

Emperical Methods for Marketing Research and Analytics Using

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About Me



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Using for AN(C)OVA

Prof. Dr. Martin Wetzels
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Course Outline

Session:

DAY 1

Session 1

Session 2

DAY 2

Session 3

Session 4

DAY 3

Session 5

Session 6

TOPIC:

INTRODUCING MULTIVARIATE ANALYSIS AND R

USING R FOR BASIC ANALYSIS

USING R FOR AN(C)OVA

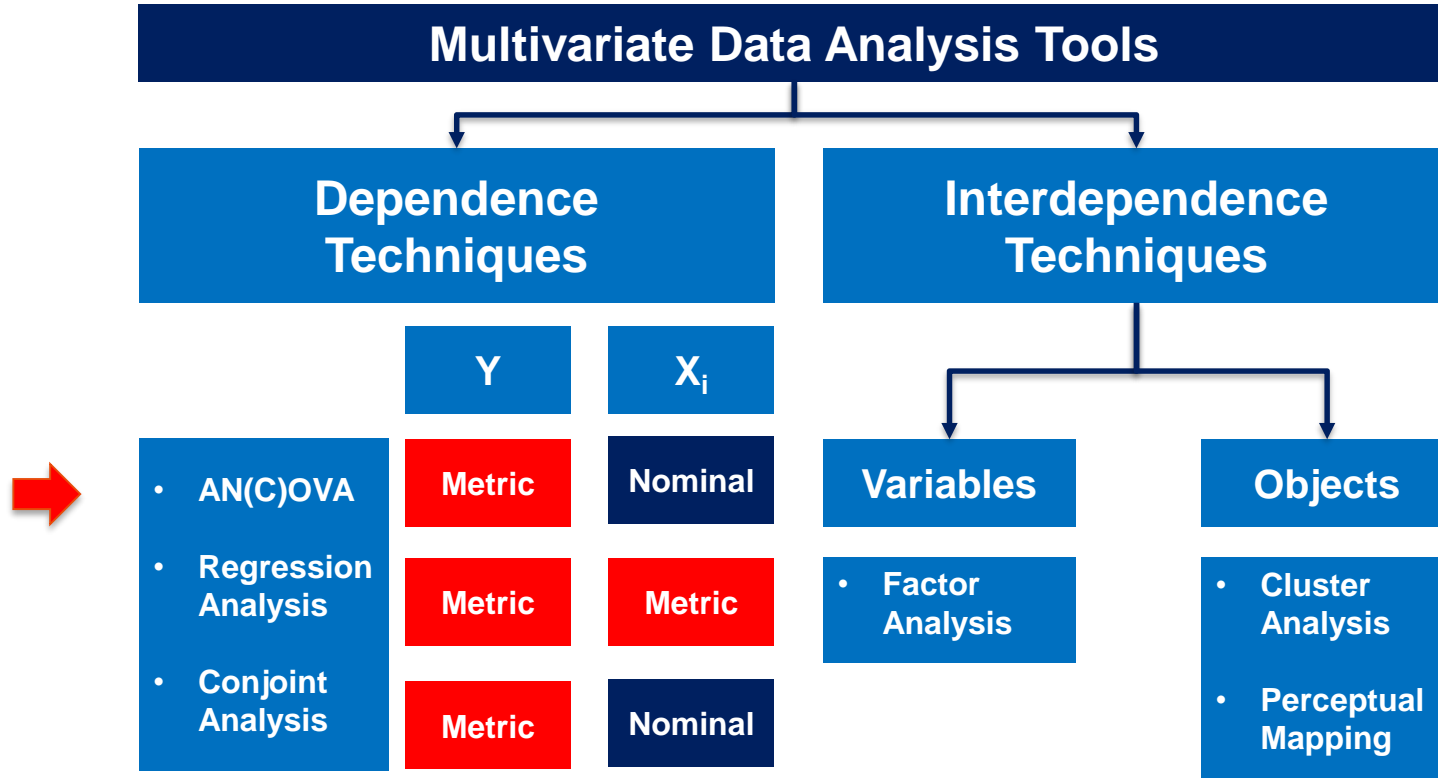
USING R FOR REGRESSION ANALYSIS

USING R FOR SCALING AND FACTOR ANALYSIS

USING R FOR SEM and PLS PATH MODELING

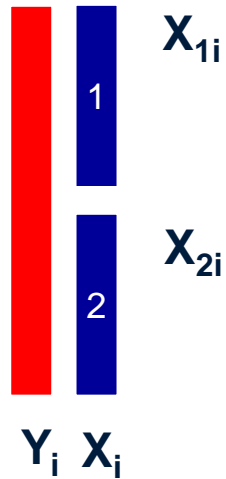
Positioning AN(C)OVA...

Hair et al. (2018); Malhotra (2010)

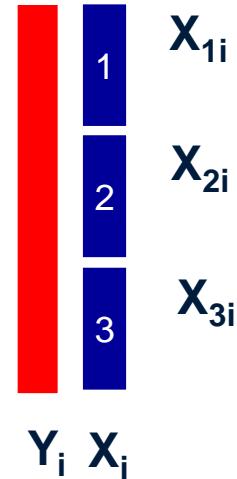


Analysis of Variance (ANOVA): Oneway ANOVA

“Two”-Sample
Independent-Samples



Oneway ANOVA



Analysis of Variance (ANOVA): Oneway ANOVA

Malhotra (2010); Pallant (2016)

- ▶ Assumptions
 - ▶ Variables
 - ▶ Y_i = **intervally scaled**
 - ▶ X_i = **nominal (“Group”)**
 - ▶ Independent samples
 - ▶ For each “group” of X, Y has the same variance (*homogeneity of variance*) and is randomly sampled from a normal distribution
- ▶ Hypotheses
 - ▶ $H_0: M_1 = M_2 = M_3 = M_j$
- ▶ Model
 - ▶ $Y_i = a + b_1 * X_i + \text{ERROR}$

Analysis of Variance (ANOVA): Oneway ANOVA

Department Store Data (Malhotra, 2010), Promotion



	store	coupon	promotion	sales	clientel	var
1	1	1	1	10	9	
2	2	1	1	9	10	
3	3	1	1	10	8	
4	4	1	1	8	4	
5	5	1	1	9	6	
6	6	1	2	8	8	
7	7	1	2	8	8	
8	8	1	2	7	8	
9	9	1	2	9	8	
10	10	1	2	6	9	
11	11	1	3	5	8	
12	12	1	3	7	9	
13	13	1	3	6	6	
14	14	1	3	4	10	
15	15	1	3	5	4	
16	16	2	1	8	10	
17	17	2	1	9	6	

Factorial design with two factors with 3 (Promotion) and 2 (Coupon) levels (= 6 cells) with 5 stores randomly assigned ($n=6*5=30$).

Experimental Design

“X” Promotion

(1=high, 2=medium, 3= low)

“X” Coupon

(1=\$20 coupon, 2=No coupon)

“X” Clientel

(Affluence of clientele, 1-10)

“Y” Sales

(normalized, scale 1-10)

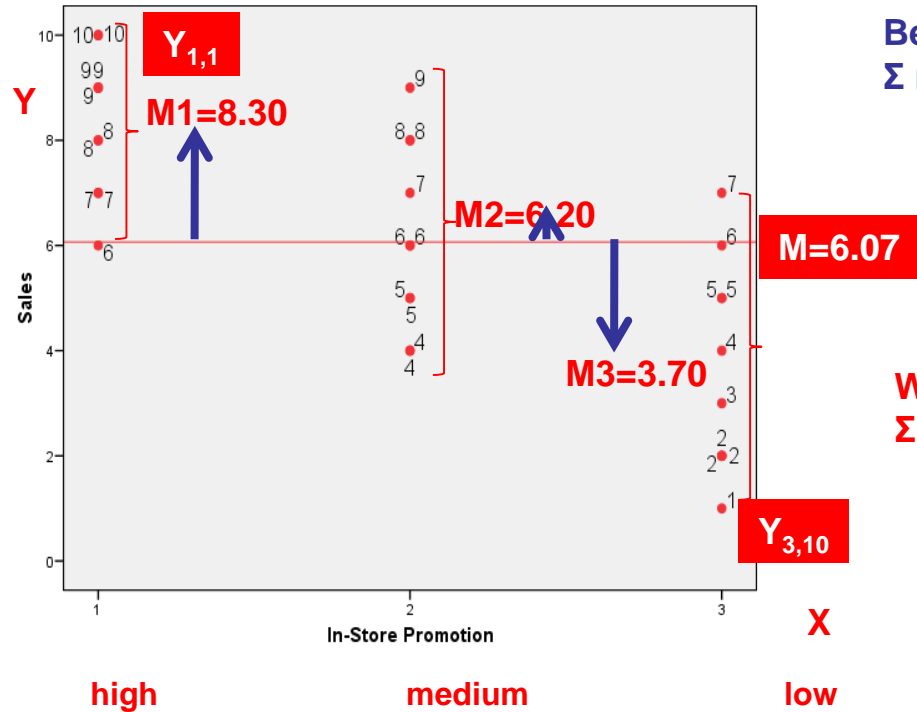
Two procedures:

- ▶ Analyze → Compare Means → Oneway ANOVA
- ▶ Analyze → General Linear Model → Univariate

Analysis of Variance (ANOVA): Oneway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)



Between SS:
 $\sum n_j \cdot (M_j - M)^2$ for all groups

Within SS (Error):
 $\sum (Y_{ij} - M_j)^2$ for all groups

Analysis of Variance (ANOVA): Oneway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

The screenshot shows the IBM SPSS Statistics Data Editor interface. The 'Analyze' menu is open, and the path 'Compare Means' > 'One-Way ANOVA...' is highlighted. A blue arrow points from the 'One-Way ANOVA...' option to the text 'Analyze → Compare Means → One-Way ANOVA' on the right. The data table below shows columns for 'store' and 'coupon'.

	store	coupon
1	1	1
2	2	1
3	3	1
4	4	1
5	5	1
6	6	1
7	7	1
8	8	1
9	9	1
10	10	1
11	11	1
12	12	1
13	13	1
14	14	1
15	15	1
16	16	2
17	17	2
18	18	2
19	19	2
20	20	2
21	21	2
22	22	2
23	23	2
24	24	2
25	25	2
26	26	2
27	27	2

Analyze → Compare Means
→
One-Way ANOVA

Analysis of Variance (ANOVA): Oneway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

The screenshot shows the 'One-Way ANOVA' dialog box. On the left, a list of variables includes 'store', 'coupon', and 'clientel'. The 'Dependent List' on the right contains 'sales'. Below it, the 'Factor' field contains 'promotion'. On the right side of the dialog, there are buttons for 'Contrasts...', 'Post Hoc...', 'Options...', and 'Bootstrap...'. At the bottom are 'OK', 'Paste', 'Reset', 'Cancel', and 'Help' buttons. A red arrow points from the text 'DV: SALES' to the 'sales' variable in the 'Dependent List'. A blue arrow points from the text 'IV: PROMOTION' to the 'promotion' variable in the 'Factor' field.

DV: SALES

IV: PROMOTION

Analysis of Variance (ANOVA): Oneway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

Descriptives and Assumptions

Descriptives

sales Sales

	N	M Mean	SD Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1 High	10	8.30	1.337	.423	7.34	9.26	6	10
2 Medium	10	6.20	1.751	.554	4.95	7.45	4	9
3 Low	10	3.70	2.003	.633	2.27	5.13	1	7
Total	30	6.07	2.532	.462	5.12	7.01	1	10

Test of Homogeneity of Variances

sales Sales

Levene Statistic	df1	df2	Sig.
1.353	2	27	.275

Homogeneity of Variance Assumption

H_0 : all variances are equal

Analysis of Variance (ANOVA): Oneway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

ANOVA Table

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	106.067	2	53.033	17.944	.000
Within Groups	79.800	27	2.956		
Total	185.867	29			

SS_x
 SS_E
 SS_y

Explained by PROMOTION
($\eta^2=106.067/185.867=0.571$)

$df_x=c-1 = 3-1 = 2$
 $df_E=N-c=30-3 =$

ANOVA

$106.067/2$

$F(2,27)=53.033/2.956$
 $=17.944$ ($p <$
 $0.001!!!$)

$79.800/27$

Error – Not Explained
by PROMOTION

Analysis of Variance (ANOVA): Oneway ANOVA

Department Store Data (Malhotra, 2010, Table 16.2)

- ▶ What to do if Levene's test is significant (i.e., homogeneity of variance is violated)?
 - ▶ Nothing! ANOVA is quite robust
 - ▶ Welch or Brown-Forsythe's F test (available in Oneway ANOVA)
 - ▶ Kruskal-Wallis test (nonparametric alternative!)

Analysis of Variance (ANOVA): Oneway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

Robust Tests of Equality of Means

sales Sales

	Statistic ^a	df1	df2	Sig.
Welch	18.090	2	17.470	.000
Brown-Forsythe	17.944	2	24.659	.000

a. Asymptotically F distributed.

Welch (1951)
 $F_W(2,17.470) = 18.090$,
 $p < 0.001$

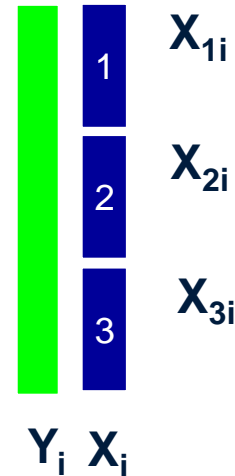
Brown and Forsyth (1974)
 $F_{BF}(2,24.659) = 17.944$, $p < 0.001$

Kruskal-Wallis Test

Department Store Data (Malhotra, 2010, Table 16.2)

- ▶ Assumptions
 - ▶ Three or more (random) samples
 - ▶ "Dependent" variable must be at least **ordinally** scaled and the "independent" variable must be **nominally** scaled
- ▶ Hypotheses
 - ▶ $H_0: \text{Median}_1 = \text{Median}_2 = \dots = \text{Median}_k$
- ▶ Extension of Wilcoxon-Mann-Whitney test for more than 2 samples

Kruskal-Wallis Test



Kruskal-Wallis Test

Department Store Data (Malhotra, 2010, Table 16.2)



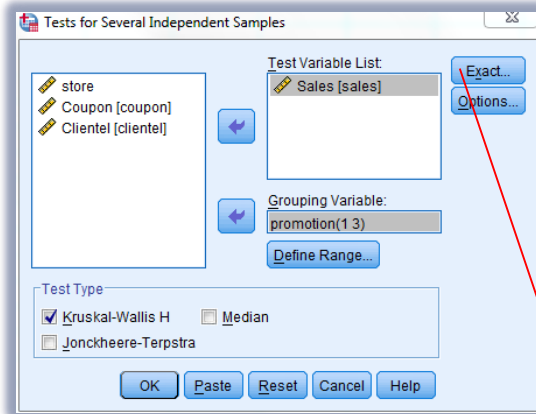
The screenshot shows the IBM SPSS Statistics Data Editor interface. The 'Analyze' menu is open, and the path 'Analyze -> Nonparametric Tests -> Legacy Dialogs -> K Independent Samples...' is highlighted. The data table has columns 'store' and 'coupon'.

	store	coupon
1	1	1
2	2	1
3	3	1
4	4	1
5	5	1
6	6	1
7	7	1
8	8	1
9	9	1
10	10	1
11	11	1
12	12	1
13	13	1
14	14	1
15	15	1
16	16	2
17	17	2
18	18	2
19	19	2
20	20	2
21	21	2
22	22	2
23	23	2
24	24	2
25	25	2
26	26	2
27	27	2

Analyze → Nonparametric Tests →
Legacy Dialogs → K Independent
Samples

Kruskal-Wallis Test

Department Store Data (Malhotra, 2010, Table 16.2)



EXACT!

		Ranks	
	In-Store Promotion	N	Mean Rank
Sales	High	10	23.50
	Medium	10	15.40
	Low	10	7.60
	Total	30	

Test Statistics ^{a, b}	
	Sales
Chi-Square	16.529
df	2
Asymp. Sig.	.000
Exact Sig.	.000
Point Probability	.000

$\chi^2(2) = 16.529$ ($p_2 < 0.001$)

ES

$\eta^2 = \chi^2 / (n-1)$

$\eta^2 = 16.529 / 29 = 0.570$

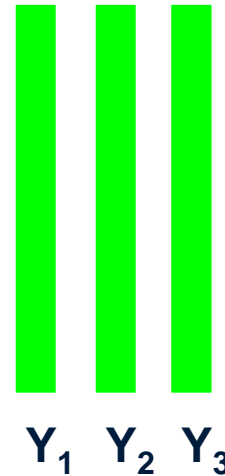
a. Kruskal Wallis Test
b. Grouping Variable:
In-Store Promotion

Friedman Test

Pallant (2016)

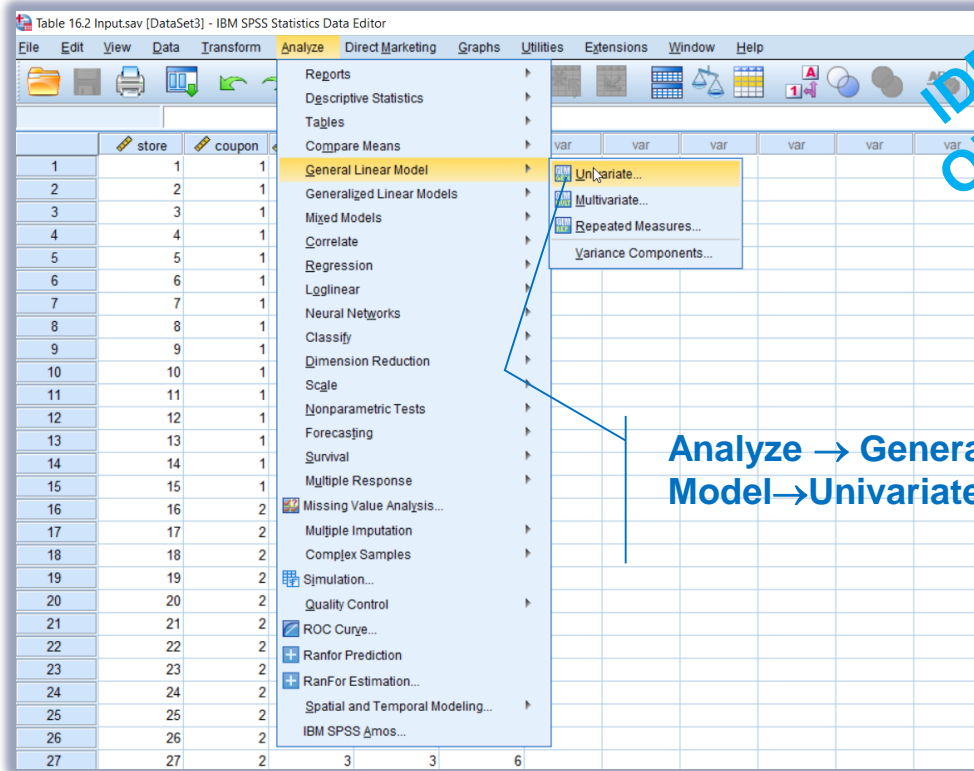
- ▶ Assumptions
 - ▶ Three or more (random) samples
 - ▶ Variables must be at least **ordinally** scaled
- ▶ Hypotheses
 - ▶ $H_0: \text{Median}_{Y_1} = \text{Median}_{Y_2} = \dots = \text{Median}_{Y_k}$
- ▶ Extension of Wilcoxon signed rank test for more than 2 samples

Friedman Test



GLM Univariate

Department Store Data (Malhotra, 2010, Table 16.2)



Analyze → General Linear Model → Univariate

GLM Univariate

Department Store Data (Malhotra, 2010, Table 16.2)



IDENTICAL TO
ONEWAY-ANOVA

Descriptives and Assumptions

Descriptive Statistics

Dependent Variable: sales

M

SD

N

promotion In-Store	Mean	Std. Deviation	N
1 High	8.30	1.337	10
2 Medium	6.20	1.751	10
3 Low	3.70	2.003	10
Total	6.07	2.532	30

Levene's Test of Equality of Error Variances^a

Dependent Variable: sales

F	df1	df2	Sig.
1.353	2	27	.275

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+promotion

Homogeneity of Variance Assumption

H₀: all variances are equal

GLM Univariate

Department Store Data (Malhotra, 2010, Table 16.2)



IDENTICAL TO
ONEWAY-ANOVA

ANOVA Table

Tests of Between-Subjects Effects

Dependent Variable: sales Sales

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	106.067 ^a	2	53.033	17.944	.000	.571
Intercept	1104.133	1	1104.133	373.579	.000	.933
promotion	106.067	2	53.033	17.944	.000	.571
Error	79.800	27	2.956			
Total	1290.000	30				
Corrected Total	185.867	29				

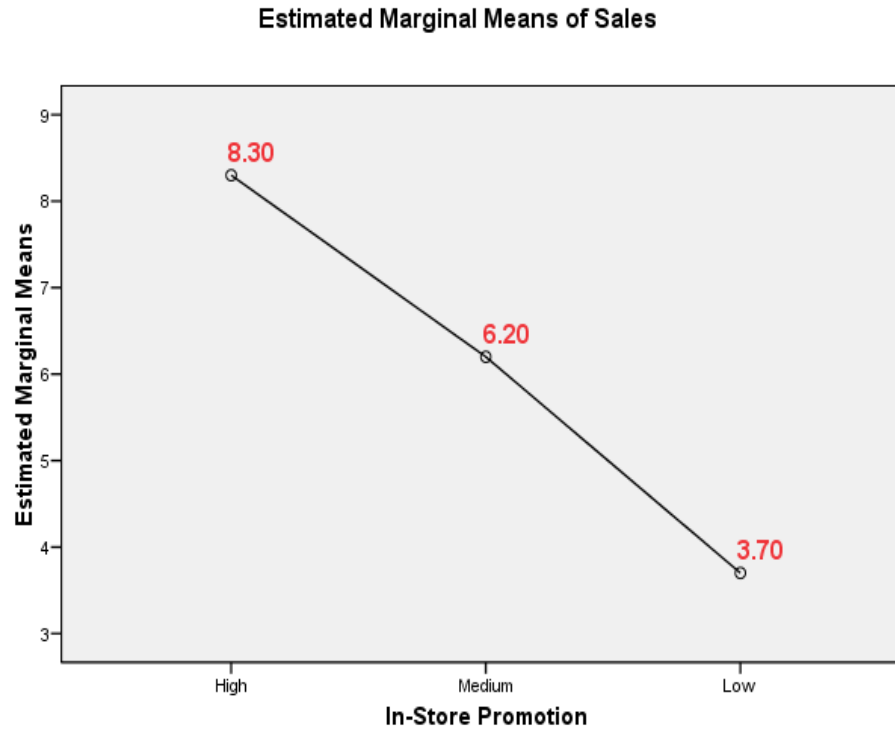
a. R Squared = .571 (Adjusted R Squared = .539)

GLM Univariate

Department Store Data (Malhotra, 2010, Table 16.2)



Mean Plot

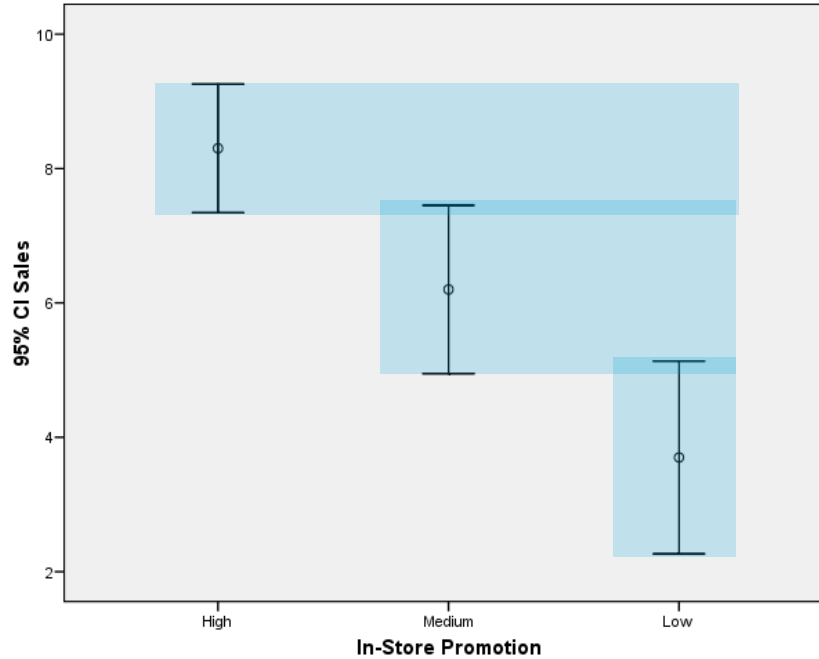


GLM Univariate

Department Store Data (Malhotra, 2010, Table 16.2)



Error Bar Graph



GLM Univariate: Post Hoc Tests



Department Store Data (Malhotra, 2010, Table 16.2)

Why do we need Post Hoc Multiple Comparison tests?

Multiple tests Inflate Familywise α !

$$FW\alpha = 1 - (1 - \alpha)^T$$

EXAMPLE

T=1, $\alpha=0.05$, $FW\alpha=0.05$

T=2, $\alpha=0.05$, $FW\alpha=0.10$

T=3, $\alpha=0.05$, $FW\alpha=0.14$

Sig. = p value!

p value for the individual tests

Multiple Comparisons

Dependent Variable: sales Sales

Tukey HSD

(I) In-Store Promotion	(J) In-Store Promotion	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1 High	2 Medium	2.10*	.769	.029	.19	4.01
	3 Low	4.60*	.769	.000	2.69	6.51
2 Medium	1 High	-2.10*	.769	.029	-4.01	-.19
	3 Low	2.50*	.769	.008	.59	4.41
3 Low	1 High	-4.60*	.769	.000	-6.51	-2.69
	2 Medium	-2.50*	.769	.008	-4.41	-.59

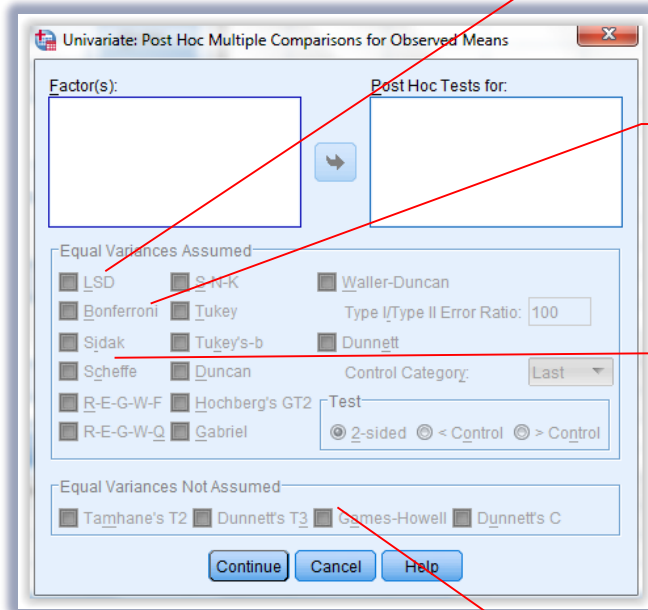
Based on observed means.

*. The mean difference is significant at the .05 level.

GLM Univariate: Post Hoc Tests

Department Store Data (Malhotra, 2010, Table 16.2)

Robust to nonnormality!



LSD

- Equal variances and sample sizes assumed
- Very liberal (t tests)!

Bonferroni, REGWQ and Tukey

- Equal variances and sample sizes assumed
- All pairwise comparisons

Scheffé

- Equal variances and sample sizes assumed
- All pairwise comparisons
- Very conservative!

Games-Howell

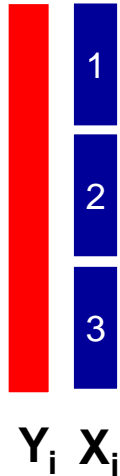
- Equal variances and sample sizes **NOT** assumed
- All pairwise comparisons



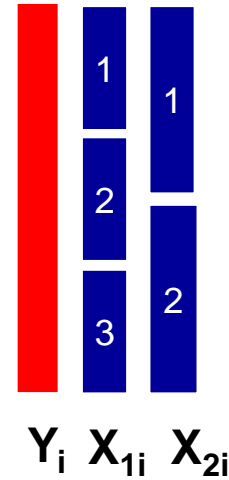
Analysis of Variance (ANOVA): Twoway ANOVA

Department Store Data (Malhotra, 2010, Table 16.2)

Oneway ANOVA



Twoway ANOVA



Analysis of Variance (ANOVA): Twoway ANOVA

Malhotra (2010); Pallant (2016)

- ▶ Assumptions
 - ▶ Variables
 - ▶ Y_i = **intervally scaled**
 - ▶ X_i = **nominal (“Group”)**
 - ▶ Independent samples
 - ▶ For each “group” of X, Y has the same variance (*homogeneity of variance*) and is randomly sampled from a normal distribution

Variables **Error term is normally distributed ($M=0$, $VAR=c$) and the error terms are uncorrelated**

- ▶ Model

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$$Y = a + b_1 * X_{1i} + b_2 * X_{2i} + b_3 * X_{1i} * X_{2i} + \text{ERROR}$$

Interaction Term

Analysis of Variance (ANOVA): Twoway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

	store	coupon	promotion	sales	clientel	var
1	1	1	1	10	9	
2	2	1	1	9	10	
3	3	1	1	10	8	
4	4	1	1	8	4	
5	5	1	1	9	6	
6	6	1	2	8	8	
7	7	1	2	8	4	
8	8	1	2	7	10	
9	9	1	2	9	6	
10	10	1	2	6	9	
11	11	1	3	5	8	
12	12	1	3	7	9	
13	13	1	3	6	6	
14	14	1	3	4	10	
15	15	1	3	5	4	
16	16	2	1	8	10	
17	17	2	1	9	6	
18	18	2	1	7	8	
19	19	2	1	7	4	
20	20	2	1	6	9	
21	21	2	2	4	6	
22	22	2	2	5	8	
23	23	2	2	5	10	
24	24	2	2	6	4	
25	25	2	2	4	9	

IVs: Coupon + Promotion

DV: Sales

Analysis of Variance (ANOVA): Twoway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

	store	coupon	promotion	sales	clientel	var
1	1	1	1	10	9	
2	2	1	1	9	10	
3	3	1	1	10	8	
4	4	1	1	8	4	
5	5	1	1	9	6	
6	6	1	2	8	8	
7	7	1	2	8	4	
8	8	1	2	7	10	
9	9	1	2	9	6	
10	10	1	2	6	9	
11	11	1	3	5	8	
12	12	1	3	7	9	
13	13	1	3	6	6	
14	14	1	3	4	10	
15	15	1	3	5	4	
16	16	2	1	8	10	
17	17	2	1	9	6	
18	18	2	1	7	8	
19	19	2	1	7	4	
20	20	2	1	6	9	
21	21	2	2	4	6	
22	22	2	2	5	8	
23	23	2	2	5	10	
24	24	2	2	6	4	
25	25	2	2	4	9	

Hypotheses:

Promotion (Main Effect)

$$H_0: M_L = M_M = M_H$$

Coupon (Main Effect)

$$H_0: M_{No} = M_{yes}$$

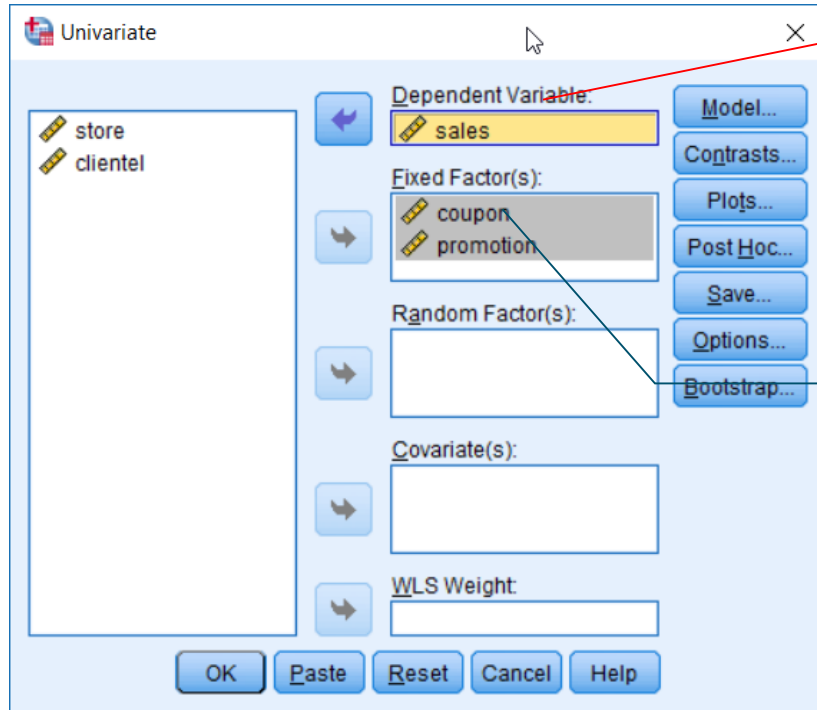
Promotion * Coupon (Interaction Effect)

$$H_0: \text{Promotion} * \text{Coupon} = 0$$

Analysis of Variance (ANOVA): Twoway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)



DV: Sales

**IVs: Coupon +
Promotion**

Analysis of Variance (ANOVA): Twoway ANOVA

Department Store Data (Malhotra, 2010, Table 16.2)



Three-way Interaction Plot

The image shows two overlapping dialog boxes from SPSS. The left dialog box is titled "Univariate" and has the following settings:

- Dependent Variable: Sales [sales]
- Fixed Factor(s): Coupon [coupon], In-Store Promotion [...]
- Random Factor(s):
- Covariate(s):
- WLS Weight:

The right dialog box is titled "Univariate: Profile Plots" and has the following settings:

- Factors: coupon, promotion
- Horizontal Axis: Factor 1
- Separate Lines: Factor 2
- Separate Plots: Factor 3
- Plots: promotion*coupon

A red arrow points from the "Plots..." button in the "Univariate" dialog to the "Factors:" list in the "Univariate: Profile Plots" dialog.

Analysis of Variance (ANOVA): Twoway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

The image shows two overlapping dialog boxes from SPSS. The 'Univariate' dialog box on the left has 'Sales [sales]' as the dependent variable and 'Coupon [coupon]' and 'In-Store Promotion [promotion]' as fixed factors. The 'Univariate: Options' dialog box on the right has '(OVERALL)', 'coupon', 'promotion', and 'coupon*promotion' listed under 'Factor(s) and Factor Interactions:'. The 'Display' section in the options dialog has several checked boxes: 'Descriptive statistics', 'Estimates of effect size', 'Observed power', 'Homogeneity tests', and 'Confidence interval adjustment' set to 'LSD(none)'. The significance level is .05 and confidence intervals are 95.0%.

store	Clientel [clientel]	Sales [sales]
1	9	
2	8	

Analysis of Variance (ANOVA): Twoway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)
Descriptives and Assumptions

Descriptive Statistics

Dependent Variable: sales Sales

		M	SD	N
promotion In-Store	coupon Coupon	Mean	Std. Deviation	N
1 High	1 Yes	9.20	.837	5
	2 No	7.40	1.140	5
	Total	8.30	1.337	10
2 Medium	1 Yes	7.60	1.140	5
	2 No	4.80	.837	5
	Total	6.20	1.751	10
3 Low	1 Yes	5.40	1.140	5
	2 No	2.00	.707	5
	Total	3.70	2.003	10
Total	1 Yes	7.40	1.882	15
	2 No	4.73	2.434	15
	Total	6.07	2.532	30

Levene's Test of Equality of Error Variances^a

Dependent Variable: sales Sales

F	df1	df2	Sig.
.689	5	24	.637

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept+promotion+coupon+promotion * coupon

**Homogeneity of
Variance
Assumption**

**H₀: all variances are
equal**

Analysis of Variance (ANOVA): Twoway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

ANOVA Table

Tests of Between-Subjects Effects

Dependent Variable: sales Sales

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	162.667 ^a	5	32.533	33.655	.000	.875
Intercept	1104.133	1	1104.133	1142.207	.000	.979
promotion	106.067	2	53.033	54.862	.000	.821
coupon	53.333	1	53.333	55.172	.000	.697
promotion * coupon	3.267	2	1.633	1.690	.206	.123
Error	23.200	24	.967			
Total	1200.000	30				
Corrected Total	185.867	29				

a. R Squared = .875 (Adjusted R Squared = .849)

**Total explained by model
Promotion, Coupon,
Promotion*Coupon**

p value

$\eta^2 = 0.571$
 $\eta^2 = 0.287$
 $\eta^2 = 0.018$

(partial) η^2

**ERROR
Not explained
by model**

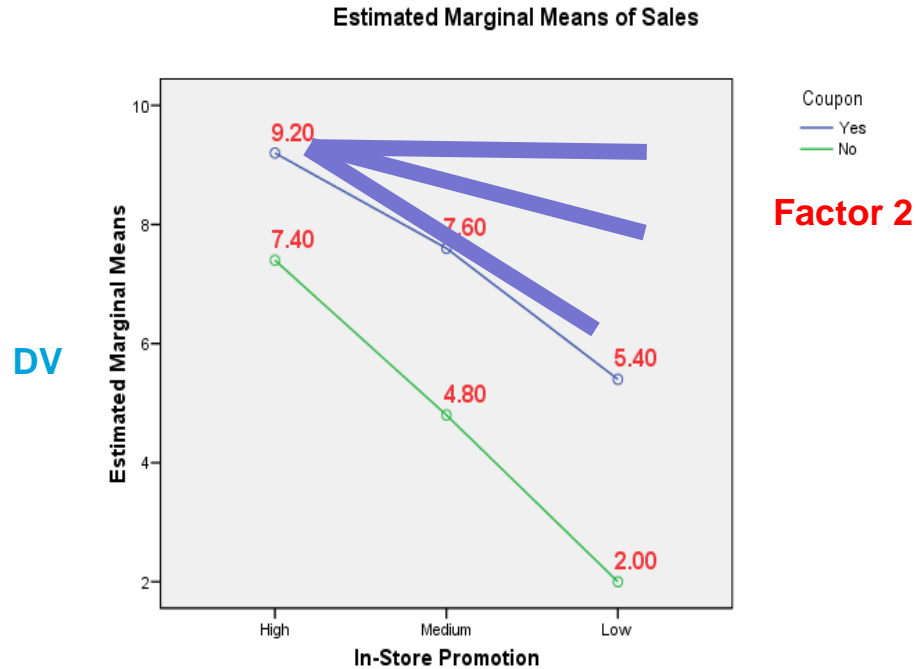
**Total including
ERROR**

Analysis of Variance (ANOVA): Twoway ANOVA



Department Store Data (Malhotra, 2010, Table 16.2)

Factor 3?



One-way ANOVA



```
7 # One way Anova (Completely Randomized Design)
8
9 attach(Table16.2)
10 # detach() # To detach data set ...
11
12 FIT.01<- aov(sales ~ factor(promotion))
13
14 library(lsr)
15 etaSquared(FIT.01, type=3, anova=TRUE)
16
17 summary(FIT.01)
18
19 # Type III === SPSS
20 # To get SPSS equivalent SPSS results
21
22 options(contrasts=c("contr.sum", "contr.poly"))
23
24 drop1(FIT.01,~, ,test="F")
25
26 # Install.packages("car") is R Base
27
28 library(car)
29
30 Anova(FIT.01, type=3)
31
32 pairwise.t.test(sales, factor(promotion), p.adj = "bonf")
33
34 TukeyHSD(FIT.01)
35
36
```

One-way ANOVA



Console

```
> summary(FIT.01)
      Df Sum Sq Mean Sq F value Pr(>F)
factor(promotion) 2  106.1      53   17.9 0.000011 ***
Residuals        27   79.8       3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> # Type III == SPSS
> # To get SPSS equivalent SPSS results
>
> options(contrasts=c("contr.sum", "contr.poly"))
>
> drop1(FIT.01,~, ,test="F")
Single term deletions

Model:
sales ~ factor(promotion)
      Df Sum of Sq  RSS  AIC F value  Pr(>F)
<none>                79.8 35.3
factor(promotion)  2     106 185.9 56.7   17.9 0.000011 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
```

One-way ANOVA



```
> pairwise.t.test(sales, factor(promotion), p.adj = "bonf")  
  
Pairwise comparisons using t tests with pooled SD  
  
data: sales and factor(promotion)  
  
  1      2  
2 0.033 -  
3 0.000007 0.009  
  
P value adjustment method: bonferroni  
>  
> TukeyHSD(FIT.01)  
Tukey multiple comparisons of means  
95% family-wise confidence level  
  
Fit: aov(formula = sales ~ factor(promotion))  
  
$`factor(promotion)`  
  diff      lwr      upr    p adj  
2-1 -2.1 -4.006 -0.1937 0.0286  
3-1 -4.6 -6.506 -2.6937 0.0000  
3-2 -2.5 -4.406 -0.5937 0.0084
```



Package **multcomp** for more options

One-way ANOVA



```
23
24
25 # Regression Approach
26
27 FIT.LM.01<-lm(sales ~ factor(promotion))
28 summary(FIT.LM.01)
29 anova(FIT.LM.01)
30 |
31 library(car)
32 Anova(FIT.LM.01, type=3)
33
34
35
```


One-way ANOVA



Console

```
> # Regression Approach
>
> FIT.LM.01<-lm(sales ~ factor(promotion))
> summary(FIT.LM.01)

Call:
lm(formula = sales ~ factor(promotion))

Residuals:
    Min       1Q   Median       3Q      Max
 -2.7   -1.3   -0.2    1.3    3.3

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)      8.3000     0.5437  15.267 8.39e-15 ***
factor(promotion)2 -2.1000     0.7688  -2.731  0.011 *
factor(promotion)3 -4.6000     0.7688  -5.983 2.21e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.719 on 27 degrees of freedom
Multiple R-squared:  0.5707, Adjusted R-squared:  0.5389
F-statistic: 17.94 on 2 and 27 DF,  p-value: 1.104e-05
```

One-way ANOVA



Console

```
> anova(FIT.LM.01)
Analysis of Variance Table

Response: sales
          Df Sum Sq Mean Sq F value    Pr(>F)
factor(promotion)  2  106.07   53.033   17.944 1.104e-05 ***
Residuals        27   79.80    2.956
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
```

One-way ANOVA



Console

```
.  
> library(car)  
> Anova(FIT.LM.01, type=3)  
Anova Table (Type III tests)  
  
Response: sales  


|                   | Sum Sq | Df | F value | Pr(>F)        |
|-------------------|--------|----|---------|---------------|
| (Intercept)       | 688.90 | 1  | 233.087 | 8.388e-15 *** |
| factor(promotion) | 106.07 | 2  | 17.944  | 1.104e-05 *** |
| Residuals         | 79.80  | 27 |         |               |

  
---  
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
> |
```

One-way ANOVA



```
38 # Descriptives and Diagnostics
39
40 print(model.tables(FIT.01,"means"),digits=3)
41
42 library(psych)
43 describeBy(sales, factor(promotion))
44
45 library(lawstat)
46 levene.test(sales,factor(promotion), location="mean")
47
48 library(car)
49 leveneTest(sales,factor(promotion), center="mean")
50
51 # bartlett.test(sales ~ factor(promotion))
52
```

One-way ANOVA



Console

```
> print(model.tables(FIT.01,"means"),digits=3)
Tables of means
Grand mean

6.066667

  factor(promotion)
factor(promotion)
  1  2  3
8 3 6.2 3.7
>
> library(psych)
> describeBy(sales, factor(promotion))
group: 1
  vars n mean  sd median trimmed  mad min max range skew kurtosis  se
1    1 10  8.3 1.34   8.5   8.38 1.48   6 10   4 -0.24   -1.4 0.42
-----
group: 2
  vars n mean  sd median trimmed  mad min max range skew kurtosis  se
1    1 10  6.2 1.75   6   6.12 2.22   4  9   5  0.17   -1.58 0.55
-----
group: 3
  vars n mean sd median trimmed  mad min max range skew kurtosis  se
1    1 10  3.7  2   3.5   3.62 2.22   1  7   6  0.22   -1.57 0.63
>
```

One-way ANOVA



Console

```
> library(lawstat)
> levene.test(sales, factor(promotion), location="mean")

classical Levene's test based on the absolute deviations from the mean (
none not applied because the location is not set to median )

data: sales
Test Statistic = 1.3532, p-value = 0.2754

>
> library(car)
> leveneTest(sales, factor(promotion), center="mean")
Levene's Test for Homogeneity of Variance (center = "mean")
      Df F value Pr(>F)
group  2  1.3532 0.2754
      27
```

One-way ANOVA

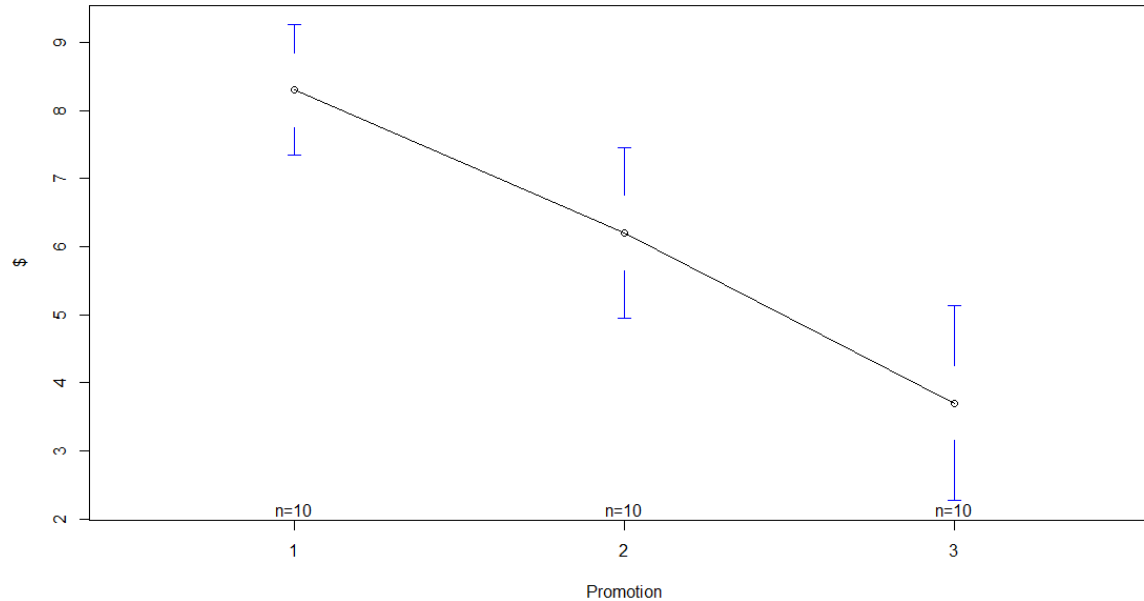


```
33  
34 # Plots  
35  
36 library(gplots)  
37  
38 plotmeans(sales ~ factor(promotion), xlab="Promotion",  
39           ylab="$", main="Mean Plot with 95% CI")  
40  
41
```

One-way ANOVA



Mean Plot with 95% CI



Welch Correction



```
> # welch correction
>
> oneway.test(sales ~ factor(promotion))

One-way analysis of means (not assuming equal variances)

data: sales and factor(promotion)
F = 18.0902, num df = 2.00, denom df = 17.47, p-value = 5.541e-05
```

Kruskal-Wallis Test



```
> # Kruskal-Wallis Test
>
> kruskal.test(sales ~ factor(promotion))

Kruskal-Wallis rank sum test

data: sales by factor(promotion)
Kruskal-Wallis chi-squared = 16.5292, df = 2, p-value = 0.0002575

>
> pairwise.wilcox.test(sales, factor(promotion), p.adj="bonf")

Pairwise comparisons using wilcoxon rank sum test

data: sales and factor(promotion)

  1      2
2 0.0431 -
3 0.0011 0.0541

P value adjustment method: bonferroni
```



Package coin for more options

Two-way ANOVA



```
> # Two-way ANOVA
>
> FIT.02<- aov(sales ~ factor(promotion)*factor(coupon))
>
> summary(FIT.02)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
factor(promotion)	2	106.07	53.03	54.86	1.12e-09	***
factor(coupon)	1	53.33	53.33	55.17	1.14e-07	***
factor(promotion):factor(coupon)	2	3.27	1.63	1.69	0.206	
Residuals	24	23.20	0.97			

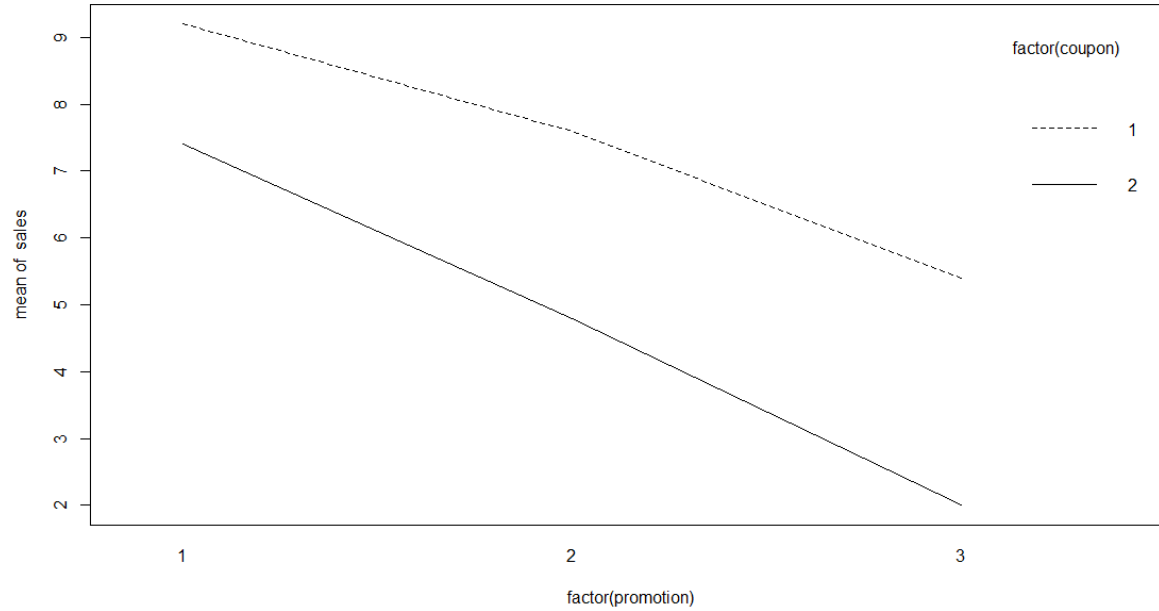
```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> print(model.tables(FIT.02,c("means"),digits=3))
Tables of means
Grand mean
6.06667

factor(promotion)
factor(promotion)
  1  2  3
8.3 6.2 3.7

factor(coupon)
factor(coupon)
  1  2
7.400 4.733

factor(promotion):factor(coupon)
factor(promotion) factor(coupon)
factor(promotion) 1  2
                  1  2
1  9.2 7.4
2  7.6 4.8
3  5.4 2.0
```

Two-way ANOVA



```

> FIT.03<- aov(sales ~ clientel + factor(promotion)*factor(coupon))
> summary(FIT.03)

```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
clientel	1	0.8	0.8	0.86	0.36
factor(promotion)	2	106.1	53.0	54.55	0.0000000019 ***
factor(coupon)	1	53.3	53.3	54.85	0.0000001568 ***
factor(promotion):factor(coupon)	2	3.3	1.6	1.68	0.21
Residuals	23	22.4	1.0		

```

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> library(car)
> Anova(FIT.03, type=3)
Anova Table (Type III tests)

Response: sales

```

	Sum Sq	Df	F value	Pr(>F)
(Intercept)	103.3	1	106.29	0.00000000043 ***
clientel	0.8	1	0.86	0.36
factor(promotion)	106.1	2	54.55	0.00000000186 ***
factor(coupon)	53.3	1	54.85	0.00000015684 ***
factor(promotion):factor(coupon)	3.3	2	1.68	0.21
Residuals	22.4	23		

```

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

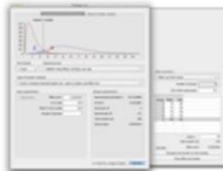
G*Power

<http://www.gpower.hhu.de/>

G*Power: Statistical Power Analyses for Windows and Mac

G*Power is a tool to compute statistical power analyses for many different t tests, F tests, χ^2 tests, z tests and some exact tests. G*Power can also be used to compute effect sizes and to display graphically the results of power analyses.

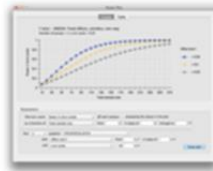
Screenshots (click to enlarge)



Main Window



Main Window (Table)



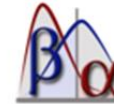
Power Plot



Power Plot (Table)

Register

Whenever we find a problem with G*Power we provide an update as quickly as we can. We will inform you about updates if you [click here](#) and add your e-mail address to our mailing list. We will only use your e-mail address to inform you about updates. We will not use your e-mail address for other purposes. We will not give your e-mail address to anyone else. You can withdraw your e-mail address from the mailing list at any time.



G*Power

<http://www.gpower.hhu.de/>

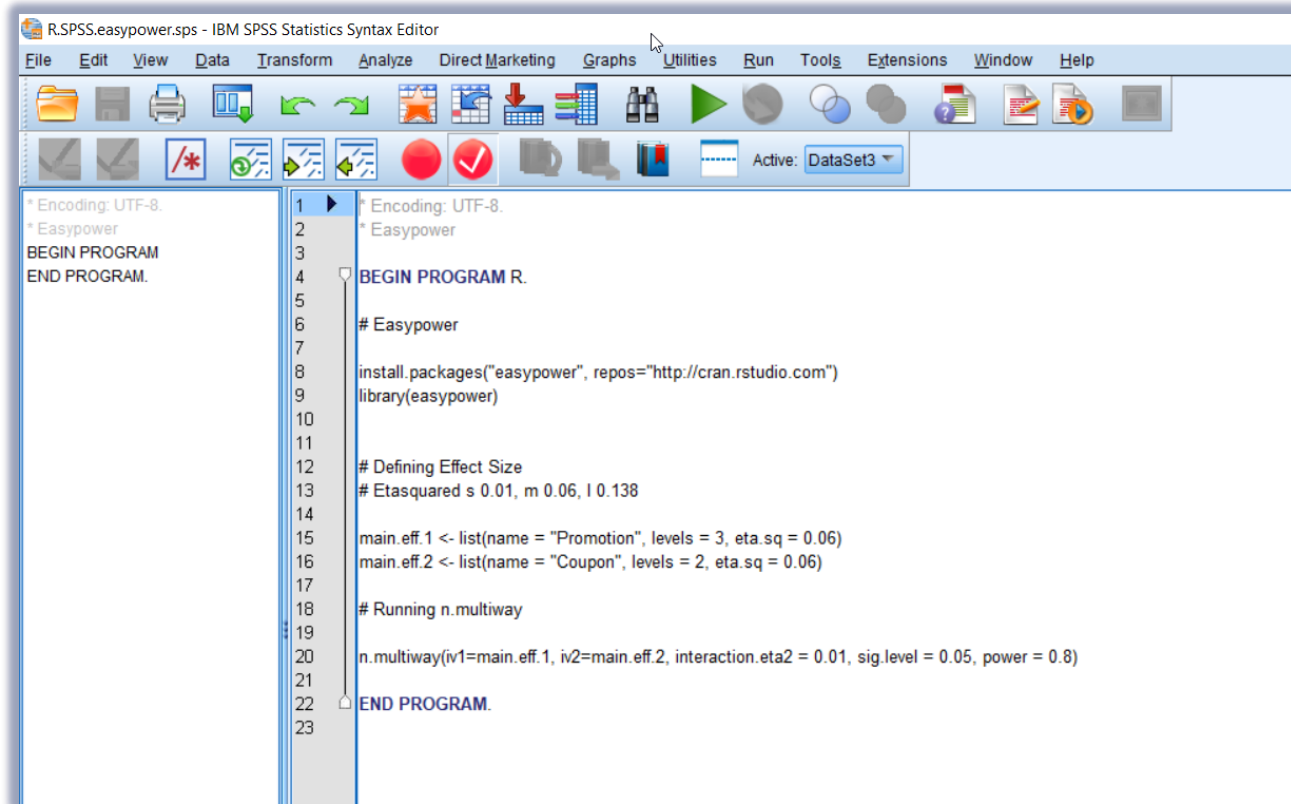
The screenshot shows the G*Power 3.1.9.2 interface. The main window displays a graph of power curves for different effects. The x-axis represents the F-value, and the y-axis represents power. A vertical green line indicates the critical F-value at 3.05678. The curves are labeled with β (blue) and α (red). Below the graph, the 'Input Parameters' section is filled out: Effect size f is 0.2526456, α err prob is 0.05, Power (1- β err prob) is 0.80, Numerator df is 2, and Number of groups is 6. The 'Output Parameters' section shows Noncentrality parameter λ as 9.8936189, Critical F as 3.0567787, Denominator df as 149, Total sample size as 155, and Actual power as 0.8026602. A red line connects the 'Numerator df' field to the list of effects on the right. In the bottom left, blue text lists: $n_{\text{Promotion}} = 155$, $n_{\text{Coupon}} = 125$, and $n_{\text{Interaction}} = 957$. The bottom right panel shows the 'Direct' method selected with a Partial η^2 of 0.06.

Numerator df:

- Main Effect Promotion (df=2 [3-1])
- Main Effects Coupon (df=1 [2-1])
- Interaction (df=2 [(2-1)*(3-1)])
- Number of groups: 6 (2*3)

f
 $\eta^2=0.06$

SPSS and R



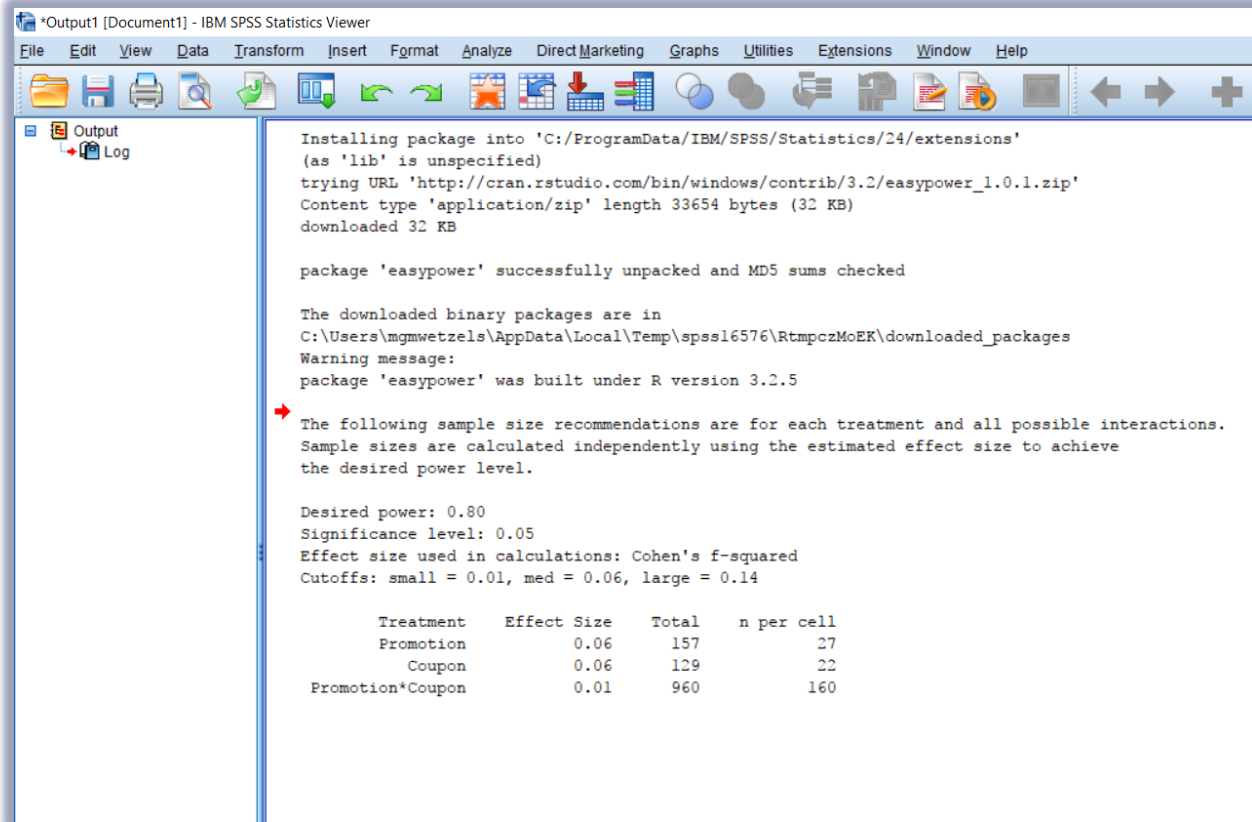
R.SPSS.easypower.sps - IBM SPSS Statistics Syntax Editor

File Edit View Data Transform Analyze Direct Marketing Graphs Utilities Run Tools Extensions Window Help

```
* Encoding: UTF-8.
* Easypower
BEGIN PROGRAM
END PROGRAM.

1 | * Encoding: UTF-8.
2 | * Easypower
3 |
4 | BEGIN PROGRAM R.
5 |
6 | # Easypower
7 |
8 | install.packages("easypower", repos="http://cran.rstudio.com")
9 | library(easypower)
10 |
11 |
12 | # Defining Effect Size
13 | # Etasquared s 0.01, m 0.06, l 0.138
14 |
15 | main.eff.1 <- list(name = "Promotion", levels = 3, eta.sq = 0.06)
16 | main.eff.2 <- list(name = "Coupon", levels = 2, eta.sq = 0.06)
17 |
18 | # Running n.multiway
19 |
20 | n.multiway(iv1=main.eff.1, iv2=main.eff.2, interaction.eta2 = 0.01, sig.level = 0.05, power = 0.8)
21 |
22 | END PROGRAM.
23 |
```


SPSS and R



*Output1 [Document1] - IBM SPSS Statistics Viewer

File Edit View Data Transform Insert Format Analyze Direct Marketing Graphs Utilities Extensions Window Help

Output
Log

```
Installing package into 'C:/ProgramData/IBM/SPSS/Statistics/24/extensions'  
(as 'lib' is unspecified)  
trying URL 'http://cran.rstudio.com/bin/windows/contrib/3.2/easypower_1.0.1.zip'  
Content type 'application/zip' length 33654 bytes (32 KB)  
downloaded 32 KB  
  
package 'easypower' successfully unpacked and MD5 sums checked  
  
The downloaded binary packages are in  
C:\Users\mgmwetzels\AppData\Local\Temp\spss16576\RtmpczMoEK\downloaded_packages  
Warning message:  
package 'easypower' was built under R version 3.2.5  
  
→ The following sample size recommendations are for each treatment and all possible interactions.  
Sample sizes are calculated independently using the estimated effect size to achieve  
the desired power level.  
  
Desired power: 0.80  
Significance level: 0.05  
Effect size used in calculations: Cohen's f-squared  
Cutoffs: small = 0.01, med = 0.06, large = 0.14  
  
Treatment   Effect Size   Total   n per cell  
Promotion   0.06         157    27  
Coupon      0.06         129    22  
Promotion*Coupon  0.01         960    160
```

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**Thank you for
Your Attention!**