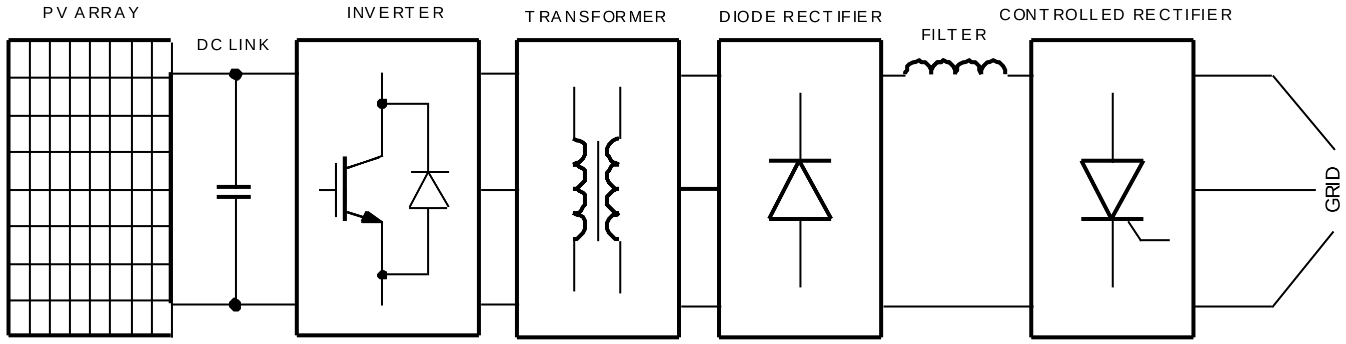


Fig. 7.70

Block diagram of an active power filter

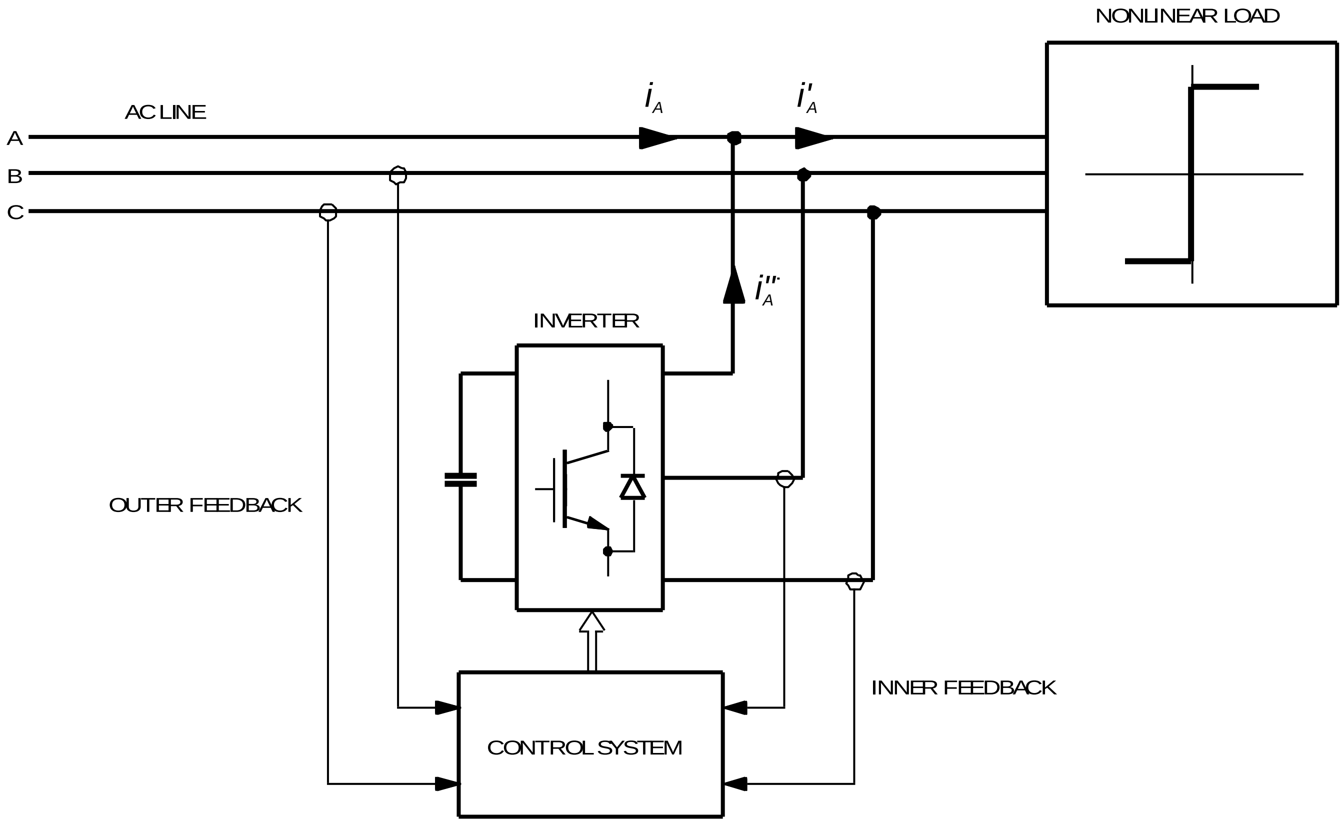


Fig. 7.71

Waveforms of voltage and current
in an active power filter

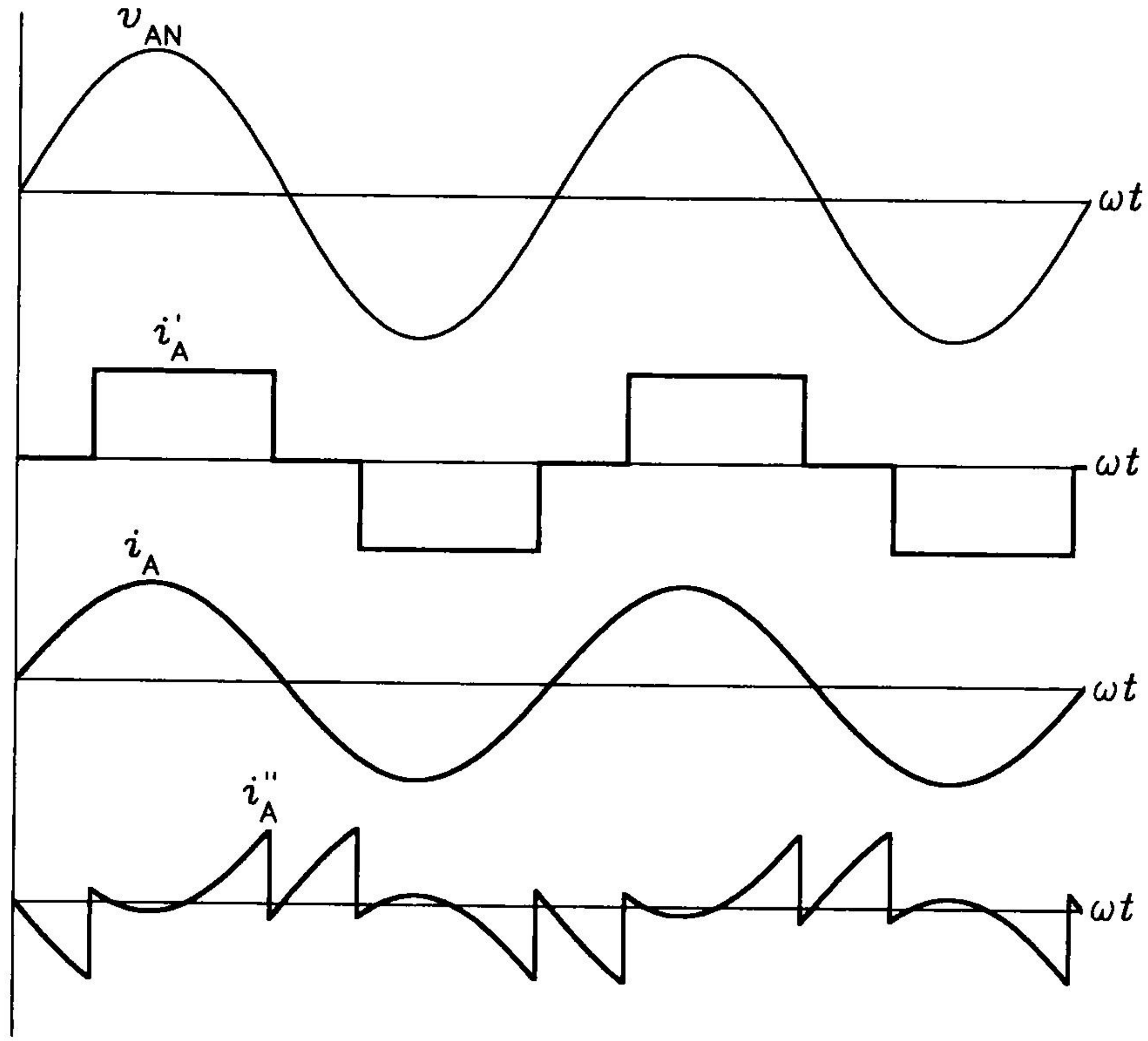


Fig. 7.72

UPS System

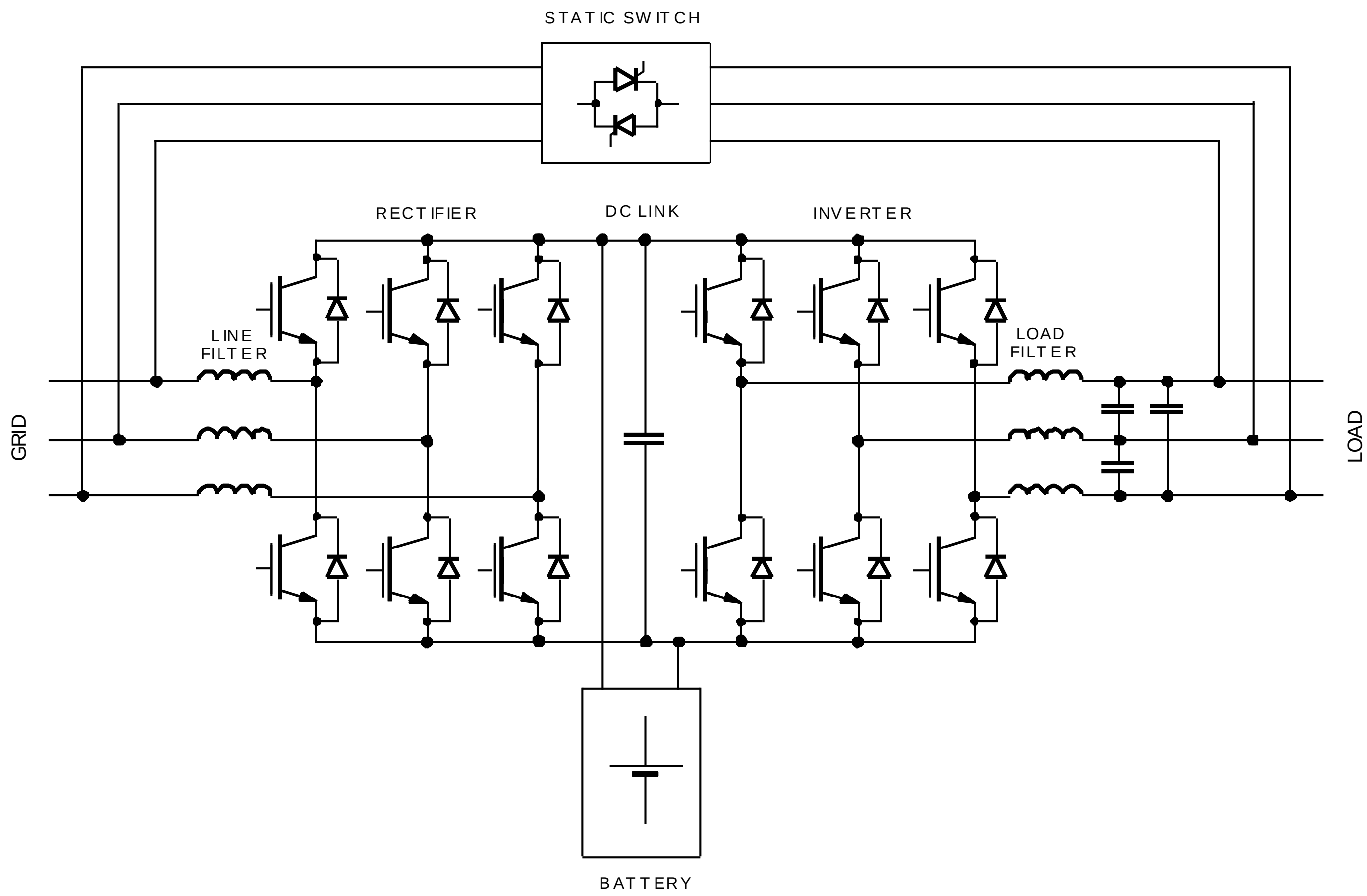


Fig. 7.73

Block diagram of an ac drive system with scalar speed control

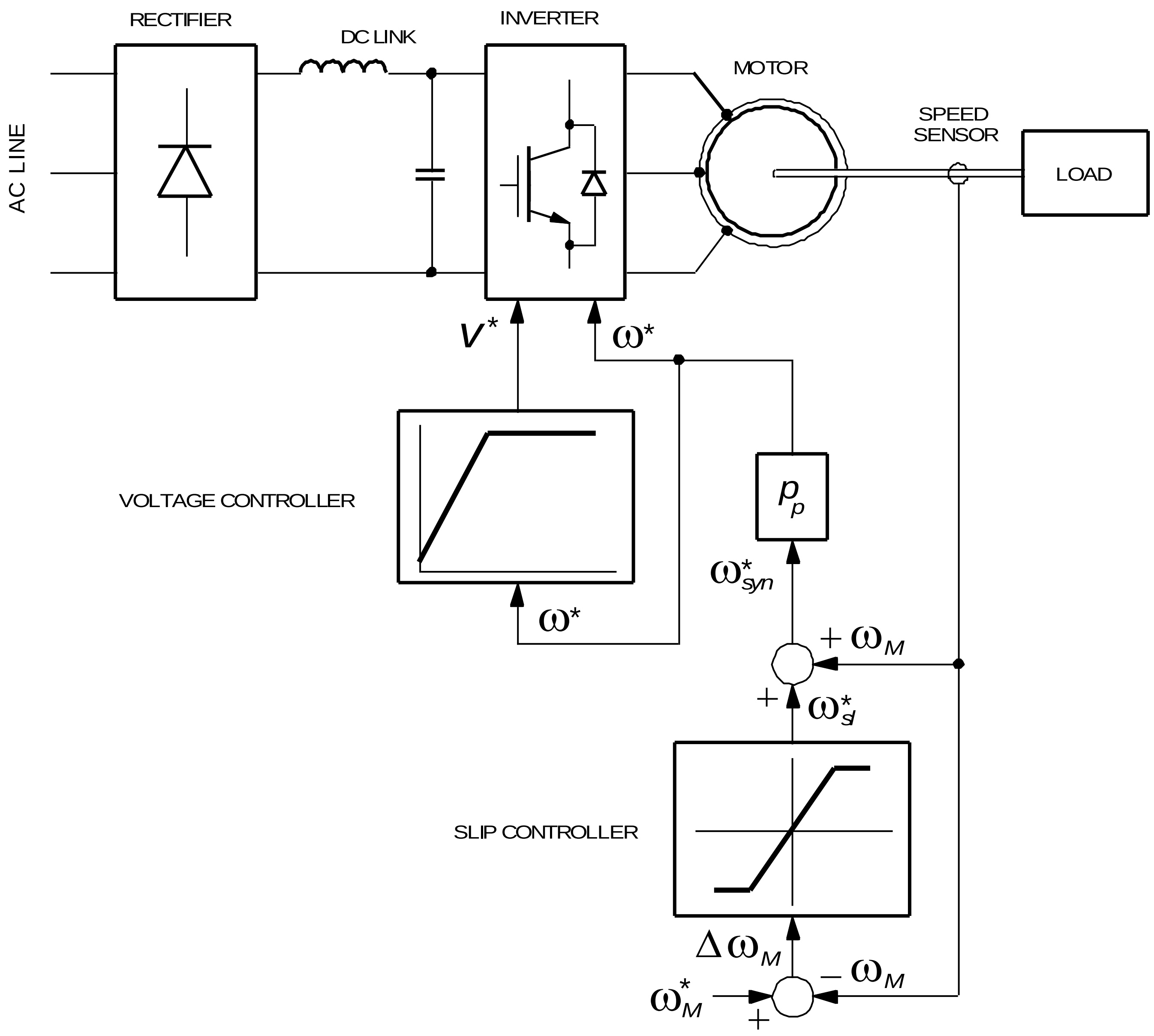


Fig. 7.74

Use of the modular frequency changer of Figure 2.24 in an ac drive: (a) system with a braking resistor, (b) system with a step-up chopper

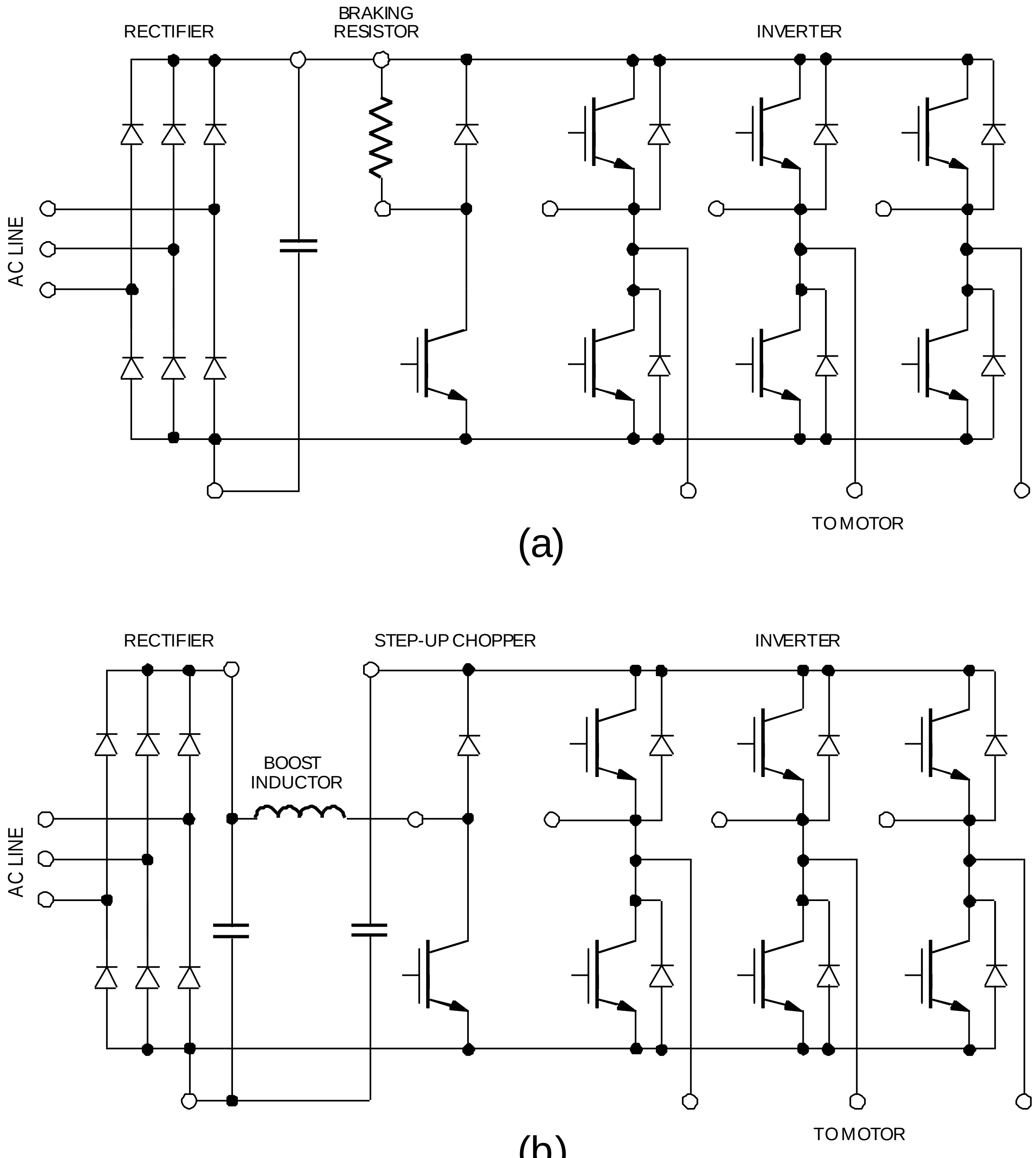


Fig. 7.75

PWM rectifier-inverter cascades for bidirectional power flow in ac motor drives: (a) current-type rectifier, inductive dc link, and current-source inverter, (b) voltage-type-rectifier, capacitive dc link, and voltage-source inverter

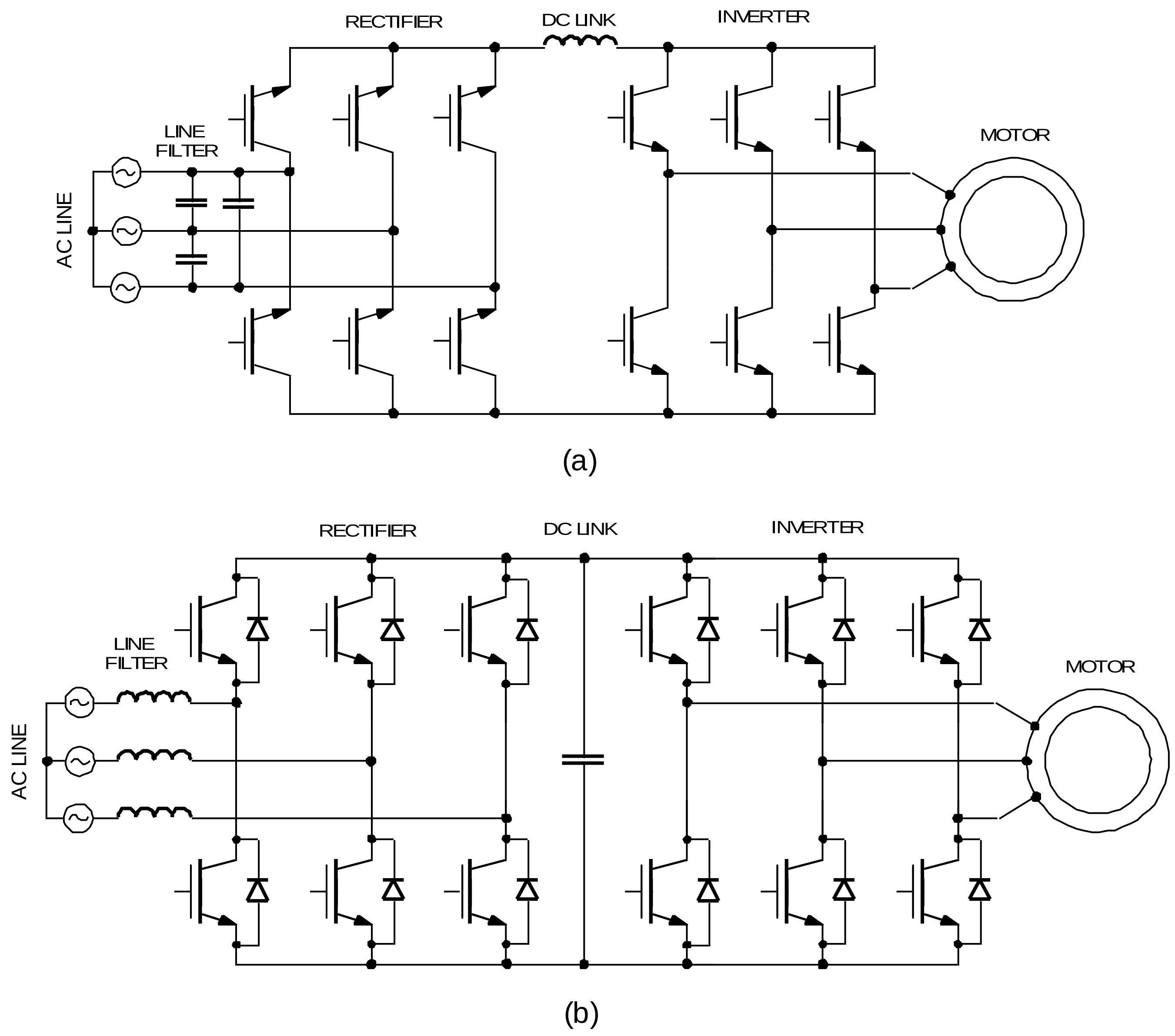


Fig. 7.76