ELEC-C1310 Automaatio- ja systeemitekniikan laboratoriotyöt

Control of an inverted pendulum

Instructor:

- **Hoang Nguyen (**hoang.kh.nguyen@aalto.fi**)**

Room for pre-hearing and the lab work: TUAS building, 3rd floor, room 3548.

The workbook "**Rotary Pendulum Workbook (Student).pdf**" is used for this lab.

Numerical values of parameters can be found in:

- "**Rotary Servo Base Unit – User Manual.pdf**"
- **"Rotary Inverted Pendulum - User Manual.pdf"**

Should you have any problems or questions, please ask the instructor!

This lab will cover chapters 1-4. No lab report is required!

IMPORTANT: the pre-hearing answers must be submitted to the instructor at least ONE day before the pre-hearing session. Only one copy is needed from each group.

Pre-hearing

Carefully read **section 2.1, section 3.1-3.2** and answer these following questions:

1. The nonlinear equations of motion for the SRV02 rotary inverted pendulum are given in equation **2.2** and equation **2.3**. Linearize these equations by the classical linearization method and by the well-known approximation of sine and cosine.
   The initial conditions for all variables are set to zero.

2. Find the linear state-space representation of the system when the input *u* is the torque applied at the servo load gear. (matrices C and D have been given)

3. What is the characteristic equation of this system?

4. What are the open-loop poles of the system? What can be said of the system based on the poles you just found?

5. Recall that the input of the state-space model you just found is the torque applied at the servo load gear. However, in the lab session, we control the servo input voltage instead.
   By using the voltage-torque relationship in Equation **2.4**, derive a new state-space model with the voltage to be the input. **Notice:** we assume that the gear's and the motor's efficiencies (η_g, η_m) are 1.
   In Matlab/Octave, write a script to find the open-loop poles of the new state-space model.

6. From this point, **only the state-space model in terms of servo input voltage** is considered.
    - What is the characteristic polynomial of A?
    - Compute the corresponding companion matrices ($\tilde{A}, \tilde{B}$) of A and B.

7. In Figure 3.1, this system has four closed-loop poles and two of them (*p1* and *p2*) are complex conjugate dominant poles. Find the location of these two dominant poles so that they satisfy the specifications given in **Section 3.1.**
   Assume that $p_3 = -30$ and $p_4 = -40$, find the desired characteristic equation.

8. Calculate companion gain $\widetilde{K}$ that assigns the poles to the locations found in question 7. Convert the companion gain $\widetilde{K}$ into controller gain K.
   (Hint: read **Section 3.2.4**)
   Notice:
    - Typo in equation 3.1: $A^n \rightarrow A^{n-1}$
    - Typo in the formula of $\tilde{T}$: $\tilde{A}^n \rightarrow \tilde{A}^{n-1}$