

Emperical Methods for Marketing Research and Analytics Using

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About Me



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Using for Regression Analysis

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Course Outline

Session:

DAY 1

Session 1

Session 2

DAY 2

Session 3

Session 4

DAY 3

Session 5

Session 6

TOPIC:

INTRODUCING MULTIVARIATE ANALYSIS AND R

USING R FOR BASIC ANALYSIS

USING R FOR AN(C)OVA

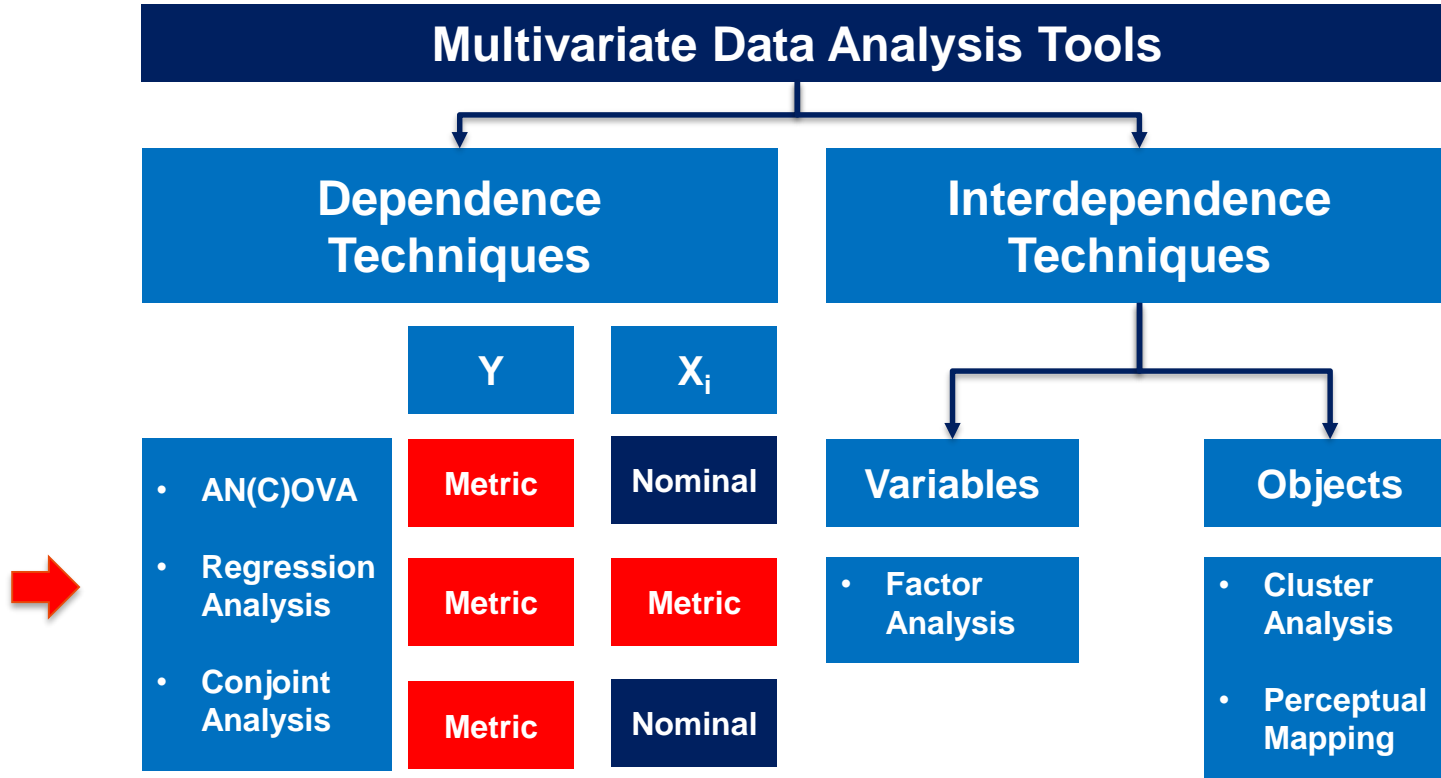
USING R FOR REGRESSION ANALYSIS

USING R FOR SCALING AND FACTOR ANALYSIS

USING R FOR SEM and PLS PATH MODELING

Positioning AN(C)OVA...

Hair et al. (2018); Malhotra (2010)



Correlation Analysis

Hair et al. (2018); Malhotra (2010)

- ▶ The **(Pearson) product moment correlation, ρ** , summarizes the strength of association between two **metric** (at least **interval**) variables [-1, 1].
- ▶ Assumptions
 - ▶ **Metric** (at least **interval**) variables
 - ▶ Independent, random sample
 - ▶ Linearity
 - ▶ (Bivariate) Normal distribution
 - ▶ Homoscedasticity
 - ▶ No causality!
 - ▶ Outliers!

Correlation Analysis

Hair et al. (2018); Malhotra (2010)

- ▶ Covariance $[-\infty, +\infty]$

$$\text{COV}_{XY} = \sum_{i=1}^n \frac{(X_i - M_X) * (Y_i - M_Y)}{n - 1}$$

- ▶ (Pearson Product-Moment) Correlation Coefficient $[-1, 1]$

$$r_{XY} = \frac{\text{COV}_{XY}}{S_X S_Y}$$

Correlation Analysis

Hair et al. (2018); Malhotra (2010)

▶ Hypotheses

- ▶ H_0 : correlation coefficient (ρ) = 0
- ▶ H_1 : correlation coefficient (ρ) \neq 0

▶ Test statistic

$$t = r * \sqrt{\frac{n-2}{1-r^2}}$$

$$df = n - 2$$

▶ Effect Size (r)

0.10 0.02

0.30 0.13

0.50 0.26

Effect Size (R^2)

small

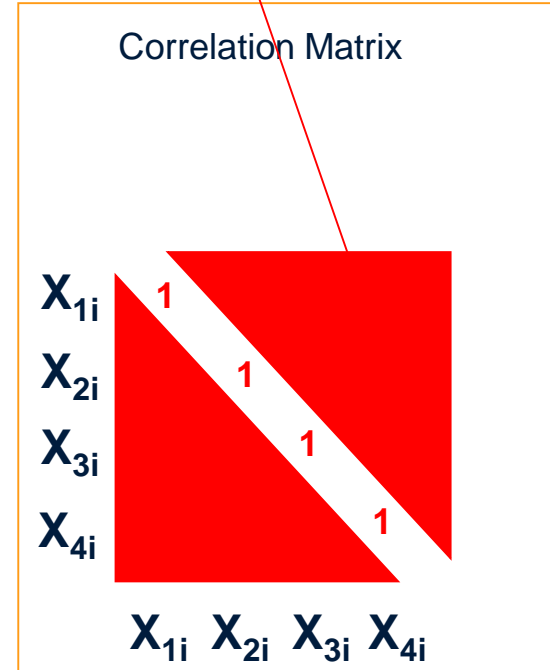
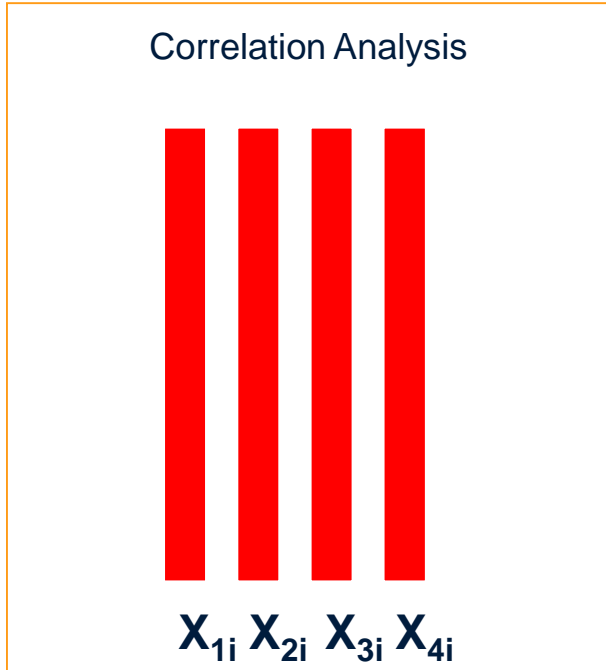
medium

large

Correlation Analysis

Hair et al. (2018); Malhotra (2010)

bivariate



SPSS Application

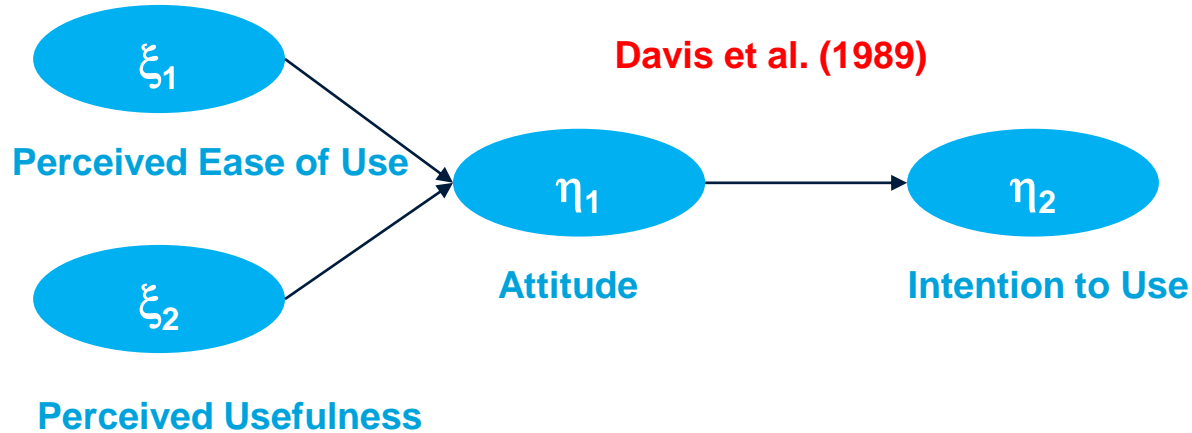
TAM Data



- ▶ Data Set *Data-01.sav* (n=439)
- ▶ Constructs and Items
 - ▶ Perceived Ease of Use (PEU, Q1-Q3)
 - ▶ Perceived Usefulness (PU, Q4-Q6)
 - ▶ Attitude (ATT, Q7-Q9)
 - ▶ Intention (INT, Q10-Q12)
- ▶ Reliability Analysis (Coefficient α)
 - ▶ PEU 0.93
 - ▶ PU 0.92
 - ▶ ATT 0.92
 - ▶ INT 0.87

SPSS Application

TAM Data



Construct	Items
PEU	Q1, Q2, Q3
PU	Q4, Q5, Q6
ATT	Q7, Q8, Q9
INT	Q10, Q11, Q12

Davis et al. (1989)

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USER ACCEPTANCE OF COMPUTER TECHNOLOGY: A COMPARISON OF TWO THEORETICAL MODELS*

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Computer systems cannot improve organizational performance if they aren't used. Unfortunately, resistance to end-user systems by managers and professionals is a widespread problem. To better predict, explain, and increase user acceptance, we need to better understand why people accept or reject computers. This research addresses the ability to predict peoples' computer acceptance from a measure of their intentions, and the ability to explain their intentions in terms of their attitudes, subjective norms, perceived usefulness, perceived ease of use, and related variables. In a longitudinal study of 107 users, intentions to use a specific system, measured after a one-hour introduction to the system, were correlated 0.35 with system use 14 weeks later. The intention-usage correlation was 0.63 at the end of this time period. Perceived usefulness strongly influenced peoples' intentions, explaining more than half of the variance in intentions at the end of 14 weeks. Perceived ease of use had a small but significant effect on intentions as well, although this effect subsided over time. Attitudes only partially mediated the effects of these beliefs on intentions. Subjective norms had no effect on intentions. These results suggest the possibility of simple but powerful models of the determinants of user acceptance, with practical value for evaluating systems and guiding managerial interventions aimed at reducing the problem of underutilized computer technology.

(INFORMATION TECHNOLOGY; USER ACCEPTANCE; INTENTION MODELS)

Conducting Regression Analysis

Hair et al. (2018); Malhotra (2010)

1. Regression Model

2. Regression Assumptions

3. Estimation Procedure

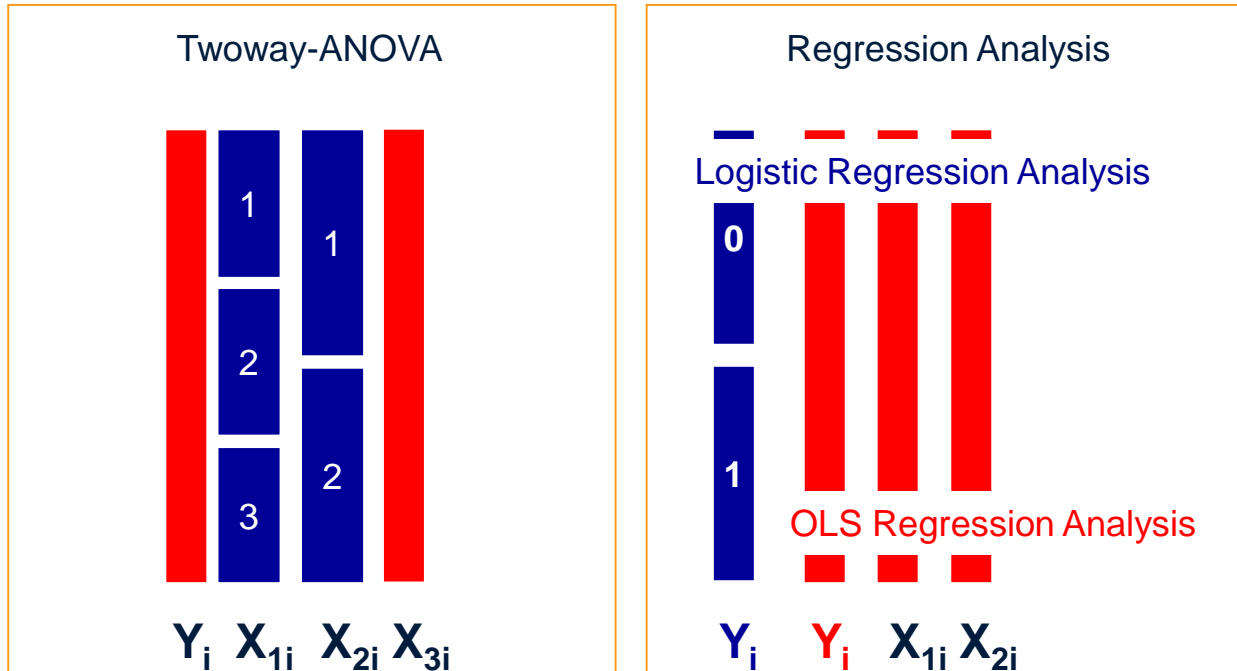
4. Significance Testing/Model Fit

5. Assessing Assumptions

6. Advanced Topics

1. Regression Model

GLM: General Linear Model



2. Regression Assumptions

Hair et al. (2018); Malhotra (2010)

▶ Assumptions

- ▶ Independent and dependent variables are **metric** (at least **interval**)
- ▶ For each combination of X_i values there is a normally distributed subpopulation of Y_i values (with equal variances)
- ▶ All Y_i values are independent
- ▶ The ERROR term has a mean of 0 (with equal variances)
- ▶ There does not exist any exact linear relationship between the X_i 's (assumption of **no multicollinearity**)
- ▶ Model is correctly specified (no omitted an/or irrelevant variables)

3. Estimation Procedure

Hair et al. (2018); Malhotra (2010)

▶ Model

- ▶ $Y_i = a + b_1 * X_{1i} + b_2 * X_{2i} + \text{ERROR}_i$

▶ Estimation of Parameters

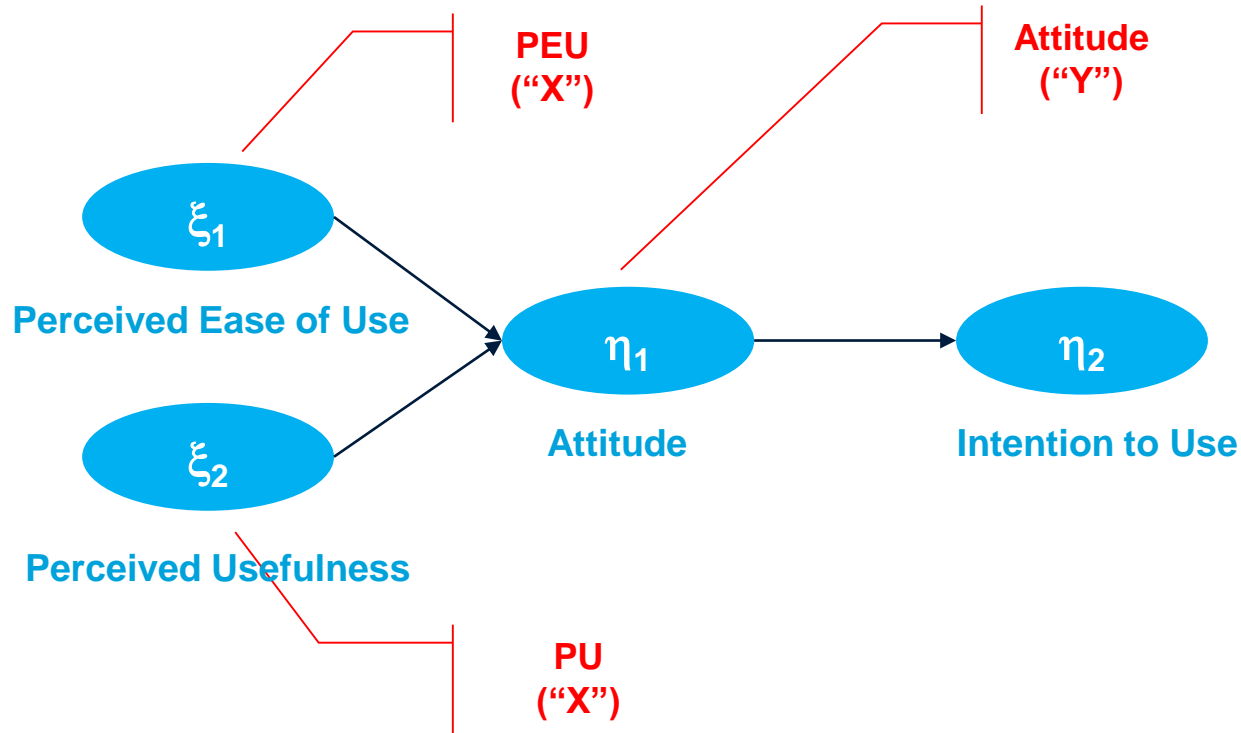
- ▶ Ordinary Least Squares Estimation
- ▶ Minimize $\Sigma (\text{Predicted } Y - \text{Observed } Y)^2$

▶ Fit Measures

- ▶ R^2 (Adjusted R^2)

Regression Analysis

TAM Data



OLS Principle

TAM Data



PEU	PU	ATT	INT	PRE_1	RES_1
3.00	4.33	4.33	4.00	3.70533	.62800
3.00	2.33	2.00	4.33	3.34763	-1.34763
2.00	3.33	2.67	4.67	3.17929	-.51262
6.00	4.67	7.00	5.33	4.80653	2.19347
3.33	2.33	2.67	3.33	3.46336	-.79670
1.00	3.33	2.00	3.00	2.83210	-.83210
4.33	3.33	4.00	4.67	3.98941	.01059
3.00	3.33	3.33	2.67	3.52648	-.19315
4.67	2.33	4.67	4.67	3.92629	.74038
4.67	3.00	3.67	4.33	4.04559	.37886
5.33	3.00	3.33	5.00	4.27698	-.94365
5.33	4.33	4.67	4.00	4.51545	.15122
4.00	3.67	5.00	3.67	3.93329	1.06671
4.33	4.67	5.33	4.00	4.22787	1.10546
4.33	4.00	4.67	4.67	4.10864	.55803

$$\text{PRED_Attitude} = 1.889 + 0.347*PEU + 0.179*PU$$

4. Significance Testing/Model Fit

Hair et al. (2018); Malhotra (2010)

- ▶ **Overall Significance of a Multiple Regression Model (F test)**

$$H_0: R^2 = 0$$

$$H_1: R^2 > 0$$

$$DF_1 = 1, DF_2 = n - k - 1$$

- ▶ **Significance of the Individual Regression Coefficients (t test)**

$$H_0: b_i = 0$$

$$H_1: b_i \neq 0$$

$$DF = n - k - 1$$

Regression Analysis

TAM Data



The screenshot shows the IBM SPSS Statistics Data Editor interface. The 'Analyze' menu is open, and the 'Regression' option is selected. The data editor displays a dataset with columns labeled q1 through q10 and rows numbered 1 through 24. The 'Regression' submenu is open, showing options such as 'Automatic Linear Modeling...', 'Linear...', 'Curve Estimation...', 'Partial Least Squares...', 'Binary Logistic...', 'Multinomial Logistic...', 'Ordinal...', 'Probit...', 'Nonlinear...', 'Weight Estimation...', '2-Stage Least Squares...', and 'Optimal Scaling (CATREG)...'. A mouse cursor is hovering over the 'Linear...' option.

	q1	q2	q6	q7	q8	q9	q10					
1	3	3	4	5	5	4	4	4				
2	3	3	2	3	3	1	2	5				
3	2	2	3	4	3	2	3	5				
4	6	6	5	5	7	7	7	6				
5	3	4				2	3	5				
6	1	1				1	3	3				
7	4	5				3	4	5				
8	3	3				3	3	4				
9	5	5				4	5	5				
10	5	4				3	4	5				
11	5	6				3	3	5				
12	5	5				5	4	4				
13	4	4				5	5	4				
14	5	4				5	5	5				
15	4	5				4	5	5				
16	4	5				2	3	2				
17	2	2				3	4	5				
18	3	3				3	4	5				
19	4	4				5	5	5				
20	3	4				5	5	4				
21	2	2				2	3	4	4			
22	4	4				4	4	6	5			
23	3	3				2	4	4	3	4	4	
24	2	3				3	3	4	4	3	3	2

Regression Analysis

TAM Data



The image shows two overlapping SPSS dialog boxes. The 'Linear Regression' dialog on the left has 'ATT' as the dependent variable and 'PEU' and 'PU' as independent variables. The 'Linear Regression: Statistics' dialog on the right has 'Estimates', 'Model fit', 'Descriptives', 'Part and partial correlations', and 'Collinearity diagnostics' checked. The 'Residuals' section has 'Outliers outside' set to 3 standard deviations. Below the dialog boxes is a data table.

3	3	5	3
4	4	4	3
5	6	5	4
3	4	4	3
3	3	2	3

Regression Analysis

TAM Data



The image shows two dialog boxes from the SPSS software interface. The left dialog box is titled "Linear Regression" and the right one is "Linear Regression: Plots".

Linear Regression Dialog:

- Dependent:** ATT
- Independent(s):** PEU, PU
- Method:** Enter
- Selection Variable:** (empty)
- Case Labels:** (empty)
- WLS Weight:** (empty)

Linear Regression: Plots Dialog:

- DEPENDENT:** *ZPRED, *ZRESID, *DRESID, *ADJPRED, *SRESID, *SDRESID
- Scatter 1 of 1:** Y: *ZPRED, X: *ZRESID
- Standardized Residual Plots:** Histogram, Normal probability plot
- Produce all partial plots

Buttons in both dialog boxes include: Previous, Next, Statistics..., Plots..., Save..., Options..., Style..., Bootstrap..., OK, Paste, Reset, Cancel, Help, Continue, Cancel, Help.

3	3	5	3	3
4	4	4	3	3
5	6	5	4	5
3	4	4	3	3
3	3	2	3	3

Regression Analysis

TAM Data



Model Summary^b

Model Fit

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.455 ^a	.207	.204	.79880

a. Predictors: (Constant), PU, PEU

b. Dependent Variable: ATT

$$R^2 = 72.751 / 350.958 = 0.207$$

F test
(p value)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	72.751	2	36.376	57.007	.000 ^b
	Residual	278.207	436	.638		
	Total	350.958	438			

Variance explained
Error variance
Total variance

a. Dependent Variable: ATT

b. Predictors: (Constant), PU, PEU

Regression Analysis

TAM Data



Coefficients
B, Beta

t test
(p value)

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
		B	Std. Error	Beta			Zero-order	Partial	Part	Tolerance	VIF
1	(Constant)	1.889	.185		10.207	.000					
	PEU	.347	.045	.352	7.631	.000	.422	.343	.325	.855	1.170
	PU	.179	.045	.185	4.001	.000	.318	.188	.171	.855	1.170

a. Dependent Variable: ATT

Correlations

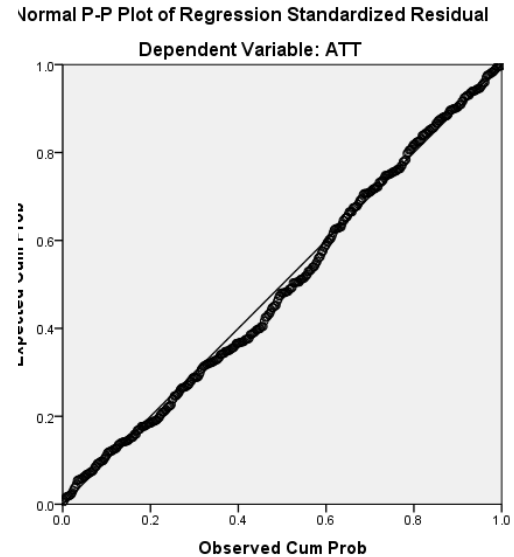
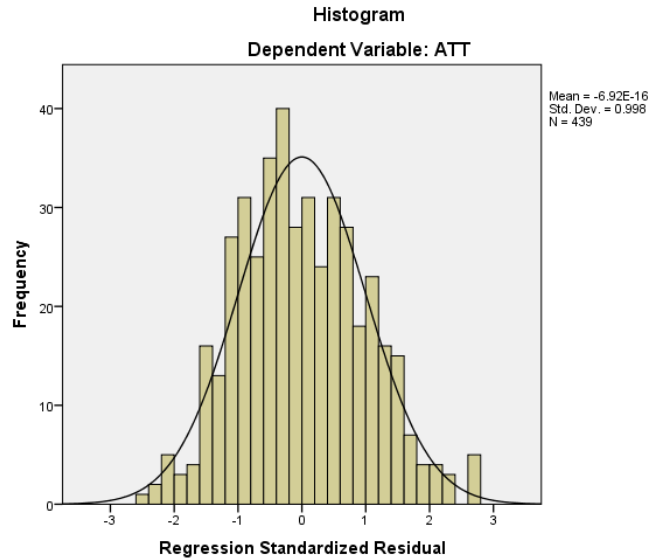
TOL = $1 - R^2$ (All other X)
VIF = $1 / \text{TOL}$
VIF > 5, Multicollinearity!

5. Assessing Assumptions

Hair et al. (2018); Malhotra (2010)



Normality Assumption

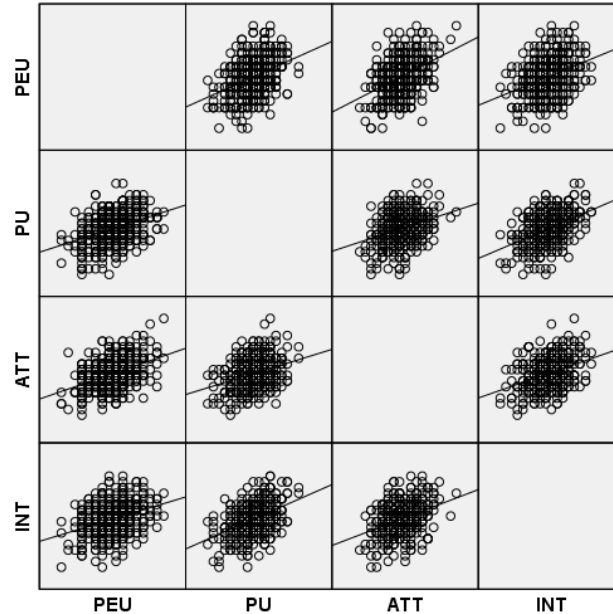


5. Assessing Assumptions

Hair et al. (2018); Malhotra (2010)



Linearity

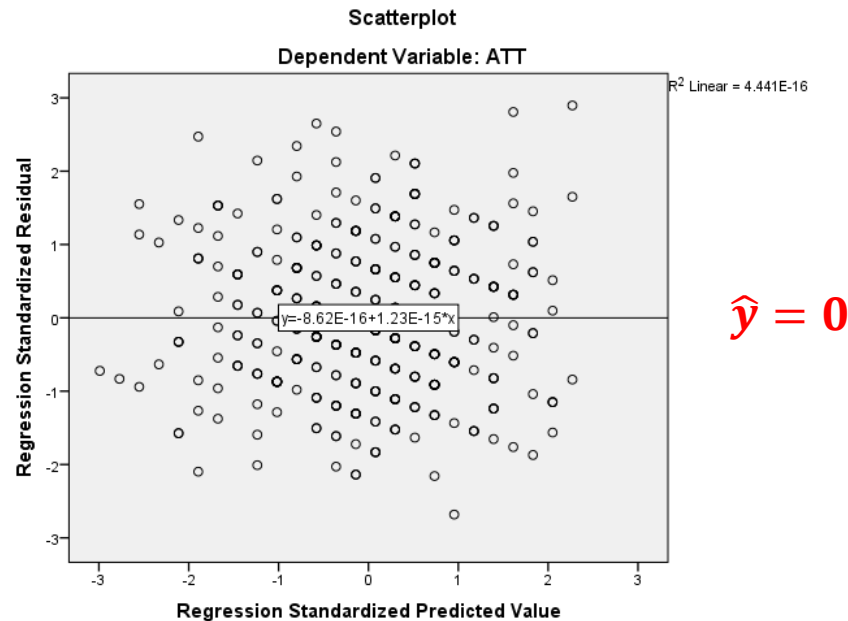


5. Assessing Assumptions

Hair et al. (2018); Malhotra (2010)

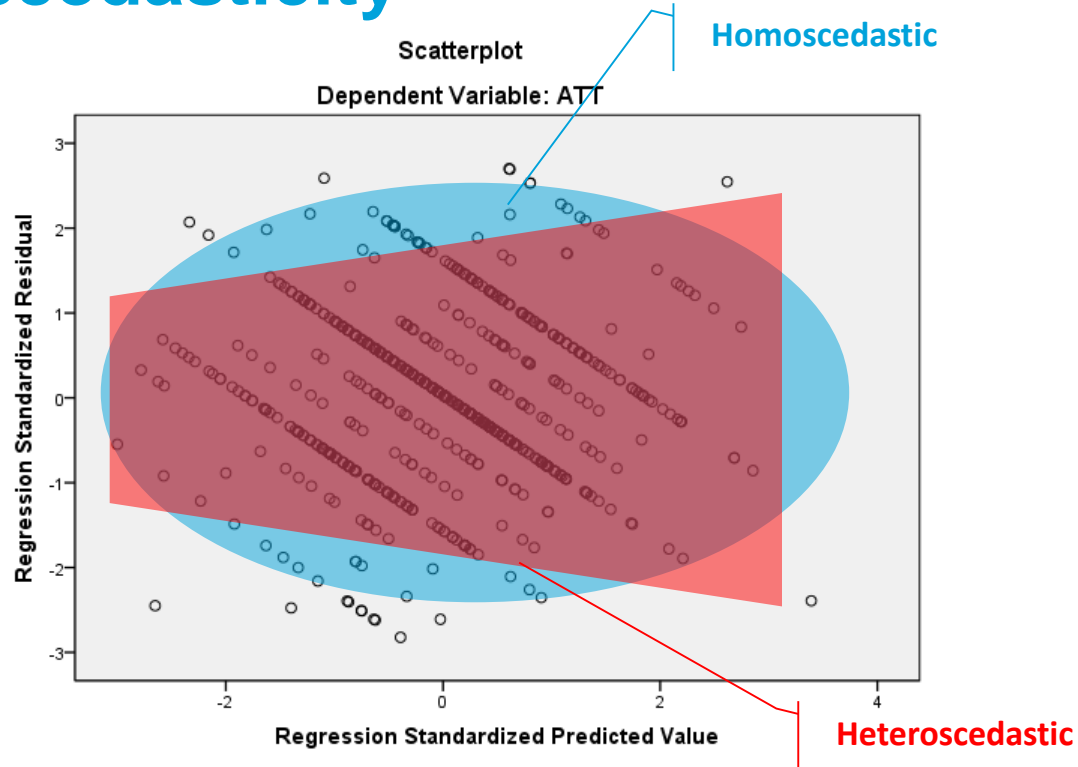


Outlier Detection: Residual Analysis



Regression Revisited...

Homoscedasticity



Correlation Analysis



```
1 # correlation
2
3 cor(Data.01.PPT)
4
5 cor(Data.01.PPT[,13:16])
6
7 library(Hmisc)
8
9 rcorr(as.matrix(Data.01.PPT[,13:16]))
10
11 library(corrgram)
12
13 corrgram(Data.01.PPT)
14
```

Correlation Analysis



```
cor(quantiles)
      q1      q2      q3      q4      q5      q6
j1  1.0000000  0.8097975  0.8138406  0.2930891  0.3428762  0.3247076
j2  0.8097975  1.0000000  0.8166044  0.3060620  0.3243314  0.3472221
j3  0.8138406  0.8166044  1.0000000  0.3239042  0.3529852  0.3680337
j4  0.2930891  0.3060620  0.3239042  1.0000000  0.8009082  0.7970698
j5  0.3428762  0.3243314  0.3529852  0.8009082  1.0000000  0.8116305
j6  0.3247076  0.3472221  0.3680337  0.7970698  0.8116305  1.0000000
j7  0.3972089  0.3690113  0.3736317  0.2921024  0.2979363  0.2478376
j8  0.3653399  0.3518016  0.3532527  0.2462745  0.2597337  0.2426744
j9  0.3904451  0.3464314  0.3557382  0.3006136  0.3236039  0.2686053
j10 0.2589751  0.2588477  0.2689082  0.3145539  0.3231403  0.3321825
j11 0.3152825  0.3161738  0.3338670  0.4183882  0.3839118  0.3861757
j12 0.3138578  0.3141935  0.3201764  0.4015143  0.3765244  0.3707145
PEU 0.9328149  0.9346877  0.9396350  0.3291739  0.3636331  0.3709495
PU  0.3439444  0.3498104  0.3739597  0.9272086  0.9356141  0.9334255
ATT 0.4139689  0.3830650  0.3886151  0.3014059  0.3165713  0.2725093
INT 0.3257088  0.3260625  0.3384005  0.4142796  0.3981133  0.4008660
      q7      q8      q9      q10     q11     q12
j1  0.3972089  0.3653399  0.3904451  0.2589751  0.3152825  0.3138578
j2  0.3690113  0.3518016  0.3464314  0.2588477  0.3161738  0.3141935
j3  0.3736317  0.3532527  0.3557382  0.2689082  0.3338670  0.3201764
j4  0.2921024  0.2462745  0.3006136  0.3145539  0.4183882  0.4015143
j5  0.2979363  0.2597337  0.3236039  0.3231403  0.3839118  0.3765244
j6  0.2478376  0.2426744  0.2686053  0.3321825  0.3861757  0.3707145
j7  1.0000000  0.8026353  0.8059340  0.2570709  0.3164261  0.3369713
j8  0.8026353  1.0000000  0.7732401  0.2332786  0.2897454  0.3177918
j9  0.8059340  0.7732401  1.0000000  0.2726496  0.3677583  0.3916562
j10 0.2570709  0.2332786  0.2726496  1.0000000  0.6810155  0.6744540
j11 0.3164261  0.2897454  0.3677583  0.6810155  1.0000000  0.7859497
j12 0.3369713  0.3177918  0.3916562  0.6744540  0.7859497  1.0000000
PEU 0.4058197  0.3811711  0.3889120  0.2803795  0.3441139  0.3378509
PU  0.2995601  0.2678096  0.3193535  0.3469233  0.4246864  0.4105496
ATT 0.9369643  0.9230840  0.9262602  0.2740098  0.3498089  0.3757832
INT 0.3257088  0.3260625  0.3384005  0.4142796  0.3981133  0.4008660
```

	PEU	PU	ATT	INT
q1	0.9328149	0.3439444	0.4139689	0.3257088
q2	0.9346877	0.3498104	0.3830650	0.3260625
q3	0.9396350	0.3739597	0.3886151	0.3384005
q4	0.3291739	0.9272086	0.3014059	0.4142796
q5	0.3636331	0.9356141	0.3165713	0.3981133
q6	0.3709495	0.9334255	0.2725093	0.4008660
q7	0.4058197	0.2995601	0.9369643	0.3331362
q8	0.3811711	0.2678096	0.9230840	0.3072647
q9	0.3889120	0.3193535	0.9262602	0.3756716
q10	0.2803795	0.3469233	0.2740098	0.8965442
q11	0.3441139	0.4246864	0.3498089	0.8996885
q12	0.3378509	0.4105496	0.3757832	0.8989238
PEU	1.0000000	0.3807257	0.4221206	0.3528830
PU	0.3807257	1.0000000	0.3184720	0.4337164
ATT	0.4221206	0.3184720	1.0000000	0.3649072
INT	0.3528830	0.4337164	0.3649072	1.0000000

Correlation Analysis



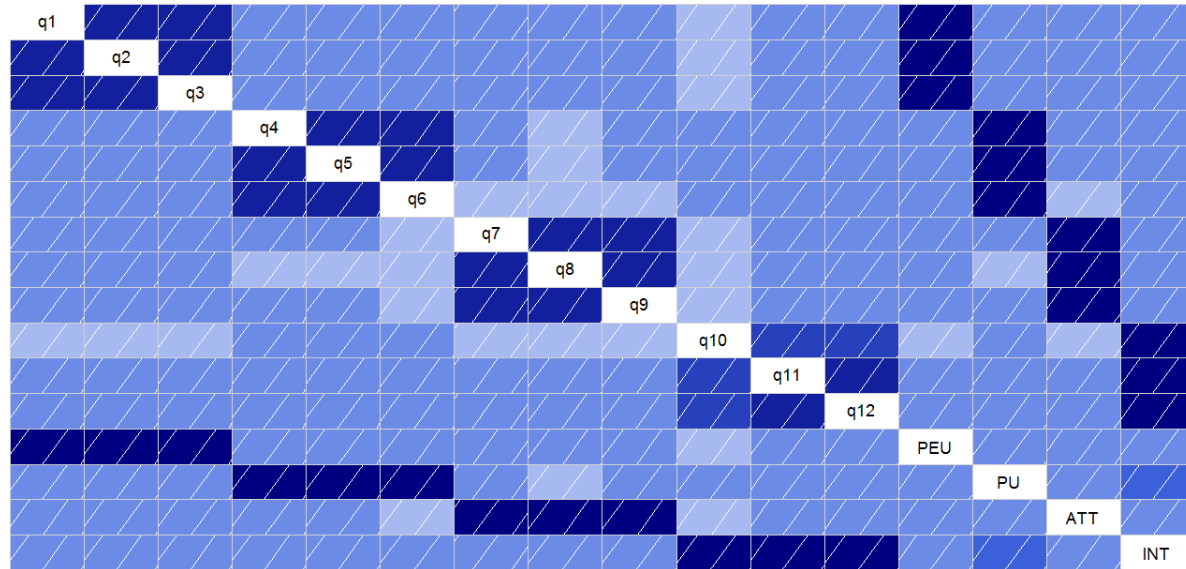
Console

```
> cor(Data.01.PPT[,13:16])
      PEU      PU      ATT      INT
PEU 1.000000 0.3807257 0.4221206 0.3528830
PU 0.3807257 1.0000000 0.3184720 0.4337164
ATT 0.4221206 0.3184720 1.0000000 0.3649072
INT 0.3528830 0.4337164 0.3649072 1.0000000
>
> library(Hmisc)
>
> rcorr(as.matrix(Data.01.PPT[,13:16]))
      PEU  PU  ATT  INT
PEU 1.00 0.38 0.42 0.35
PU 0.38 1.00 0.32 0.43
ATT 0.42 0.32 1.00 0.36
INT 0.35 0.43 0.36 1.00

n= 439

P
      PEU  PU  ATT  INT
PEU      0  0  0
PU      0  0  0
ATT      0  0  0
INT      0  0  0
```

Correlation Analysis



Regression Analysis



```
15 # Regression Analysis
16
17 library(car)
18
19 R00<-lm(ATT ~ PEU + PU, data=Data.01.PPT)
20
21 summary(R00)
22
23 vif(R00)
24
25 plot(R00)
26
```

Regression Analysis



Console

```
Call:
lm(formula = ATT ~ PEU + PU, data = Data.01.PPT)

Residuals:
    Min       1Q   Median       3Q      Max
-2.05603 -0.56699 -0.03866  0.54699  2.19347

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.8887     0.1850  10.207 < 2e-16 ***
PEU          0.3472     0.0455   7.631 1.48e-13 ***
PU           0.1789     0.0447   4.001 7.40e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7988 on 436 degrees of freedom
Multiple R-squared:  0.2073, Adjusted R-squared:  0.2037
F-statistic: 57.01 on 2 and 436 DF, p-value: < 2.2e-16

>
> vif(R00)
      PEU      PU
1.169525 1.169525
```

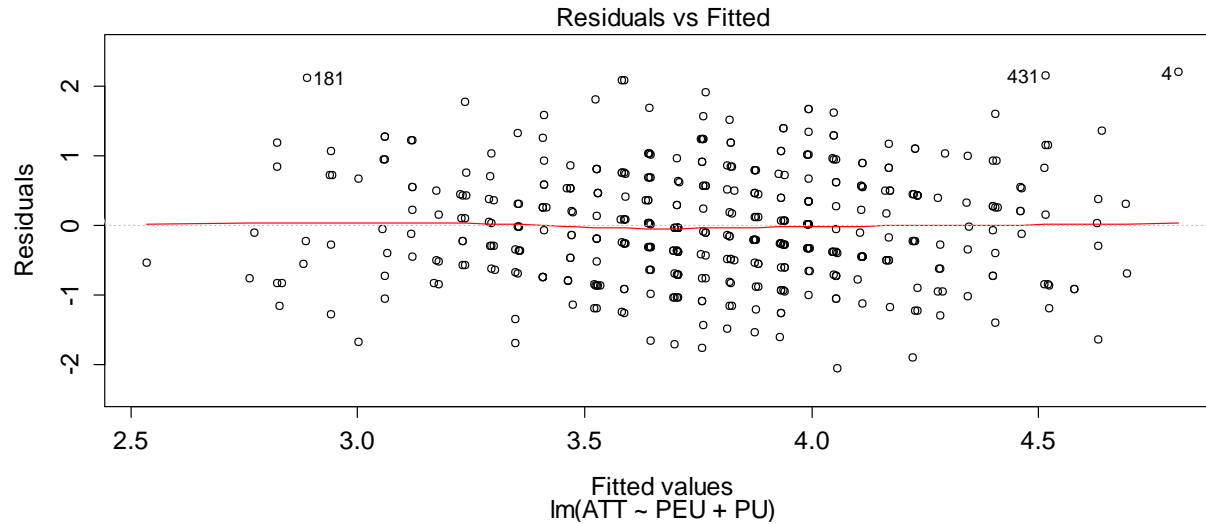
Regression Analysis



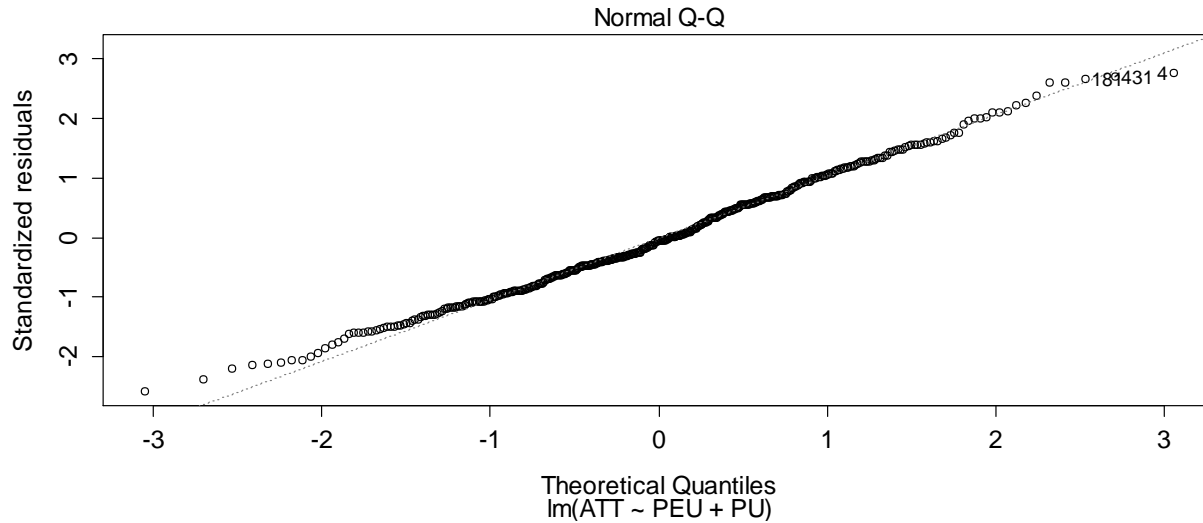
Console

```
> # Standardized Coefficients
>
> library(QuantPsyc)
> lm.beta(R00)
      PEU      PU
0.3518752 0.1845040
>
> # Alternatively
>
> R00$coefficients[["PEU"]]*(sd(Data.01.PPT$PEU)/sd(Data.01.PPT$ATT))
[1] 0.3518752
>
> R00$coefficients[["PU"]]*(sd(Data.01.PPT$PU)/sd(Data.01.PPT$ATT))
[1] 0.184504
```

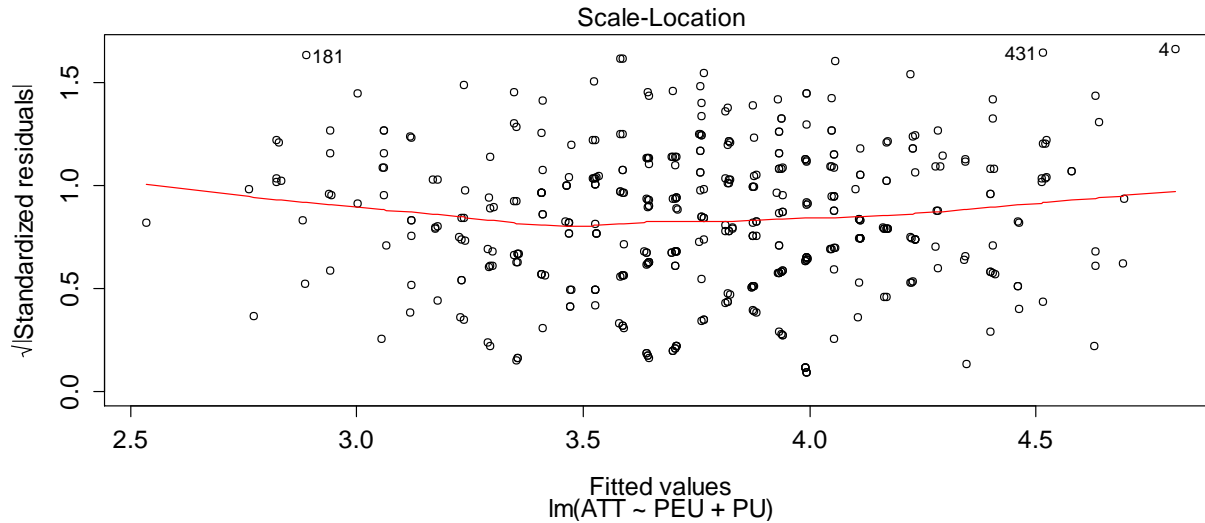
Regression Analysis



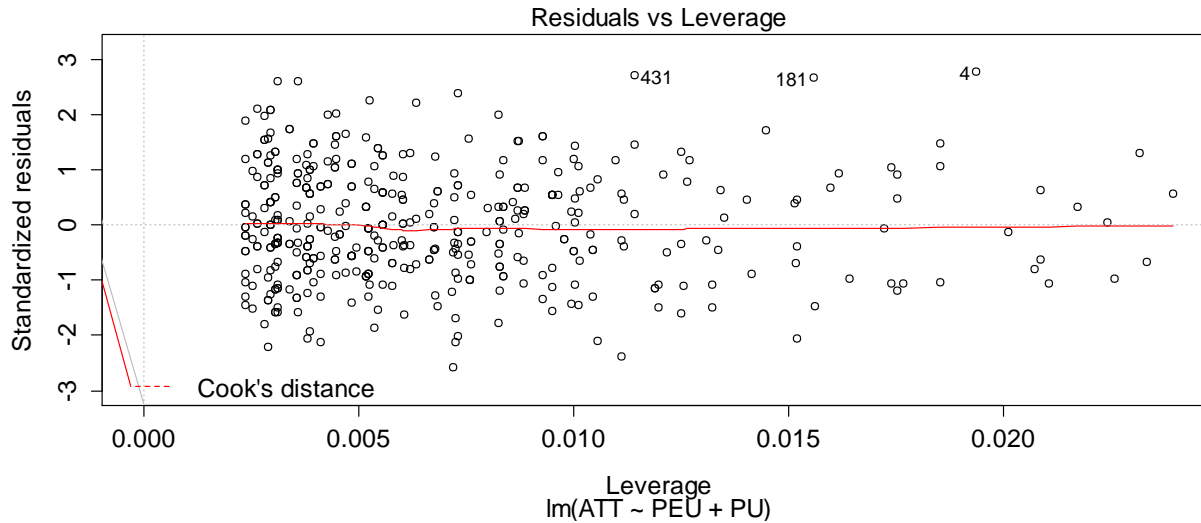
Regression Analysis



Regression Analysis



Regression Analysis



6. Advanced Topics: Nominal Variables

Malhotra (2010); Pallant (2016)

- ▶ Can nominal data be used in regression analysis?
 - ▶ Yes, how?
 - ▶ By using dummy variables (0,1)

Two categories	
	Dummy
A	1
B	0

Two categories	
	Effect
A	1
B	-1

Three Categories		
	Dummy1	Dummy2
A	1	0
B	0	1
C	0	0

Three Categories		
	Effect1	Effect2
A	1	0
B	0	1
C	-1	-1

R package:
dummies

6. Advanced Topics: Nominal Variables

Malhotra (2010); Pallant (2016)



The screenshot shows the IBM SPSS Statistics Data Editor interface. The 'Transform' menu is open, and the 'Create Dummy Variables' option is highlighted. A red arrow points from the text 'Create Dummy Variables' to this menu item. The data grid shows a table with 27 rows and 5 columns. The first column contains values from 1 to 27. The second column contains values 1, 2. The third column contains values 1, 1. The fourth column contains values 9, 7, 7, 6, 4, 10, 4, 9, 8, 4, 6, 8, 10, 4, 9, 4, 6, 6, 4, 9, 4, 6, 6, 4, 4, 6, 4. The fifth column contains values 9, 8, 4, 9, 6, 10, 4, 9, 8, 4, 6, 8, 10, 4, 9, 4, 6, 6, 4, 9, 4, 6, 6, 4, 4, 6, 4.

Row	Col 1	Col 2	Col 3	Col 4	Col 5
1	1	1	1	9	9
2	2	2	1	7	8
3	3	2	1	7	4
4	4	2	1	6	9
5	5	2	2	4	6
6	6	2	2	5	8
7	7	2	2	5	10
8	8	2	2	6	4
9	9	2	2	4	9
10	10	2	3	2	4
11	11	2	3	3	6
12	12	2	3	3	6
13	13	2	3	3	6
14	14	2	3	3	6
15	15	2	3	3	6
16	16	2	3	3	6
17	17	2	3	3	6
18	18	2	3	3	6
19	19	2	3	3	6
20	20	2	3	3	6
21	21	2	3	3	6
22	22	2	3	3	6
23	23	2	3	3	6
24	24	2	3	3	6
25	25	2	3	3	6
26	26	2	3	3	6
27	27	2	3	3	6

6. Advanced Topics: Nominal Variables

Malhotra (2010); Pallant (2016)



Department Store Data (Malhotra, 2010, Table 16.2)

ANOVA^a

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	106.067	2	53.033	17.944	.000 ^b
	Residual	79.800	27	2.956		
	Total	185.867	29			

a. Dependent Variable: sales Sales

b. Predictors: (Constant), D2, D1

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3.700	.544		6.806	.000
	D1	4.600	8.30769	.871	5.983	.000
	D2	2.500	6.20769	.473	3.252	.003

a. Dependent Variable: sales Sales

Promotion	D1	D2
1	1	0
2	0	1
3	0	0

6. Advanced Topics: Nominal Variables



```
74 # Dummy variables
75
76 A.01<-lm(sales ~ factor(promotion), data=Table16.2)
77 summary(A.01)
78
79 summary(lm(lm(sales ~ promotion, data=Table16.2)))
80
81 Table16.2$prom.1<-factor(Table16.2$promotion)
82 Table16.2$prom.2<-factor(Table16.2$promotion)
83
84 contrasts(Table16.2$prom.1)<-contr.treatment(3, base=3)
85 contrasts(Table16.2$prom.2)<-contr.sum(3, base=3)
86
87 summary(lm(lm(sales ~ prom.1, data=Table16.2)))
88
89 summary(lm(lm(sales ~ prom.2, data=Table16.2)))
90
```

6. Advanced Topics: Nominal Variables

Console

```
> summary(lm(sales ~ prom.1, data=Table16.2))

Call:
lm(formula = sales ~ prom.1, data = Table16.2)

Residuals:
    Min       1Q   Median       3Q      Max
-2.7   -1.3   -0.2    1.3    3.3

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   3.7000     0.5437   6.806 2.61e-07 ***
prom.11       4.6000     0.7688   5.983 2.21e-06 ***
prom.12       2.5000     0.7688   3.252 0.00307 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.719 on 27 degrees of freedom
Multiple R-squared:  0.5707,    Adjusted R-squared:  0.5389
F-statistic: 17.94 on 2 and 27 DF,  p-value: 1.104e-05
```

6. Advanced Topics: Nominal Variables

Console

```
> summary(lm(sales ~ prom.2, data=Table16.2))

Call:
lm(formula = sales ~ prom.2, data = Table16.2)

Residuals:
    Min       1Q   Median       3Q      Max
-2.7    -1.3    -0.2     1.3     3.3

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   6.0667     0.3139  19.328 < 2e-16 ***
prom.21       2.2333     0.4439   5.031 2.8e-05 ***
prom.22       0.1333     0.4439   0.300  0.766
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.719 on 27 degrees of freedom
Multiple R-squared:  0.5707,    Adjusted R-squared:  0.5389
F-statistic: 17.94 on 2 and 27 DF,  p-value: 1.104e-05
```

6. Advanced Topics: Equality of Coefficients

Testing Equality of Coefficients

General Approach

$$(1) Y_i = a + b_1 * X_{1i} + b_2 * X_{2i} + b_3 * X_{3i} + \text{ERROR (unconstrained model)}$$

Assume $H_0: b_1 = b_2$

$$(2) Y_i = a + b_1 * X_{1i} + b_1 * X_{2i} + b_3 * X_{3i} + \text{ERROR (constrained model)} \Leftrightarrow$$

$$(3) Y_i = a + b_1 * (X_{1i} + X_{2i}) + b_3 * X_{3i} + \text{ERROR (constrained model)} \Leftrightarrow$$

Compare (1) and (3) using:

$$F(j, n-k-1) = \frac{(R^2_U - R^2_C) * (n-k-1)}{(1-R^2_U) * j}$$

n=number of observations

k=number of variables in unconstrained model

j=number of constraints

6. Advanced Topics: Equality of Coefficients

Hayes (2013); Hayes and Matthes (2009)



Table 17.1 Input.Equality.sav [DataSet2] - IBM SPSS Statistics Data Editor

	resp	attitude	duration	importance	dandimp	var	var	var	var	var	va
1	1	6	10	3	13						
2	2	9	12	11	23						
3	3	8	12	4	16						
4	4	3	4	1	5						
5	5	10	12	11	23						
6	6	4	6	1	7						
7	7	5	8	7	15						
8	8	2	2	4	6						
9	9	11	18	8	26						
10	10	9	9	10	19						
11	11	10	17	8	25						
12	12	2	2	5	7						
13											
14											
15											
16											
17											
18											
19											
20											

dandimp=duration + importance

6. Advanced Topics: Equality of Coefficients



UNconstrained model

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.972 ^a	.945	.933	.860

a. Predictors: (Constant), Importance, Duration

$$F(j, n-k-1) = \frac{(R^2_U - R^2_C) * (n-k-1)}{(1-R^2_U) * j}$$

Constrained model ($b_1=b_2$)

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.965 ^a	.931	.924	.912

a. Predictors: (Constant), dandimp

$$F(1, 9) = \frac{0.945 - 0.931 * (12 - 2 - 1)}{(1 - 0.945) * 1}$$

$$F(1, 9) = 2.29 \quad (p=0.16)$$

Coefficients^a

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	.296	.601		.492	.633
	dandimp	.408	.035	.965	11.637	.000

$b_1=b_2$

a. Dependent Variable: Attitude

6. Advanced Topics: Equality of Coefficients



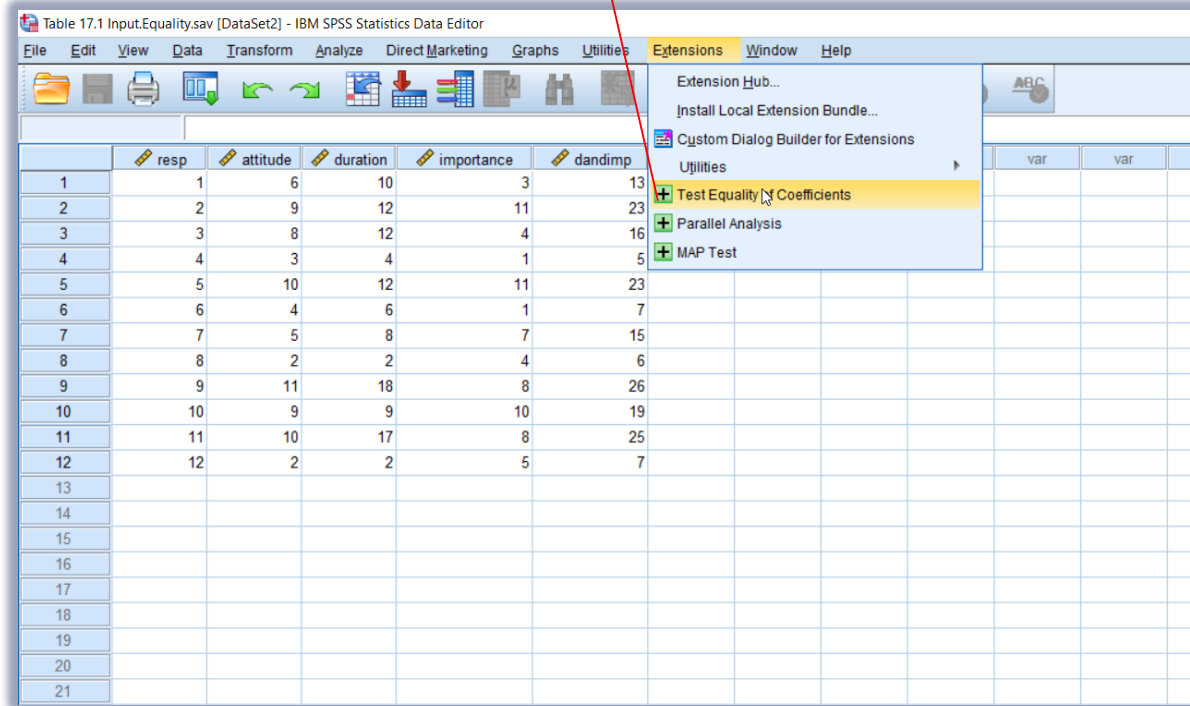
SPSS Macro
TestEqualityofCoefficients.sps

```
* Encoding: UTF-8.
New File.
DATA LIST
/* R2U: R2 Unconstra...
/* Enter data between...
0.945 0.931 12 2 1
Compute
Compute
Compute
Compute
PRINT
PRINT
EXECUTE.

1 * Encoding: UTF-8.
2 New File.
3
4 DATA LIST FREE / R2U R2C N K J.
5
6 /* R2U: R2 Unconstrained model */
7 /* R2C: R2 Constrained model */
8 /* N: Number of observations */
9 /* K: Number of independent variables in unconstrained model */
10 /* J: Number of constraints */
11
12 /* Enter data between "BEGIN DATA." and "END DATA." */
13 BEGIN DATA.
14 0.945 0.931 12 2 1
15 END DATA.
16
17 F stat    DF1    DF2    p value
18 2.29      1      9      .164430
19
20
21
22
23 PRINT / F (F10.2) DF1 (F8.0) DF2 (F8.0) P (F16.6).
24 EXECUTE.
25
```

6. Advanced Topics: Equality of Coefficients

Extension (*.spe)
Equality of Coefficients



The screenshot shows the IBM SPSS Statistics Data Editor interface. The title bar reads 'Table 17.1 Input.Equality.sav [DataSet2] - IBM SPSS Statistics Data Editor'. The menu bar includes File, Edit, View, Data, Transform, Analyze, Direct Marketing, Graphs, Utilities, Extensions, Window, and Help. The 'Extensions' menu is open, showing options: Extension Hub..., Install Local Extension Bundle..., Custom Dialog Builder for Extensions, Utilities, Test Equality of Coefficients (highlighted), Parallel Analysis, and MAP Test. The main data grid contains the following data:

	resp	attitude	duration	importance	dandimp
1	1	6	10	3	13
2	2	9	12	11	23
3	3	8	12	4	16
4	4	3	4	1	5
5	5	10	12	11	23
6	6	4	6	1	7
7	7	5	8	7	15
8	8	2	2	4	6
9	9	11	18	8	26
10	10	9	9	10	19
11	11	10	17	8	25
12	12	2	2	5	7
13					
14					
15					
16					
17					
18					
19					
20					
21					

6. Advanced Topics: Equality of Coefficients

Hayes (2013); Hayes and Matthes (2009)



```
1 * Encoding: UTF-8.
2 * Test Equality of Coefficients ===.
3
4 BEGIN PROGRAM R.
5
6 # Load packages
7
8 install.packages("car", repos="http://cran.rstudio.com")
9 library(car)
10
11 # Transfer data to R data frame
12 testData<-spssdata.GetDataFromSPSS()
13
14
15 # Estimate regression model
16
17 M00<-lm(attitude ~ duration + importance, data=testData)
18
19 # Restriction
20
21 linearHypothesis(M00, "duration = importance")
22
23 END PROGRAM.
```

6. Advanced Topics: Equality of Coefficients

Hayes (2013); Hayes and Matthes (2009)



```
Hypothesis:
duration - importance = 0

Model 1: restricted model
Model 2: attitude ~ duration + importance

  Res.Df  RSS Df Sum of Sq    F Pr(>F)
1      10 8.3149
2       9 6.6524  1    1.6625 2.2492 0.1679
```

```
> anova(R01,R00)
Analysis of Variance Table

Model 1: ATT ~ I(PEU + PU)
Model 2: ATT ~ PEU + PU
  Res.Df  RSS Df Sum of Sq    F Pr(>F)
1     437 281.43
2     436 278.21  1     3.2195 5.0456 0.02519 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Baron and Kenny (1986)

Journal of Personality and Social Psychology
1986, Vol. 51, No. 6, 1173–1182

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0022-3514/86/\$00.75

The Moderator–Mediator Variable Distinction in Social Psychological Research: Conceptual, Strategic, and Statistical Considerations

Reuben M. Baron and David A. Kenny
University of Connecticut



In this article, we attempt to distinguish between the properties of moderator and mediator variables at a number of levels. First, we seek to make theorists and researchers aware of the importance of not using the terms *moderator* and *mediator* interchangeably by carefully elaborating, both conceptually and strategically, the many ways in which moderators and mediators differ. We then go beyond this largely pedagogical function and delineate the conceptual and strategic implications of making use of such distinctions with regard to a wide range of phenomena, including control and stress, attitudes, and personality traits. We also provide a specific compendium of analytic procedures appropriate for making the most effective use of the moderator and mediator distinction, both separately and in terms of a broader causal system that includes both moderators and mediators.

Preacher and Hayes (2004)

Behavior Research Methods, Instruments, & Computers
2004, 36 (4), 717-731

SPSS and SAS procedures for estimating indirect effects in simple mediation models

KRISTOPHER J. PREACHER

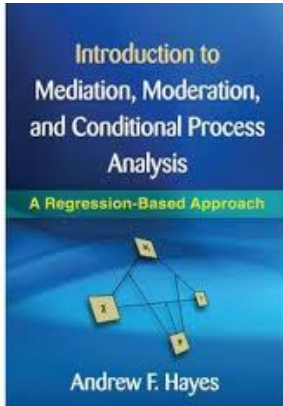
University of North Carolina, Chapel Hill, North Carolina

and

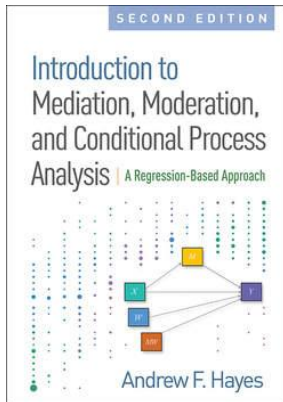
ANDREW F. HAYES

Ohio State University, Columbus, Ohio

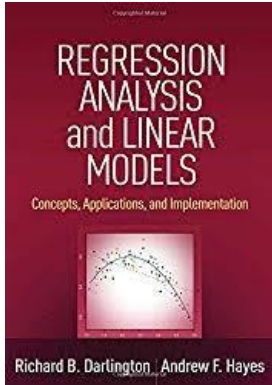
Researchers often conduct mediation analysis in order to indirectly assess the effect of a proposed cause on some outcome through a proposed mediator. The utility of mediation analysis stems from its ability to go beyond the merely descriptive to a more functional understanding of the relationships among variables. A necessary component of mediation is a statistically and practically significant indirect effect. Although mediation hypotheses are frequently explored in psychological research, formal significance tests of indirect effects are rarely conducted. After a brief overview of mediation, we argue the importance of directly testing the significance of indirect effects and provide SPSS and SAS macros that facilitate estimation of the indirect effect with a normal theory approach and a bootstrap approach to obtaining confidence intervals, as well as the traditional approach advocated by Baron and Kenny (1986). We hope that this discussion and the macros will enhance the frequency of formal mediation tests in the psychology literature. Electronic copies of these macros may be downloaded from the Psychonomic Society's Web archive at www.psychonomic.org/archive/.



Hayes, A. F. (2013). *Introduction to Mediation, Moderation, and Conditional Process Analysis: A Regression-Based Approach*. New York, NY: The Guilford Press.



Hayes, A. F. (2018). *Introduction to Mediation, Moderation, and Conditional Process Analysis: A Regression-Based Approach*. New York, NY: The Guilford Press.



Darlington, R. B., & Hayes, A. F. (2016). Regression Analysis and Linear models: Concepts, Applications, and Implementation. New York, NY: The Guilford Press.

PROCESS Macro

<http://www.processmacro.org/>

HOME DOWNLOAD WORKSHOPS FAQ PAPERS

The PROCESS macro for SPSS and SAS

f t e y

***** DIRECT AND INDIRECT EFFECTS *****

Effect	SE	t	P	LLCI	ULCI
Direct effect of X on Y					
Effect					
→ -.2457	.1539	-1.5968	.1132	-.5507	.0593

Conditional indirect effect(s) of X on Y at values of the moderator(s)

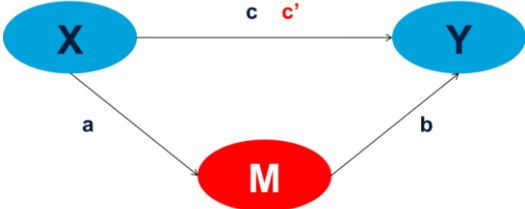
Mediator	Effect	Boot SE	BootLLCI	BootULCI
ptsd lonely	-.0435	.0537	-.2155	.0183
ptsd	-.0325	.0486	-.1903	.0183
ptsd	-.2314	.0364	-.0512	.0238
ptsd	.6258	.0774		
ptsd	1.0544	.1103		

values for qu...

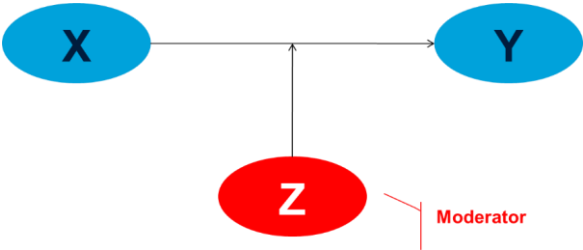
PROCESS is an observed variable OLS and logistic regression path analysis modeling tool for SPSS and SAS. It is widely used through the social, business, and health sciences for estimating direct and indirect effects in single and multiple mediator models (parallel and serial), two and three way interactions in moderation models along with simple slopes and regions of significance for probing interactions, and conditional indirect effects in moderated mediation models with a single or multiple mediators or moderators. The use of **PROCESS** is described and documented in *Introduction to Mediation, Moderation, and*

6. Advanced Topics

Mediation

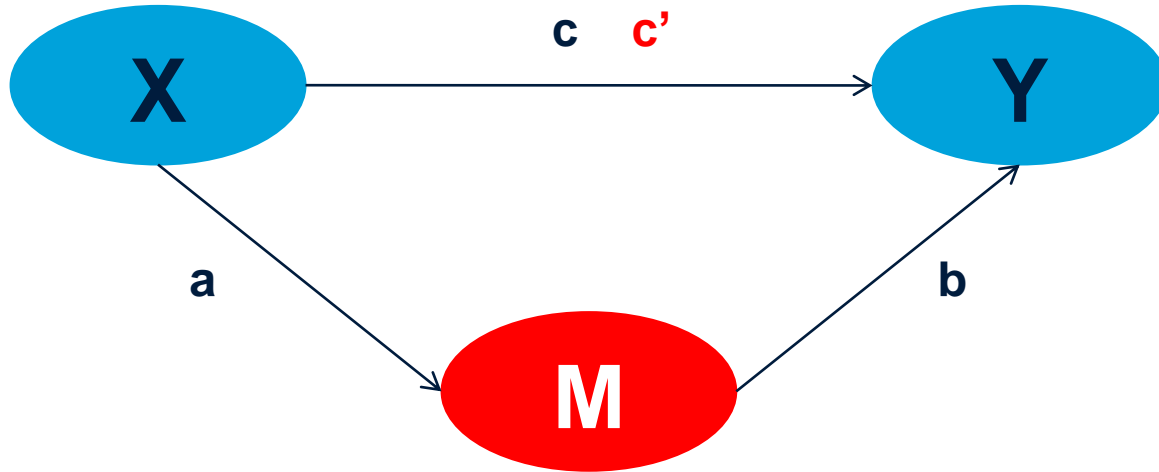


Moderation



Mediation

Preacher and Hayes (2004)



$c' = 0 \rightarrow$ **COMPLETE (FULL) mediation**

$c' \neq 0 \rightarrow$ **PARTIAL mediation**

Mediation

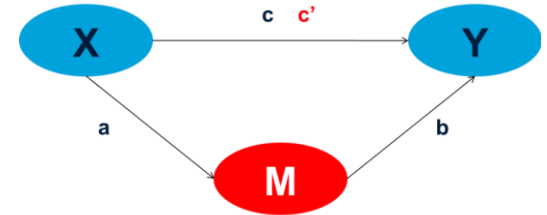
Preacher and Hayes (2004)

► Regression Approach to Mediation

1. $Y = i_1 + c*X + \text{ERROR}_1$
2. $M = i_2 + a*X + \text{ERROR}_2$
3. $Y = i_3 + c'*X + b*M + \text{ERROR}_3$

► Sobel (1982) test

- $H_0: a*b = 0$ ($c - c'$)



Mediation (Preacher and Hayes, 2004)

Mediation Data



Mediation.sav [DataSet2] - IBM SPSS Statistics Data Editor

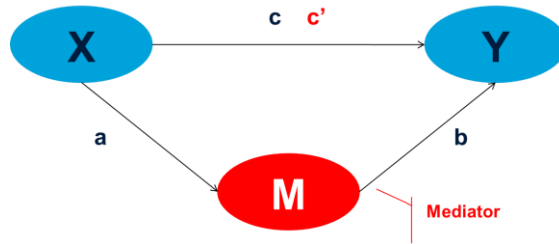
	Y	X	M	var	var	var	var	var	var	var	var
1	5	4	4								
2	5	4	5								
3	5	3	4								
4	5	5	4								
5	6	3	4								
6	5	5	5								
7	5	6	5								
8	6	6	6								
9	6	3	5								
10	6	5	5								
11	5	5	5								
12	6	4	5								
13	5	4	4								
14	6	3	4								
15	5	3	4								
16	5	5	4								
17	6	3	4								
18	6	5	5								
19	5	3	4								
20	6	5	6								
21	6	5	5								
22	6	5	5								
23	6	5	5								

Y DV
X IV
M Mediator

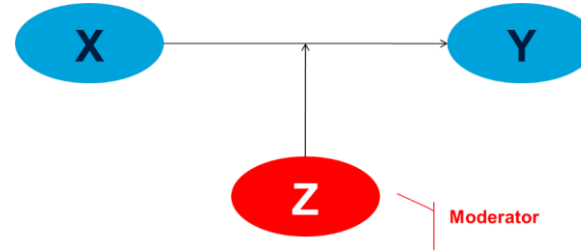
6. Advanced Topics: Mediation And Moderation

Hayes (2013); Hayes and Matthes (2009); Preacher and Hayes (2004)

Mediation

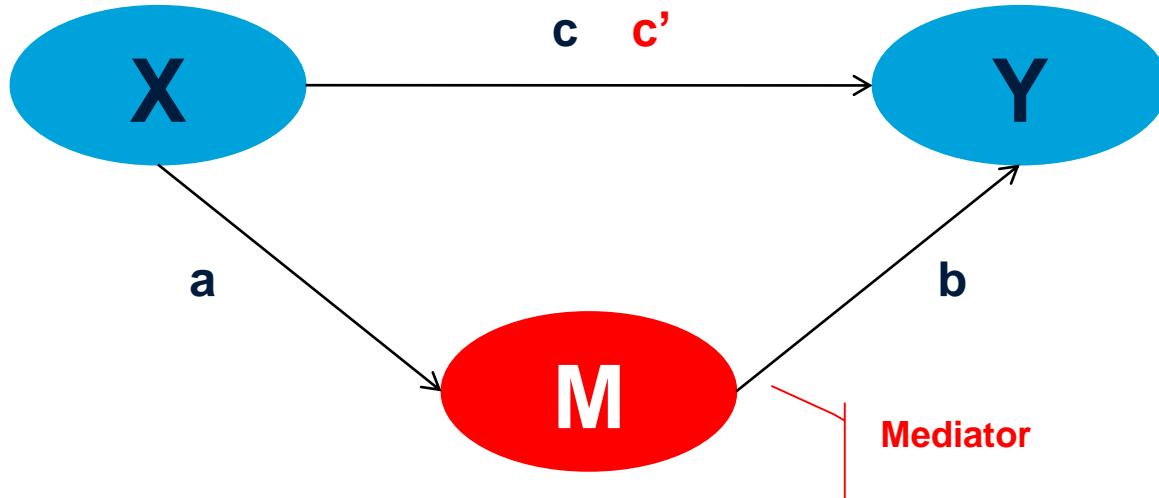


Moderation



6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)

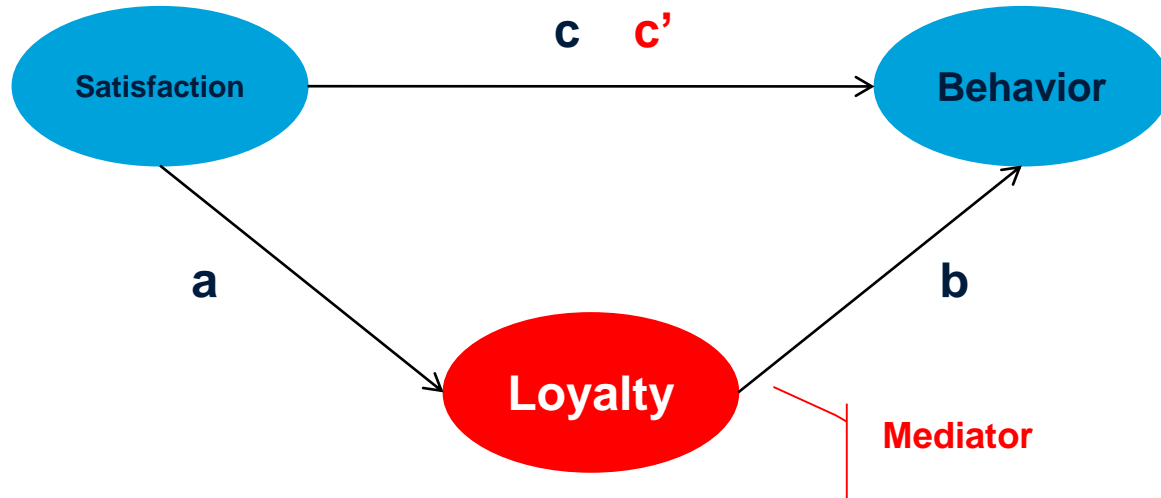


$c' = 0 \rightarrow$ COMPLETE (FULL) mediation

$c' \neq 0 \rightarrow$ PARTIAL mediation

6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)



6. Advanced Topics: Mediation

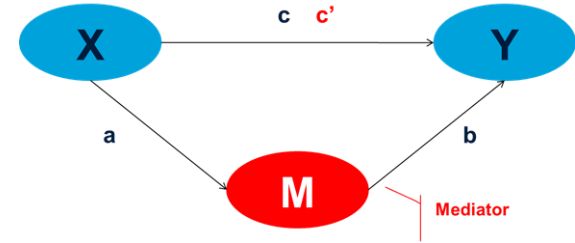
Hayes (2013); Preacher and Hayes (2004)

► Regression Approach to Mediation

1. $Y = i_1 + c*X + \text{ERROR}_1$
2. $M = i_2 + a*X + \text{ERROR}_2$
3. $Y = i_3 + c'*X + b*M + \text{ERROR}_3$

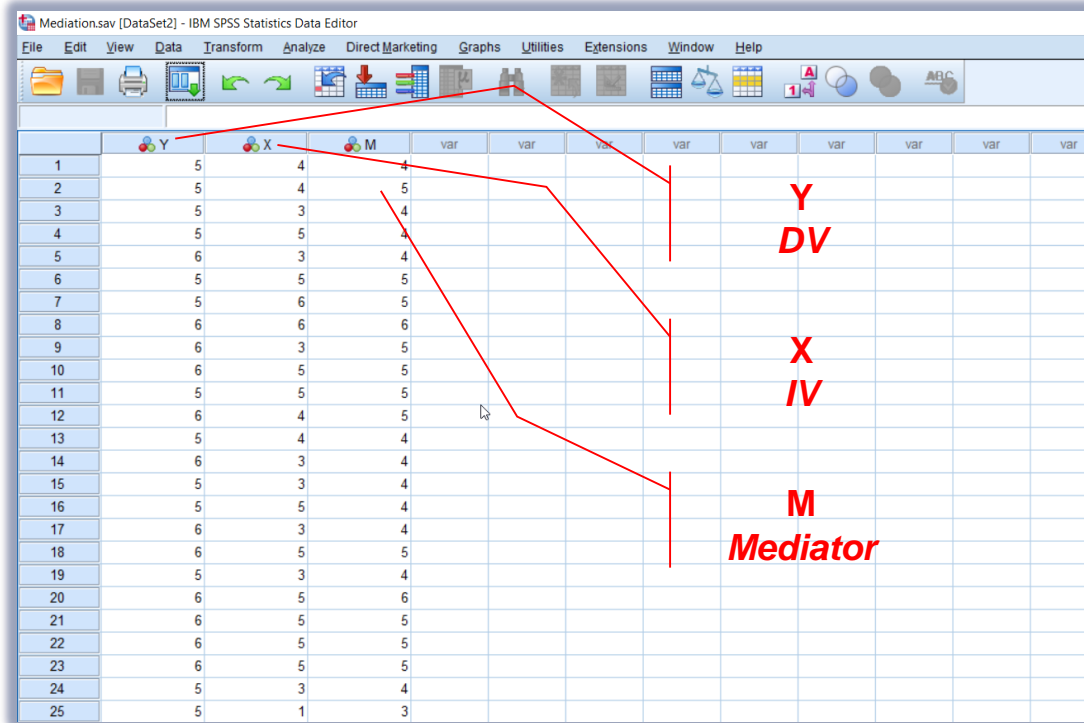
► Sobel (1982) test

- $H_0: a*b = 0$ ($c - c' = 0$)



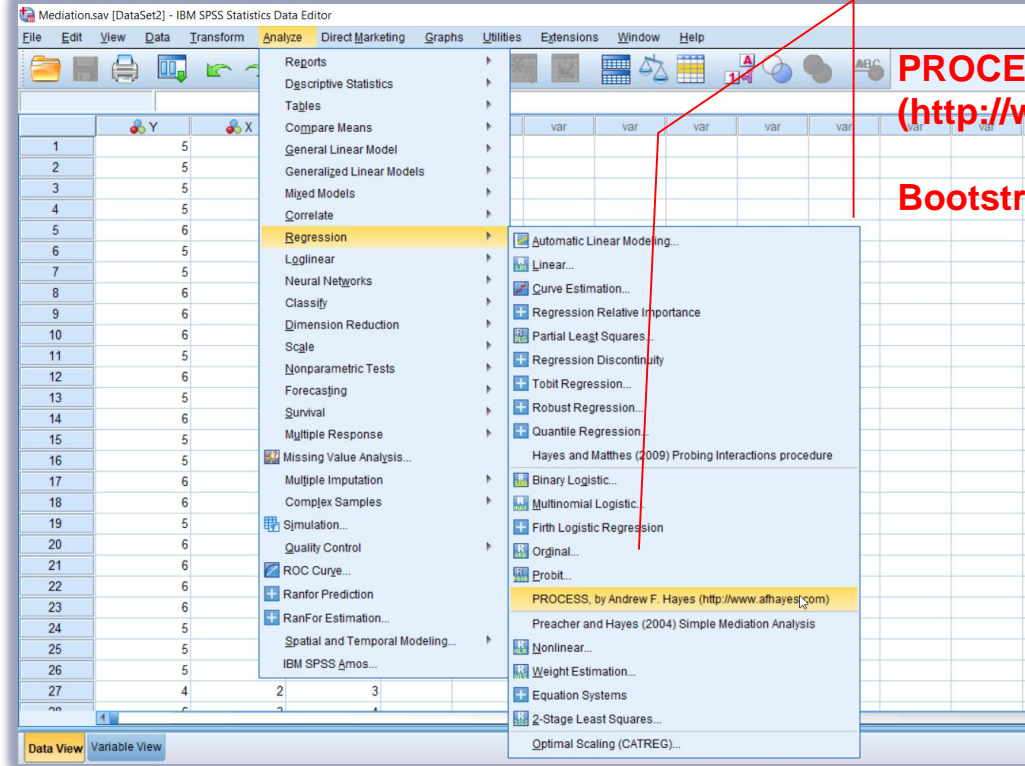
6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)



6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)



Custom Dialogs (*.spd)

PROCESS Macro
(<http://www.afhayes.com/>)

Bootstrapping!

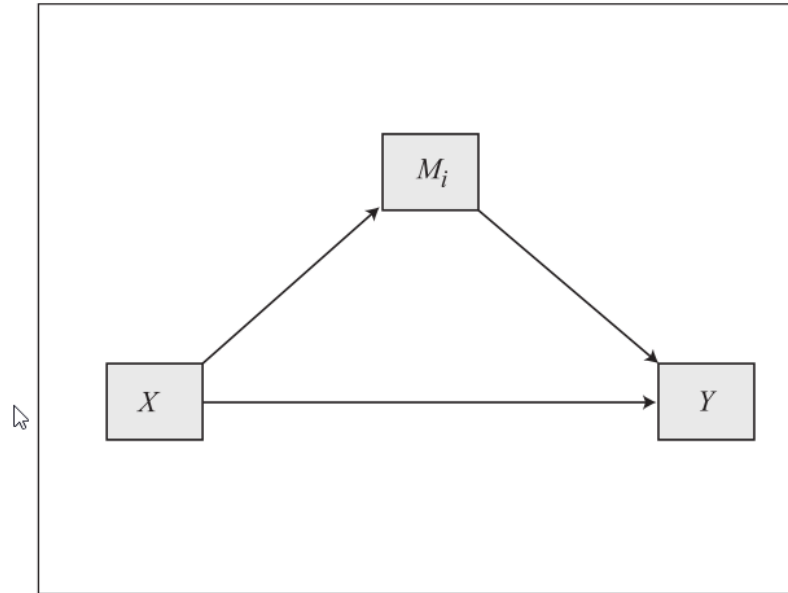
6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)

[templates.pdf](#)

Model 4

Conceptual Diagram



6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)



The image shows two overlapping dialog boxes from the PROCESS software. The larger box on the left is the 'PROCESS Procedure for SPSS' dialog, and the smaller box on the right is the 'PROCESS Options' dialog. A red arrow points from the 'Options' button in the main dialog to the 'PROCESS Options' dialog.

PROCESS Procedure for SPSS (Left Dialog):

- Data File Variables:** Empty list.
- Outcome Variable (Y):** Y
- Independent Variable (X):** X
- M Variable(s):** M
- Model Number:** 4
- Bootstrapping for indirect effects:**
 - Bootstrap Samples: 5000
 - Bootstrap CI method: Bias Corrected
- Confidence level for confidence intervals:** 95%
- Covariate(s) in model(s) of...:** ...both M and Y
- Proposed Moderator W, Z, V, Q:** Empty text boxes.
- Buttons:** About, Options, Conditioning, Multicategorical, Long names, OK, Paste, Reset, Cancel.

PROCESS Options (Right Dialog):

- Mean center for products
- Heteroscedasticity-consistent SEs
- OLS/ML confidence intervals
- Generate data for plotting (model 1, 2, and 3 only)
- Effect size (models 4 and 6)
- Sobel test (model 4 only)
- Total effect model (models 4 and 6 only)
- Compare indirect effects (models 4 and 6 only)
- Print model coefficient covariance matrix
- Decimal places in output:** 4
- Buttons:** Continue, Cancel.

6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)



```
*****
Outcome: M
Model Summary
      R      R-sq      MSE      F      df1      df2      p
      .7607   .5787   .3098  271.9727   1.0000  198.0000   .0000

Model
      coeff      se      t      p      LLCI      ULCI
constant  2.5025   .1300  19.2568   .0000   2.2462   2.7588
X          a .5132   .0311  16.4916   .0000   .4518   .5746

*****
Outcome: Y
Model Summary
      R      R-sq      MSE      F      df1      df2      p
      .5871   .3446   .3253   51.7991   2.0000  197.0000   .0000

Model
      coeff      se      t      p      LLCI      ULCI
constant  3.2430   .2257  14.3674   .0000   2.7979   3.6882
M          b .4494   .0728   6.1707   .0000   .3058   .5930
X          c .0275   .0491   .5598   .5763  -.0694   .1244
;
```

6. Advanced Topics: Mediation



Hayes (2013); Preacher and Hayes (2004)

```
***** TOTAL EFFECT MODEL *****
Outcome: Y

Model Summary
      R      R-sq      MSE      F      df1      df2      p
      .4669      .2180      .3863      55.1864      1.0000      198.0000      .0000

Model
      coeff      se      t      p      LLCI      ULCI
constant  c  4.3677      .1451      30.0994      .0000      4.0815      4.6538
X          .2581      .0347      7.4288      .0000      .1896      .3266

***** TOTAL, DIRECT, AND INDIRECT EFFECTS *****

Total effect of X on Y
      c  Effect      SE      t      p      LLCI      ULCI
      .2581      .0347      7.4288      .0000      .1896      .3266

Direct effect of X on Y
      c' Effect      SE      t      p      LLCI      ULCI
      .0275      .0491      .5598      .5763      -.0694      .1244

Indirect effect of X on Y
      Effect      Boot SE      BootLLCI      BootULCI
M      .2306      .0420      .1525      .3166

Normal theory tests for indirect effect
      Effect      se      z      p
      .2306      .0400      5.7701      .0000
```

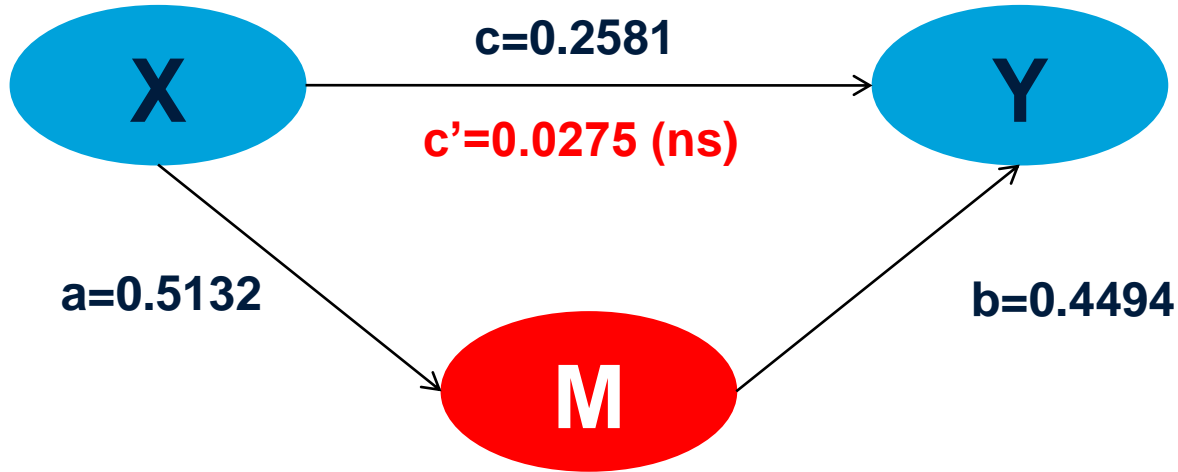
Bootstrap Results (5000 resamples [10000])

Sobel test

$a*b (0.5132*0.4494) = c-c' (0.2581-0.0275)=0.2306$

6. Advanced Topics: Mediation

Hayes (2013); Preacher and Hayes (2004)



$c' = 0 \rightarrow$ COMPLETE (FULL) mediation
 $c' \neq 0 \rightarrow$ PARTIAL mediation

Regression Analysis



```
64 # Mediation
65
66 library(mediation)
67
68 MM<-lm(M ~ X, data=Mediation)
69 MY<-lm(Y ~ X + M, data=Mediation)
70 MED1<-mediate(MM,MY, treat='X', mediator='M')
71 summary(MED1)
72
73 library(bda)
74
75 mediation.test(Mediation$M, Mediation$X, Mediation$Y)
76
77 library(QuantPsyc)
78
79 Data.Mediation<-Mediation
80 names(Data.Mediation)<-c('y','x','m')
81
82 proximal.med(Data.Mediation)
83
84 library(boot)
85 med.boot <- boot(Data.Mediation, proxInd.ef, R=1000)
86 sort(med.boot$t)[c(25,975)] #95% CI
87 plot(density(med.boot$t)) # Distribution of bootstapped indirect effect
88 summary(med.boot$t)
89
```

Regression Analysis



Console

```
> # Mediation
>
> options(scipen=999)
>
> library(mediation)
>
> MM<-lm(M ~ X, data=Mediation)
> MY<-lm(Y ~ X + M, data=Mediation)
> MED1<-mediate(MM,MY, treat='X', mediator='M')
> summary(MED1)
```

Causal Mediation Analysis

Quasi-Bayesian Confidence Intervals

	Estimate	95% CI Lower	95% CI Upper	p-value
ACME	0.2317	0.1533	0.3122	0.00
ADE	0.0253	-0.0684	0.1301	0.63
Total Effect	0.2570	0.1914	0.3230	0.00
Prop. Mediated	0.9053	0.5620	1.3323	0.00

Sample Size Used: 200

simulations: 1000

Regression Analysis



Console

```
> library(bda)
>
> mediation.test(Mediation$M, Mediation$X, Mediation$Y)
              Sobel          Aroian          Goodman
z.value 5.779376771925725 5.770079189740873 5.788719444249967
p.value 0.000000007497785 0.000000007923429 0.000000007092505
>
> library(QuantPsyc)
>
> Data.Mediation<-Mediation
> names(Data.Mediation)<-c('y','x','m')
> proximal.med(Data.Mediation)

```

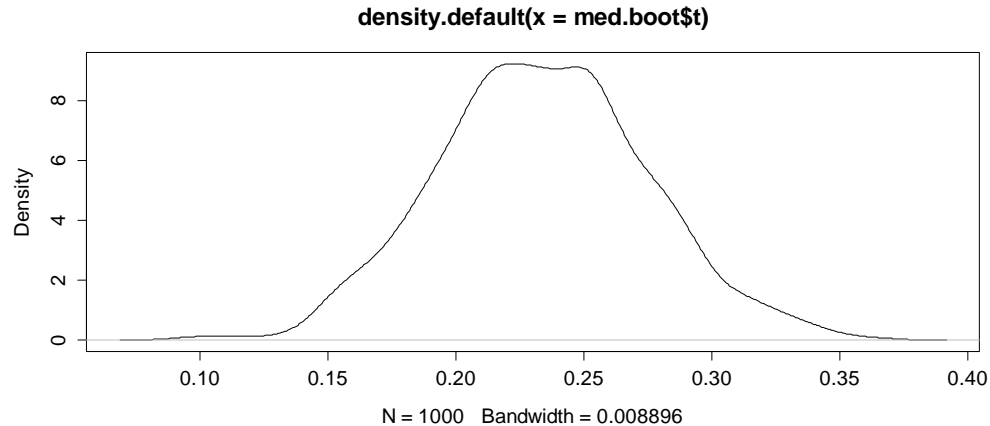
	Effect	SE	t-ratio	Med.Ratio
a	"0.513190797699425"	"0.0311183270153198"	"16.4915934409577"	"--"
b	"0.449396851151261"	"0.07282753989585"	"6.17069932327714"	"--"
t	"0.258127031757939"	"0.0347470100395941"	"7.42875520696038"	"--"
t'	"0.0275007032320145"	"0.0491301595620519"	"0.559751962484079"	"--"
ab	"0.230626328525925"	"0.0399050516391716"	"5.77937677192573"	"0.893460583943012"
Aroian	"--"	"0.0399693524026456"	"--"	"--"
Goodman	"--"	"0.0398406470976945"	"--"	"--"

Regression Analysis



Console

```
> library(boot)
> med.boot <- boot(Data.Mediation, proxInd.ef, R=1000)
> sort(med.boot$t)[c(25,975)] #95% CI
[1] 0.1541803 0.3175103
> plot(density(med.boot$t)) # Distribution of bootstapped indirect effect
```



Regression Analysis

Console



```
> library(psych)
> MED.1<-mediate(Y ~ X + (M), data=Mediation, n.iter=50)
> mediate.diagram(MED.1, digits=10)
> print(MED.1, digits=10, short=FALSE)
```

Mediation/Moderation Analysis

Call: mediate(y = Y ~ X + (M), data = Mediation, n.iter = 50)

The DV (Y) was Y . The IV (X) was X . The mediating variable(s) = M .

Total effect(c) of X on Y = 0.258127 S.E. = 0.03474701 t = 7.428755 df= 197 with p = 3.25063e-12
Direct effect (c') of X on Y removing M = 0.0275007 S.E. = 0.04913016 t = 0.559752 df= 197 with p = 0.5762845
Indirect effect (ab) of X on Y through M = 0.2306263
Mean bootstrapped indirect effect = 0.2328739 with standard error = 0.0433557 Lower CI = 0.1449465 Upper CI = 0.3094526
R = 0.5870605 R2 = 0.34464 F = 51.79908 on 2 and 197 DF p-value: 8.380536e-19

Full output

```
Total effect estimates (c)
  Y      se      t  df      Prob
X 0.258127 0.03474701 7.428755 197 3.25063e-12

Direct effect estimates (c')
  Y      se      t  df      Prob
X 0.0275007 0.04913016 0.559752 197 5.762845e-01
M 0.4493969 0.07282754 6.170699 197 3.796747e-09

'a' effect estimates
  M      se      t  df      Prob
X 0.5131908 0.03111833 16.49159 198 5.06913e-39

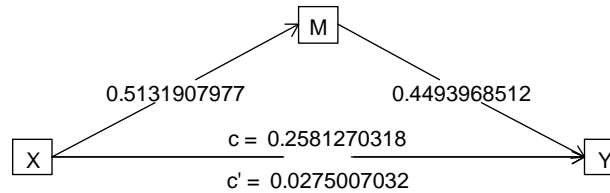
'b' effect estimates
  Y      se      t  df      Prob
M 0.4493969 0.07282754 6.170699 197 3.796747e-09

'ab' effect estimates
  Y      boot      sd      lower      upper
X 0.2306263 0.2328739 0.0433557 0.1449465 0.3094526
>
```

Regression Analysis

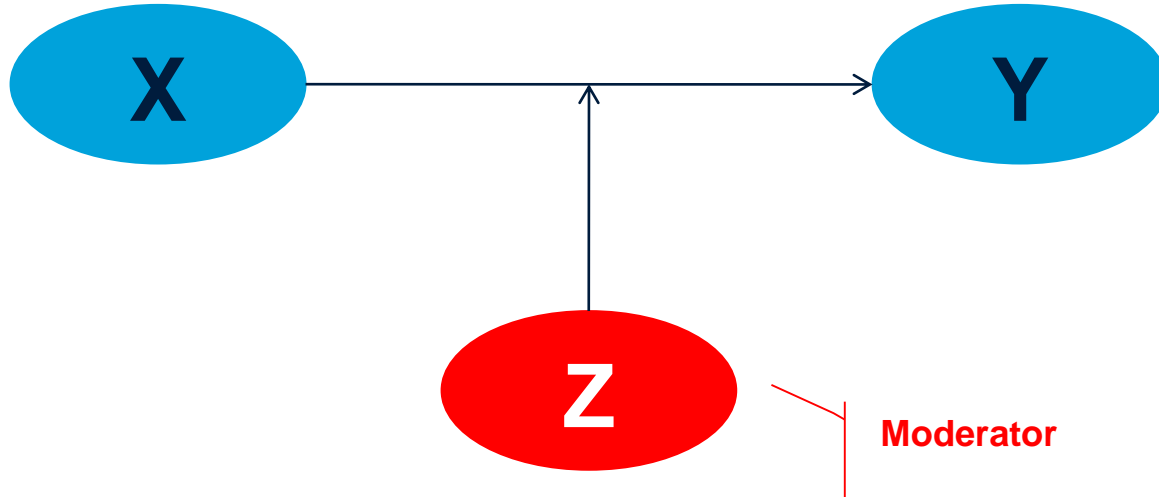


Mediation model



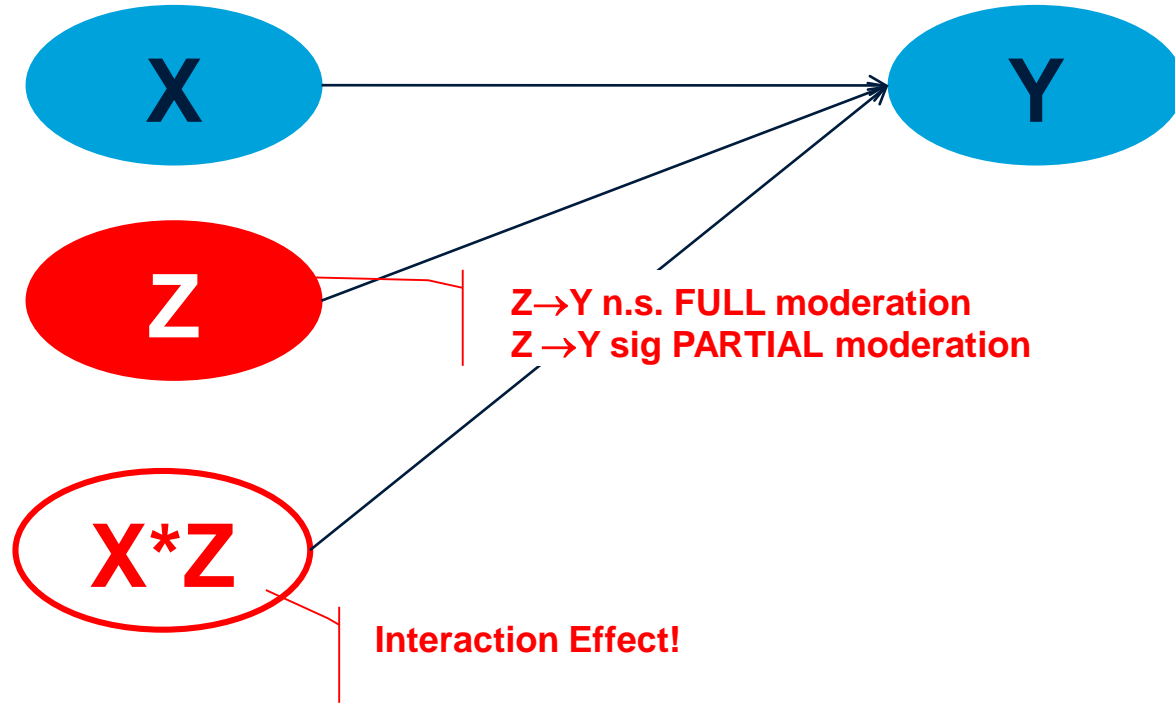
Moderation

Hayes and Matthes (2009)



Moderation

Hayes and Matthes (2009)



6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009)

► Equations:

$$Y_i = a + b_1 * X_i + b_2 * Z_i + b_3 * Z_i * X_i + \text{ERROR}$$

$$Y_i = a + (b_1 + b_3 * Z) * X_i + b_2 * Z_i + \text{ERROR}$$

What happens if $b_3 = 0$?

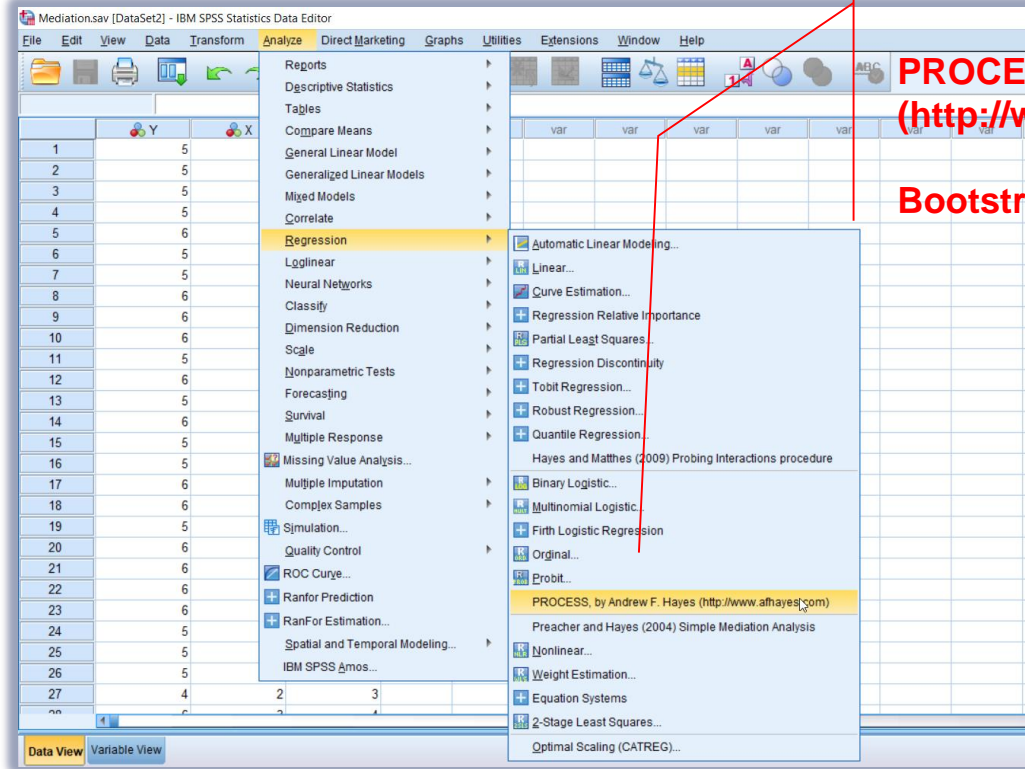
$$Y_i = a + (b_1 + 0 * Z) * X_i + b_2 * Z_i + \text{ERROR}$$

$$Y_i = a + b_1 * X_i + b_2 * Z_i + \text{ERROR}$$

No Moderation!

6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009)



Custom Dialogs (*.spd)

PROCESS Macro
(<http://www.afhayes.com/>)

Bootstrapping!

6. Advanced Topics: Moderation

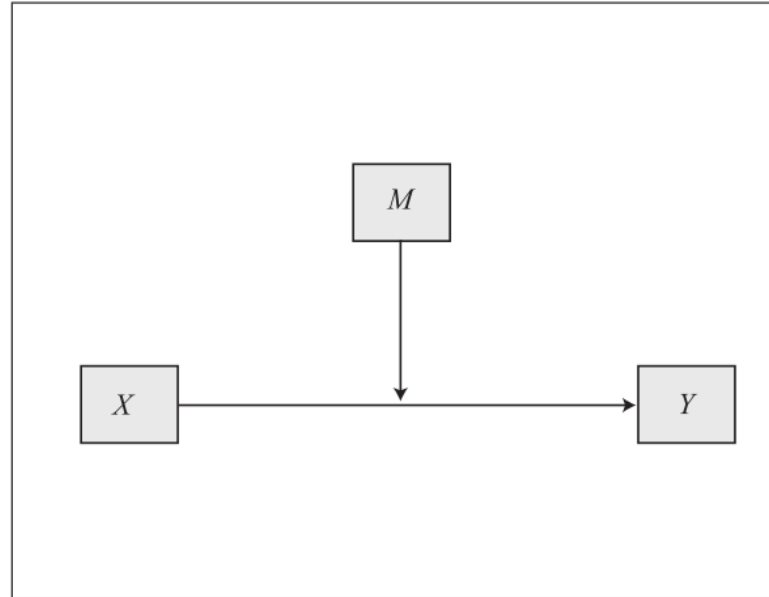
Hayes (2013); Preacher and Hayes (2004)



Model 1

[templates.pdf](#)

Conceptual Diagram



6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009)

The image displays three screenshots of the PROCESS procedure for SPSS interface, illustrating the configuration for moderation analysis.

PROCESS Procedure for SPSS, written by Andrew F. Hayes (www.afhayes.com)

- Data File Variables:** Empty list.
- Outcome Variable (Y):** Y
- Independent Variable (X):** X
- M Variable(s):** Z
- Model Number:** 1
- Bootstrapping for indirect effects:**
 - Bootstrap Samples: 5000
 - Bootstrap CI method: Percentile, Bias Corrected
- Confidence level for confidence intervals:** 95%
- Covariate(s) in model(s) of...:** ...both M and Y, ...M only, ...Y only
- Buttons:** About, Options, Conditioning, Multicategorical, Long names, OK, Paste, Reset, Cancel.

PROCESS Options

- Mean center for products
- Heteroscedasticity-consistent SEs
- OLS/ML confidence intervals
- Generate data for plotting (model 1, 2, and 3 only)
- Effect size (models 4 and 6)
- Sobel test (model 4 only)
- Total effect model (models 4 and 6 only)
- Compare indirect effects (models 4 and 6 only)
- Print model coefficient covariance matrix
- Decimal places in output:** 4
- Buttons:** Continue, Cancel.

Conditioning choices

- Conditioning:**
 - Pick-a-Point:** Mean and +/- 1 SD from Mean, Percentiles
 - Johnson-Neyman (Models 1 and 3 only)
- Buttons:** Continue, Cancel.

Copyright 2016 by Andrew F. Hayes. Do not use the PASTE button.

Spiller et al. (2013): Floodlight Analysis

6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009)



```
*****
Outcome: Y
Model Summary
      R      R-sq      MSE      F      df1      df2      p
      .8687      .7546      .4623      200.8704      3.0000      196.0000      .0000

Model
      coeff      se      t      p      LLCI      ULCI
constant      -.1238      .5556      -.2227      .8240      -1.2196      .9720
Z              .0956      .1315      .7272      .4680      -.1637      .3549
X              .4689      .1296      3.6188      .0004      .2134      .7245
int_1          .0786      .0300      2.6159      .0096      .0193      .1379

Product terms key:

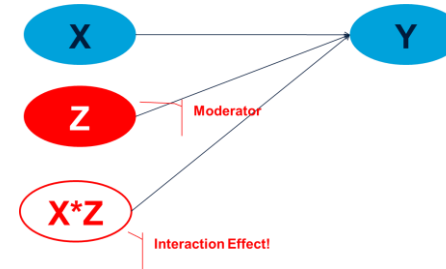
int_1  X          X  Z

R-square increase due to interaction(s):
      R2-chng      F      df1      df2      p
int_1      .0086      6.8430      1.0000      196.0000      .0096

*****

Conditional effect of X on Y at values of the moderator(s):
      Z      Effect      se      t      p      LLCI      ULCI
2.9473      .7006      .0528      13.2648      .0000      .5964      .8048
4.2250      .8010      .0399      20.0643      .0000      .7223      .8798
5.5027      .9015      .0578      15.5837      .0000      .7874      1.0156

Values for quantitative moderators are the mean and plus/minus one SD from mean.
Values for dichotomous moderators are the two values of the moderator.
*****
```



Incremental F test

Conditional Effects

6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009)

SPSS.PROCESS.Moderation.Plot.sps



```
1 * Encoding: UTF-8.
2 DATA LIST FREE/X Z Y.
3 BEGIN DATA.
4
5 Data for 3.0 3.0353 2.9473 2.2845
6 Paste to 4.2 4.2600 2.9473 3.1426
7 5.4 5.4847 2.9473 4.0006
8 DATA LIST 3.0 3.0353 4.2 4.2250 2.7116
9 BEGIN DATA 4.2 4.2600 4.2250 3.6926
10 5.4 5.4847 4.2250 4.6736
11 3.0 3.0353 5.5027 3.1386
12 4.2 4.2600 5.5027 4.2426
13 5.4 5.4847 5.5027 5.3466
14 3.0
15 END DATA.
16
17 GRAPH /SCATTERPLOT=X WITH Y BY Z.
18
19 GRAPH /LINE(MULTIPLE)=mean(Y) BY X BY Z.
20
```

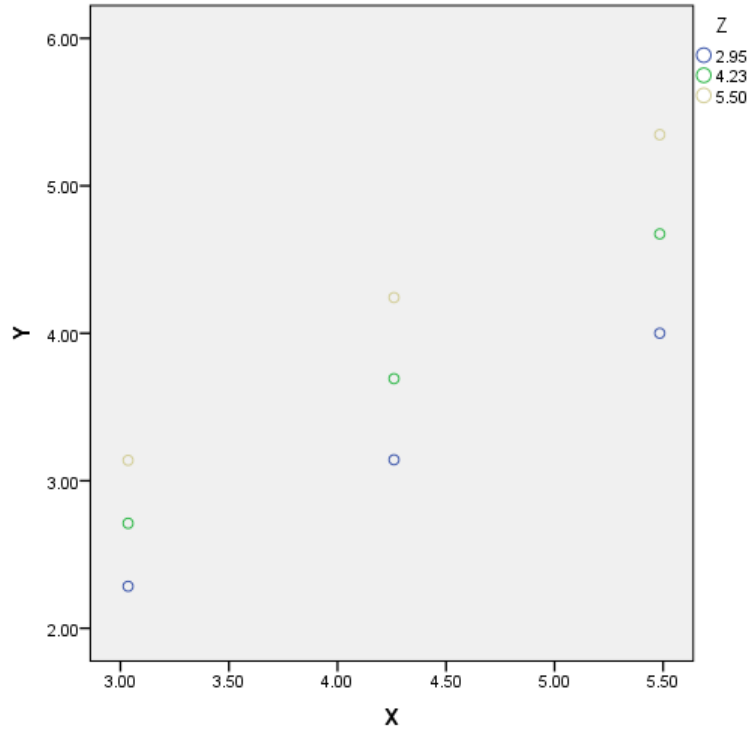
```
END DATA.
GRAPH/SCATTERPLOT=X WITH Y BY Z.
```

6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009)

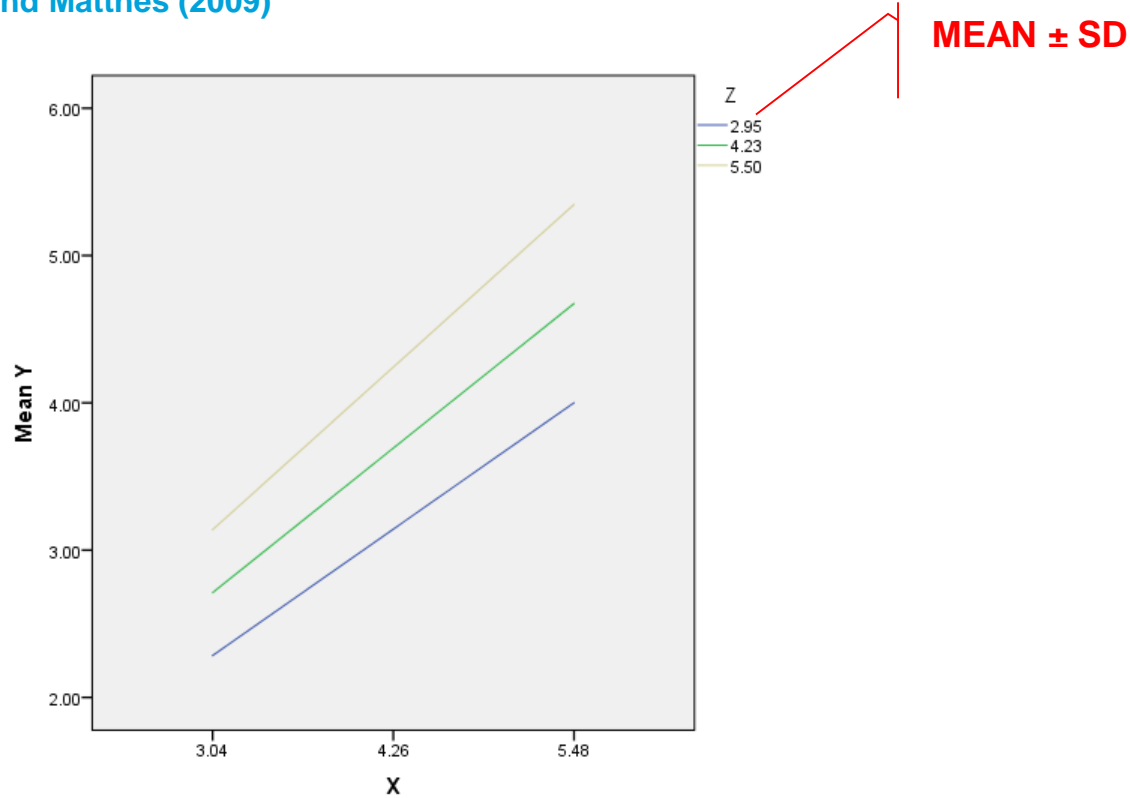


MEAN \pm SD



6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009)



Spiller et al. (2013)

STEPHEN A. SPILLER, GAVAN J. FITZSIMONS, JOHN G. LYNCH JR.,
and GARY H. McCLELLAND*

It is common for researchers discovering a significant interaction of a measured variable X with a manipulated variable Z to examine simple effects of Z at different levels of X . These "spotlight" tests are often misunderstood even in the simplest cases, and it appears that consumer researchers are unsure how to extend them to more complex designs. The authors explain the general principles of spotlight tests, show that they rely on familiar regression techniques, and provide a tutorial demonstrating how to apply these tests across an array of experimental designs. Rather than following the common practice of reporting spotlight tests at one standard deviation above and below the mean of X , it is recommended that when X has focal values, researchers should report spotlight tests at those focal values. When X does not have focal values, it is recommended that researchers report ranges of significance using a version of Johnson and Neyman's test the authors term a "floodlight."

Keywords: moderated regression, spotlight analysis, simple effects tests

Spotlights, Floodlights, and the Magic
Number Zero: Simple Effects Tests in
Moderated Regression

Regression Analysis



```
92 # Moderation
93
94 library(QuantPsyc)
95
96 MOD1<-moderate.lm(X, Z, Y, Moderation, mc=FALSE)
97 summary(MOD1)
98
99 GMOD1<-sim.slopes(MOD1, Moderation$Z, zsd=2, mcz=FALSE)
100
101 graph.mod(GMOD1, X, Y, Moderation)
102
```

Regression Analysis



Console

```
Call:
lm(formula = y ~ mcx * mcz, na.action = na.omit)

Residuals:
    Min       1Q   Median       3Q      Max
-1.68679 -0.39206  0.01796  0.42799  2.19791

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  3.69259    0.04854  76.080 < 0.0000000000000002 ***
mcx          0.80104    0.03992  20.064 < 0.0000000000000002 ***
mcz          0.43046    0.03817  11.277 < 0.0000000000000002 ***
mcx:mcz      0.07861    0.03005   2.616    0.00959 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6799 on 196 degrees of freedom
Multiple R-squared:  0.7546, Adjusted R-squared:  0.7508
F-statistic: 200.9 on 3 and 196 DF,  p-value: < 0.00000000000000022
```

Regression Analysis



Console

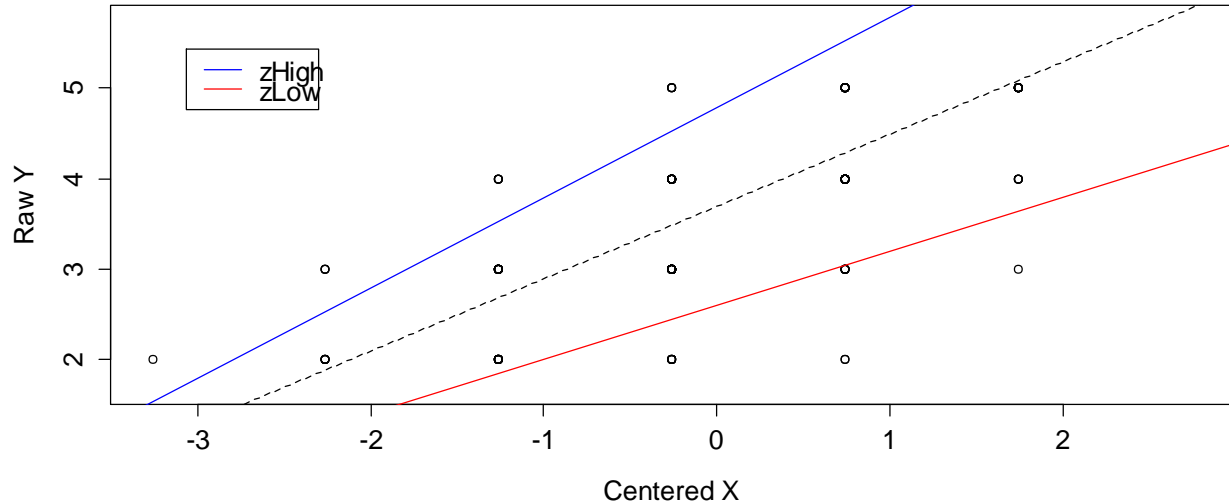
```
Call:
lm(formula = y ~ mcx * mcz, na.action = na.omit)

Residuals:
    Min       1Q   Median       3Q      Max
-1.68679 -0.39206  0.01796  0.42799  2.19791

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.12376    0.55564  -0.223  0.823970
mcx          0.46893    0.12958   3.619  0.000377 ***
mcz          0.09560    0.13146   0.727  0.467960
mcx:mcz      0.07861    0.03005   2.616  0.009592 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6799 on 196 degrees of freedom
Multiple R-squared:  0.7546, Adjusted R-squared:  0.7508
F-statistic: 200.9 on 3 and 196 DF, p-value: < 0.000000000000000022
```

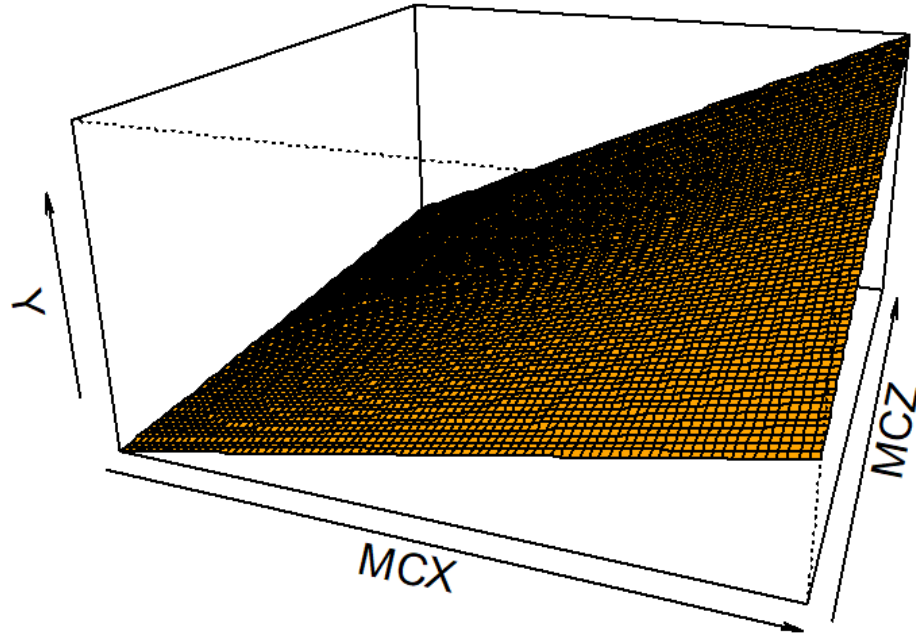
Regression Analysis



Regression Analysis



 Graphics



Regression Analysis



Console

```
> library(psych)
> MOD2<-mediate(Y ~ X*Z, data=Moderation)
> print(MOD2, digits=10)

Mediation/Moderation Analysis
Call: mediate(y = Y ~ X * Z, data = Moderation)

The DV (Y) was Y . The IV (X) was X Z X*Z . The mediating variable(s) = .
DV = Y
```

	slope	se	t	p
X	0.8010410	0.03992372	20.064290	2.107443e-49
Z	0.4304616	0.03817242	11.276767	4.698036e-23
X*Z	0.0786056	0.03004897	2.615917	9.591751e-03

```
with R2 = 0.754574
R = 0.8686622 R2 = 0.754574 F = 200.8704 on 3 and 196 DF p-value: 1.590401e-59
```


Regression Analysis



```
113 # Rockchalk
114 |
115 library(rockchalk)
116
117 R.rc<-lm(Y ~ X*Z, data=Moderation)
118 PS.rc<-plotslopes(R.rc, plotx="X", modx="Z", modxvals="std.dev")
119 TS.rc<-testslopes(PS.rc)
120 plot(TS.rc)
121
122
```

6. Advanced Topics: Moderation

Hayes (2013); Hayes and Matthes (2009); Spiller et al. (2013)

► Equations:

$$Y_i = a + b_1 * X_i + b_2 * Z_i + b_3 * Z_i * X_i + \text{ERROR}$$

$$Y_i = a + (b_1 + b_3 * Z) * X_i + b_2 * Z_i + \text{ERROR}$$

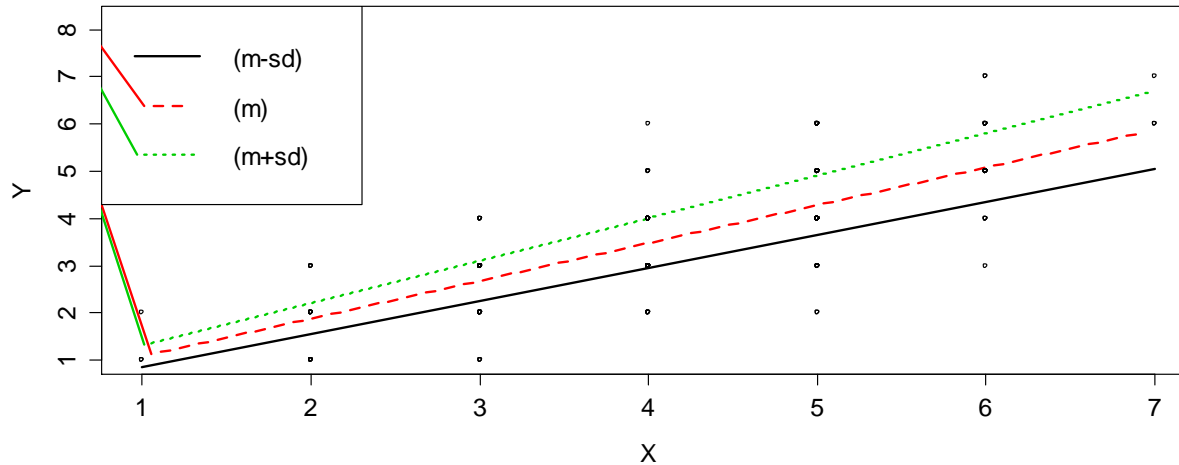
What happens if $b_3 = 0$?

$$Y_i = a + (b_1 + 0 * Z) * X_i + b_2 * Z_i + \text{ERROR}$$

$$Y_i = a + b_1 * X_i + b_2 * Z_i + \text{ERROR}$$

No Moderation!

Regression Analysis



Moderation

Hayes (2013); Hayes and Matthes (2009)

Z (M=4.23; SD=1.27)

Y.MEAN=-0.1238+0.4689*X+0.0956*4.23+0.0786*4.23*X

Y.MEAN=0.2806+0.8014*X

Y+1*SD=-0.1238+0.4689*X+0.0956*5.50+0.0786*5.50*X

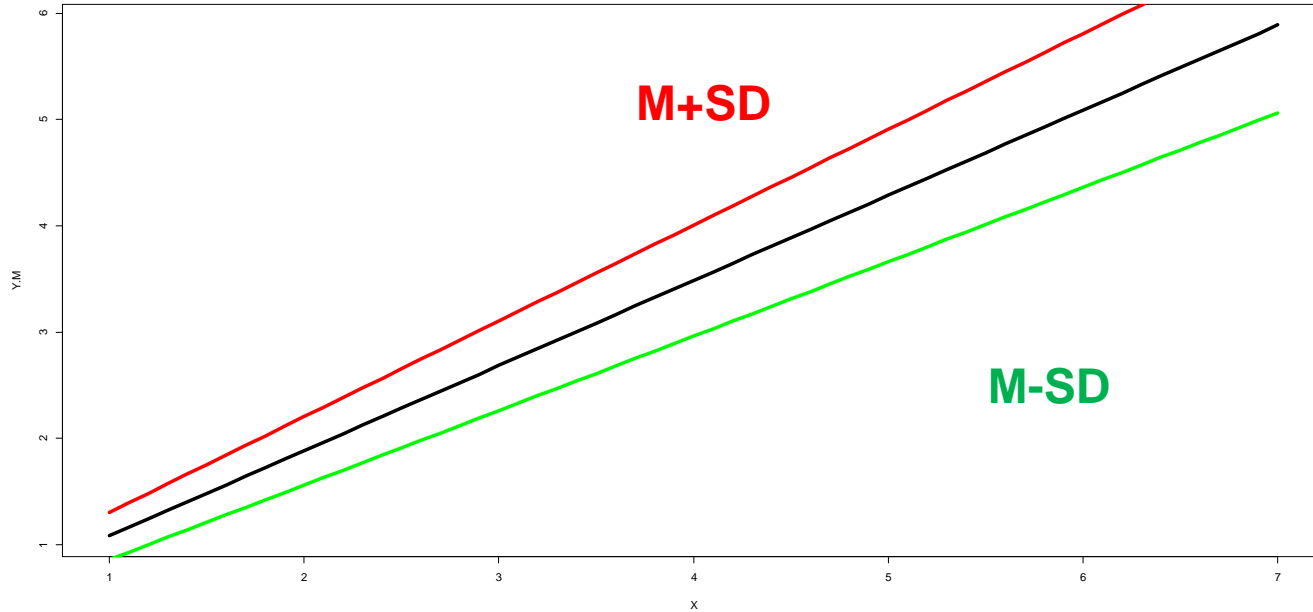
Y+1*SD=0.4020+0.9012*X

Y-1*SD=-0.1238+0.4689*X+0.0956*2.95+0.0786*2.95*X

Y-1*SD=0.1582+0.7008*X

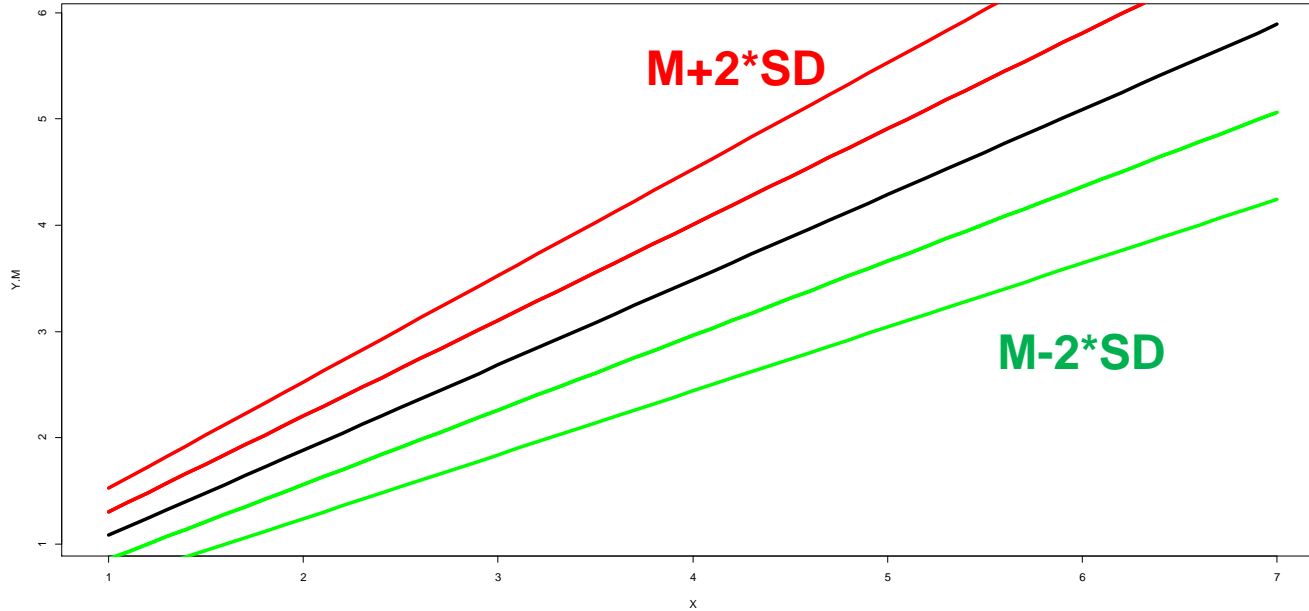
Moderation

Hayes (2013); Hayes and Matthes (2009)



Moderation

Hayes (2013); Hayes and Matthes (2009)

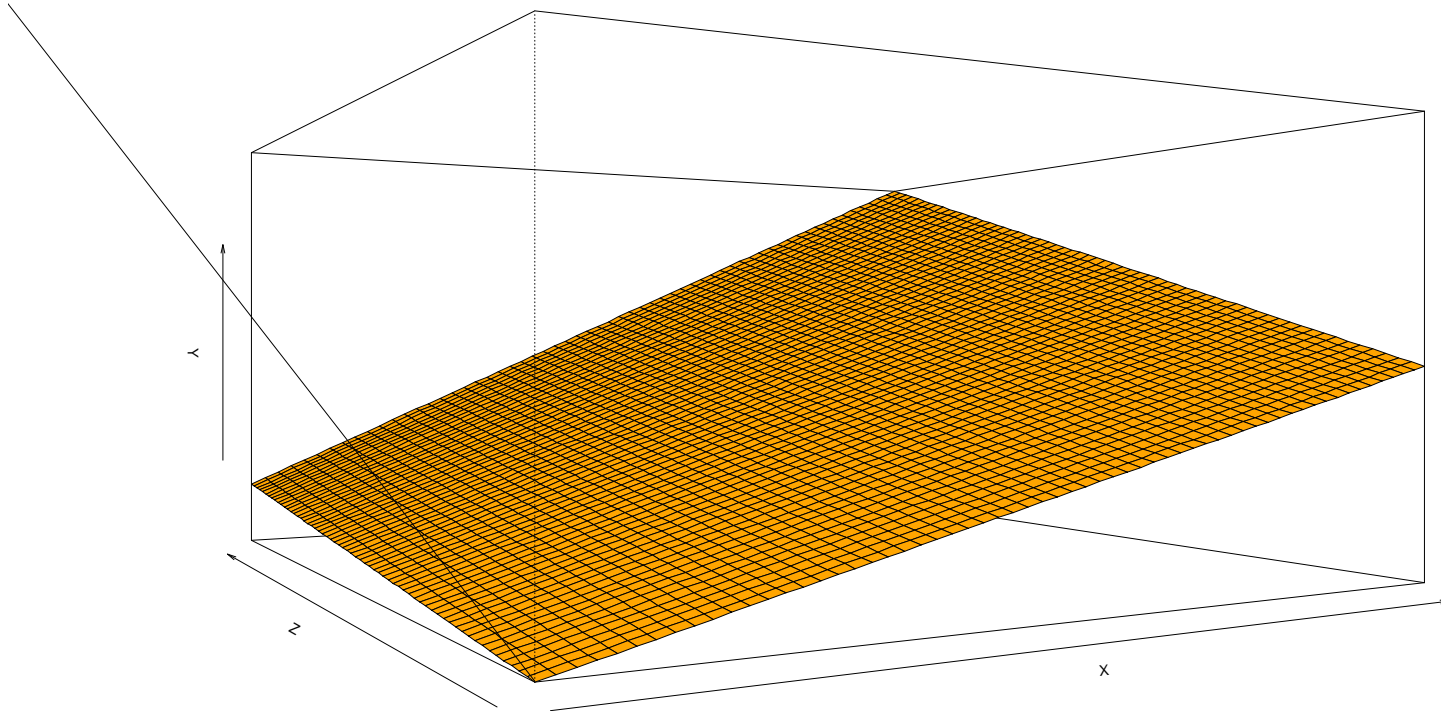


Moderation

Hayes (2013); Hayes and Matthes (2009)



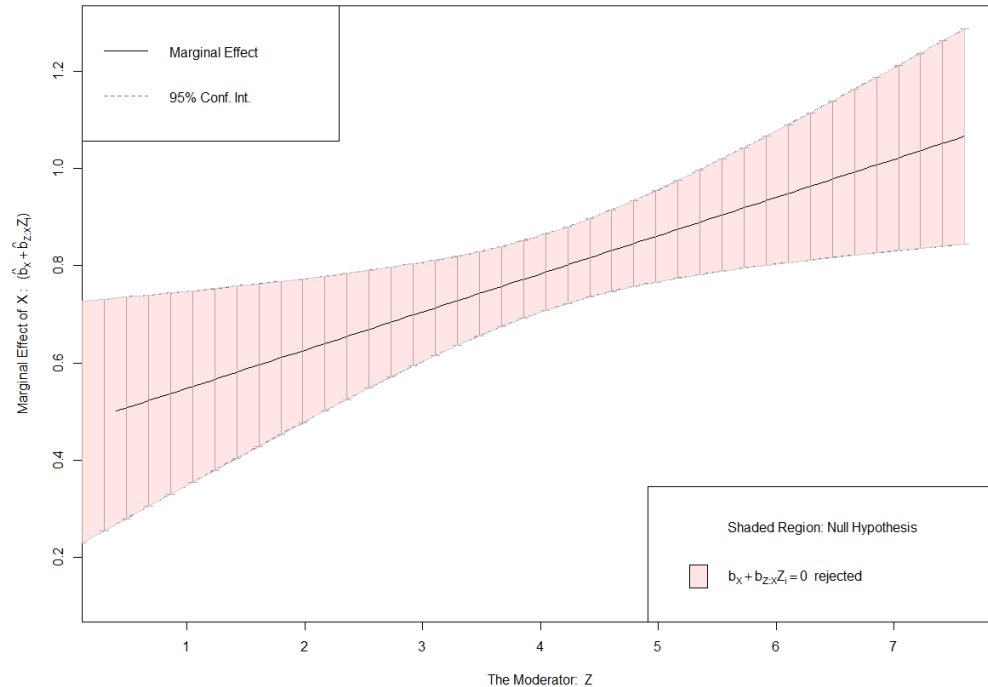
Surface Plot



Regression Analysis



R Graphics



Regression Analysis



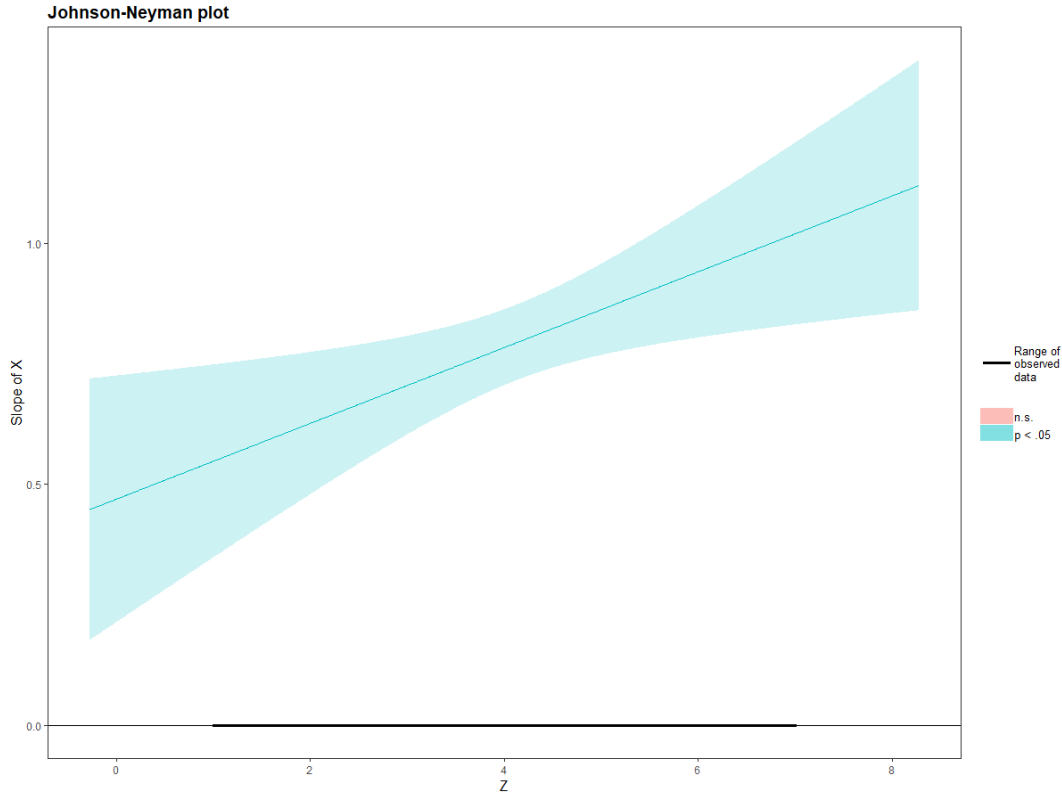
```
> # jtools  
>  
> library(jtools)  
>  
> MOD3<-lm(Y ~ X*Z, data=Moderation)  
>  
> johnson_neyman(model = MOD3, pred = X, modx = Z)
```

JOHNSON-NEYMAN INTERVAL

when Z is **OUTSIDE** the interval [-36.88, -1.57], the slope of X is $p < .05$.

Note: The range of observed values of Z is [1.00, 7.00]

Regression Analysis



G*Power

<http://www.gpower.hhu.de/>

G*Power: Statistical Power Analyses for Windows and Mac

G*Power is a tool to compute statistical power analyses for many different t tests, F tests, χ^2 tests, z tests and some exact tests. G*Power can also be used to compute effect sizes and to display graphically the results of power analyses.

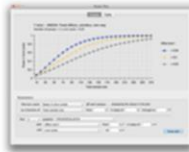
Screenshots (click to enlarge)



Main Window



Main Window (Table)



Power Plot

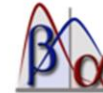


Power Plot (Table)

Kontrast



Suchbegriff

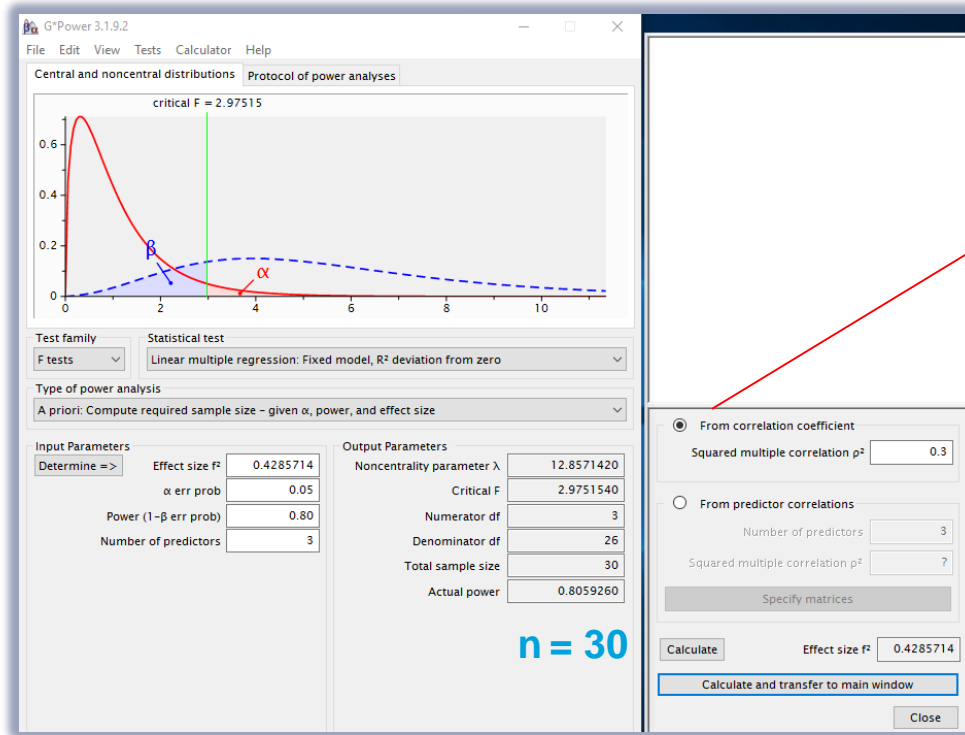


Register

Whenever we find a problem with G*Power we provide an update as quickly as we can. We will inform you about updates if you [click here](#) and add your e-mail address to our mailing list. We will only use your e-mail address to inform you about updates. We will not use your e-mail address for other purposes. We will not give your e-mail address to anyone else. You can withdraw your e-mail address from the mailing list at any time.

G*Power Sample Size Determination

<http://www.gpower.hhu.de/>

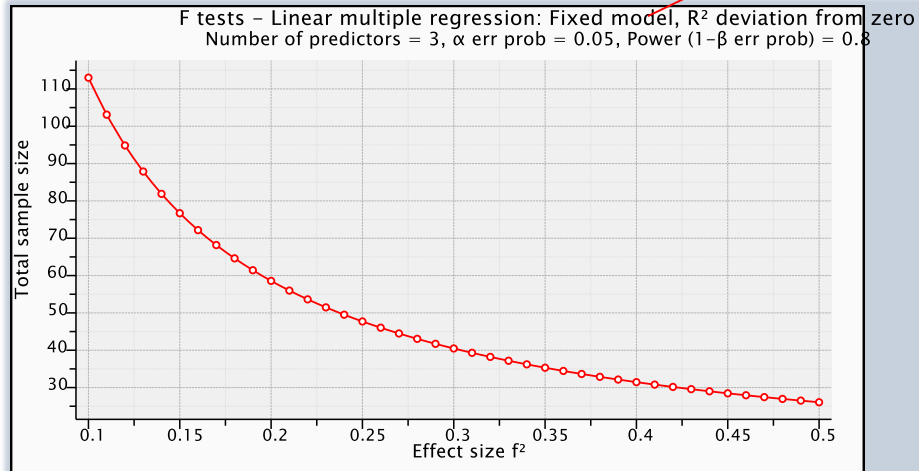


$R^2 = 0.30$
 $f^2 = 0.4286$
3 IVs

G*Power Sample Size Determination

<http://www.gpower.hhu.de/>

X-Y plot for
range of
values



IBM SPSS and R

```
* Encoding: UTF-8.  
* pwr  
  
BEGIN PROGRAM R.  
  
# pwr Regression  
  
install.packages("pwr", repos="http://cran.rstudio.com")  
  
library(pwr)  
  
# R2=0.30  
# u=3 (independent variables)  
# n=v+u+1  
  
pwr.f2.test(u=3, f2=0.30/(1-0.30), sig.level=0.05, power=0.80)  
  
END PROGRAM.
```

```
Multiple regression power calculation
```

```
u = 3  
v = 25.65333  
f2 = 0.4285714  
sig.level = 0.05  
power = 0.8
```

$$n = v + u + 1 = 26 + 3 + 1 = 30$$

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