Exercise problems of Topic 1

Write your answers clearly so that the answer proceeds logically and includes necessary intermediate steps and sufficient explanations. Your answer should be understandable without oral explanations, too. See further instructions for systematic problems solving in MyCourses.

The exercise problem answers are to be returned during the contact sessions to the course teachers either handwritten (on paper) or typescripted (shown on screen). For other return methods, contact the teachers.

Return your answers one by one when a teacher is free. You may also ask help and instruction.

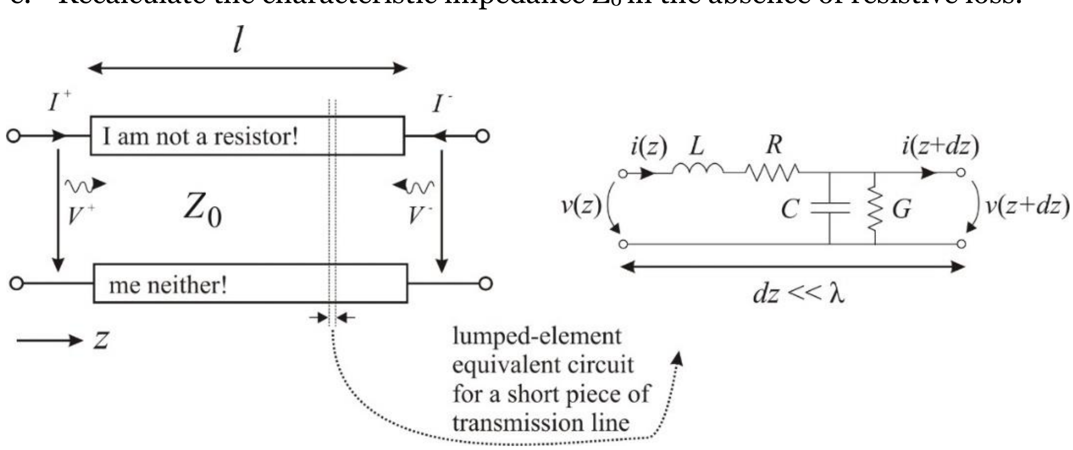
Be prepared to explain and justify your answer to the teacher. The purpose of this returning method is to enhance your learning through two-way communication and constructive feedback given by the teacher. The teacher will grade your answer in the scale of 0-3 points.

Note that at least two (2) of the problems must be returned latest on Thu 17 Jan and two (2) more latest on Thu 24 Jan. If you cannot meet this, you lose a chance to earn those points. However, if you have a good reason not the meet the DL, contact the teachers well in advance. The optimal return rate is about three (3) returned problems per week ☺

Exercise problem 1.1. Solve and answer the following small tasks.

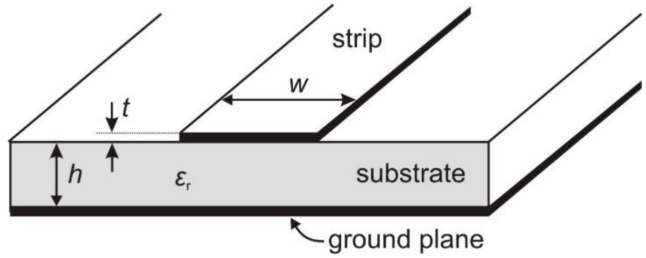
A low-loss transmission line (see the figure below) has the following per unit length equivalent circuit parameters: L = 0.75 µH/m, C = 300 pF/m, R = 1 Ω/m, G = 0.001 S/m at 5 GHz.

a. Why and when the *transmission line theory* is used instead of ordinary circuit theory?
b. Let us have the propagation constant $\gamma = \alpha + j\beta$. Write down the general form for the voltage $V(z)$ and current $I(z)$ waves along the transmission lines – i.e., the solutions of the "telegraph equations" are asked. Explain, what general forms of $V(z)$ and $I(z)$ physically mean.
c. Define briefly, what the "characteristic impedance" means. Determine the value of the characteristic impedance Z_0 of the transmission line.
d. Calculate the loss per unit length [dB/m].
e. Recalculate the characteristic impedance Z_0 in the absence of resistive loss.



Exercise problem 1.2. Solve and answer the following tasks.

A 50-Ω microstrip line will be implemented on an FR-4 substrate whose relative dielectric constant is ε_r = 4.3, the substrate thickness h = 1.5 mm and the loss tangent tan δ = 0.02. The thickness of the copper is marked t (e.g., 35 μm) and the copper conductivity $\sigma = 6{\cdot}10^7$ S/m. See the figure.



Think first, why microstrip lines are useful and important (you do not need to write an answer).

a. Sketch a figure of the cross section of a microstrip line on your answer sheet (see an example below). A wave propagates in the sketch perpendicular to the paper and away from the reader. Sketch into your figure the shape of the electric and magnetic near fields of the microstrip line structure when the voltage is applied between the strip and the ground plane. Especially, pay attention to the "fringing" fields.

b. Based on the part a., answer with justifications why the effective relative permittivity $\varepsilon_{r,eff}$ satisfy the inequality $1 < \varepsilon_{r,eff} < \varepsilon_r$?

c. Calculate the effective permittivity $\varepsilon_{r,eff}$, the width w of the 50-Ω strip, and the wavelength in the line **at 1 GHz**. You can assume that the thickness of the strip is t = 0 μm.

Exercise problem 1.3. Solve and answer the following problems. Write all the intermediate phases and good explanations to your answers.

Open AWR Design Environment circuit simulator (you can ask a computer and help from the teachers!). Open a transmission line calculator tool: "Tools" → "TXLine...".

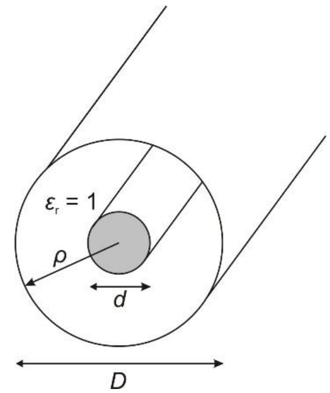
a. Check your answers ($\varepsilon_{r,eff}$, w and λ at 1 GHz) of part c. of Problem 1.2 with the circuit simulator. Use the values as given in the Problem 1.2 but set the metal thickness t = 35 μm. Does the simulator give the same results as you calculated in Problem 1.2? If not, explain why.

b. Simulate the attenuation [dB/m] of the same 50-Ω microstrip line at the following frequencies: 0.1, 0.2, 0.5, 1, 2, 5, and 10 GHz. Use w as in a. part, ε_r = 4.3, h = 1.5 mm, tan δ = 0.02, metal thickness t = 35 μm and its conductivity $\sigma = 6{\cdot}10^7$ S/m (pure copper).

c. Where do the losses come from? Based on the part b., how do the attenuation change as a function of frequency? Can you find any explanation for that?

Exercise problem 1.4. Solve and answer the following tasks.

In this problem you will learn, why 50 Ω is typically used as the characteristic impedance of transmission lines (some systems like TV receiver use 75 Ω - you will also learn, why).



Let us consider an air-filled coaxial cable (ε_r = 1) with the diameters of the inner and outer conductors d and D, respectively. ρ is the polar coordinate of the cylinder. See the figure.

a. The maximum power handling capacity P_{max} of the coaxial cable is limited by the electric field breakdown of the line. The breakdown the electric field of the air-filled coaxial cable takes place when $E_{max} = E(\rho{=}0.5{\cdot}d) = 3{\cdot}10^6$ V/m. Show that the maximum power handling capacity P_{max} of the coaxial cable can be calculated from the formula

$$P_{\max} = \frac{\pi E_{\max}^2}{4\eta} \cdot d^2 \ln\frac{D}{d}.$$

Find the ratio of D/d that maximizes P_{max} and calculate the corresponding characteristic impedance Z_0 of the coaxial cable.

Hints: The maximum power P_{max}, the electric field strength $E(\rho)$, and the characteristic impedance Z_0 of a coaxial cable can be calculated as

$$P_{\max} = \frac{U_{\max}^{\;2}}{2Z_0},\ E(\rho) = \frac{U}{\rho \ln(D/d)},\ and\ Z_0 = \frac{\eta}{2\pi}\ln\left(\frac{D}{d}\right)$$

where U is the voltage between the inner and outer conductors and η is the wave impedance.

b. Next, show that the attenuation of the coaxial cable can calculated from the formula

$$\alpha_c = \frac{R_s}{2\eta} \cdot \frac{D+d}{Dd \ln\left(\frac{D}{d}\right)},$$

in which R_s is the surface loss resistance. Find the ratio of D/d that minimizes α_c and calculate the corresponding characteristic impedance Z_0 of the coaxial cable.

Hint: The attenuation constant α_c of the coaxial cable is derived in Pozar Chapter 2.7 and it can be calculated as

$$\alpha_c = \frac{R_s}{Z_0}\frac{D+d}{4\pi Dd},$$

where R_s is the surface loss resistance (independent of d and D).

Conclude your answer. Why some receive-only systems (such as TV receiver) use 75 Ω as the characteristic impedance? Why typically 50 Ω is used?

Exercise problem 1.5. Solve and answer the following tasks.

Let us consider two cascaded transmission lines with the characteristic impedances Z_1 = 50 Ω and Z_2 = 150 Ω. Let there be a forward (positive z direction) travelling wave in line Z_1 with the amplitude of V^+ = 1 V. See the figure below ("+" = forward = positive z direction, "-" = reverse = negative z direction).

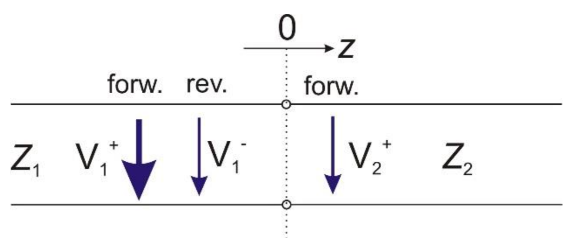
a. Explain with justification, what happens when the forward travelling wave V_1^+ reaches the interface of the two transmission lines (in the location z = 0).

b. Calculate the voltage reflection coefficient $\rho = V_1^-/V_1^+$ between the two transmission lines.

c. What is the total voltage $V_{tot} = V_1^+ + V_1^-$ in the location z = 0? How about the voltage V_2^+?

d. As you noticed (?) in part c., $V_2^+ > V_1^+$, how is this possible? Or is it possible at all?

Hint: Calculate the forward powers P_1^+ and P_2^+?



Exercise problem 1.6. Solve and answer the following tasks.

A generator is connected to a lossless transmission line as shown below. Z_0 = 50 Ω, Z_L = 20 – j 50 Ω, l = 1.5 λ, and U = 10 V (peak value).

a. Explain with justifications, why the amplitude V^+ of the forward (positive z direction) travelling wave is 5 V.

b. Find ρ_L in the location z = 0 and write the voltage $V(z)$ and current $I(z)$ functions along the line as a function of z ($-l \le z \le 0$). Assume a lossless line: $\gamma = j\beta$.

c. Plot (e.g., with Matlab) the magnitudes (absolute value) of the voltage and current functions for $-l \le z \le 0$ in the same figure. For better readability, use a different scale for the voltage and the current.